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Global Stability of a Three-Species System with Attractive Prey-Taxis

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Abstract

This paper reports the global asymptotic stability of a three-species predator-prey system involving the prey-taxis. With the assumptions, we establish the global asymptotic stability results of its equilibria, respectively. Our results illustrate that 1) the global asymptotic stability of the semi-trivial equilibrium does not involve the prey-taxis coefficients χ,ξ; 2) the global asymptotic stability of two boundary equilibria relies on a single prey-taxis coefficient χ and ξ, respectively; 3) the global asymptotic stability of the unique positive equilibrium depends on two prey-taxis coefficients χ and ξ.

Keywords

Predator-Prey Model, Global Asymptotic Stability, Prey-Taxis, Lyapunov Function

1. Introduction

In the past few decades, predator-prey systems involving the prey-taxis have attracted more and more scholars to investigate them. Chen *et al.* [1] reported stationary patterns of a predator-prey model with prey-taxis and investigated the stability of the nonconstant steady states by employing the Crandall-Rabinowitz bifurcation theory. Tu *et al.* [2] considered the asymptotic behaviors of a parabolic-elliptic chemotaxis system with competitive kinetics and loop of a predator-prey model. Bell and Haskell [3] established the global existence of positive classical solutions and the existence of nontrivial steady states via the bifurcation theory of a predator-prey system. The global existence and uniform boundedness of solutions to a predator-prey system with prey-taxis for general functional responses in any spatial dimensions have been investigated by Ahn and Yoon [4]. The existence of the unique global bounded classical solution is proven, and the steady-state bifurcation, the Hopf bifurcation, and Hopf/steady-state mode interaction are studied via the Lyapunov-Schmidt procedure by Qiu *et al.* [5]. We recommend more existing results about the predator-prey systems with directed prey-taxis, see Refs. [6] [7] [8] [9] [10], etc.

In this present paper, we focus on a predator-prey model with two predators and one prey as well as the prey-taxis as follows.

$$\begin{cases} \partial_t u = \partial_{xx} u - \chi \partial_x \cdot \left(\dfrac{u}{(1+\delta w)^2} \partial_x w \right) + \dfrac{\beta_1 uw}{\alpha_1 + w + su} - \dfrac{\rho_1 uw}{\alpha_1 + w + su} - \delta_1 u, & x \in \Omega,\ t > 0, \\ \partial_t v = \partial_{xx} v - \xi \partial_x \cdot \left(\dfrac{v}{(1+\delta w)^2} \partial_x w \right) + \dfrac{\beta_2 vw}{\alpha_2 + w} - \dfrac{\rho_2 vw}{\alpha_2 + w} - \delta_2 v, & x \in \Omega,\ t > 0, \\ \partial_t w = d \partial_{xx} w + rw\left(1 - \dfrac{w}{K}\right) - \dfrac{\mu_1 uw}{\alpha_1 + w + su} - \dfrac{\mu_2 vw}{\alpha_2 + w}, & x \in \Omega,\ t > 0, \\ \partial_\nu u = \partial_\nu v = \partial_\nu w = 0, & x \in \partial\Omega,\ t \geq 0, \\ u(x,0) = u_0(x) \geq 0,\ v(x,0) = v_0(x) \geq 0,\ w(x,0) = w_0(x) \geq 0, & x \in \Omega, \end{cases} \tag{1}$$

where $u = u(x,t), v = v(x,t)$ and $w = w(x,t)$ are predator and prey densities at position x and time t, respectively. $\Omega \subset \mathbb{R}^N$ is a bounded domain with its smooth boundary $\partial\Omega$; constant d describes the diffusive rate of prey; For $j = 1, 2$, β_j are the ratios of biomass conversion of predators species; ρ_j represent the rates of toxic substances produced by per unit biomass about predators due to prey species are toxic corresponding to the predators; δ_j are the natural mortality of the predators u and v; α_j describe the half-saturation constant of the predators; s describes the measure of mutual interference among the predator u; two constants r and K in the third equation are the intrinsic growth rate and the maximum environmental capacity of prey species, respectively. Moreover, $-\chi \partial_x \cdot \left(\dfrac{u}{(1+\delta w)^2} \partial_x w \right)$ and $-\xi \partial_x \cdot \left(\dfrac{v}{(1+\delta w)^2} \partial_x w \right)$ are prey-taxis terms. They imply the tendency of predators moving toward the positive direction of the increasing gradient of prey population as $\chi > 0$ and $\xi > 0$. If $\chi < 0$ and $\xi < 0$, we say that predators move toward the opposite direction of the increasing gradient of prey population to avoid group defense by a large number of prey species or volume-filling effect in predator species [11]. Consequently, $\chi, \xi > 0$ and $\chi, \xi < 0$ corresponding to attractive and repulsive prey-taxis, respectively. Moreover, $\dfrac{u}{(1+\delta w)^2}$ and $\dfrac{v}{(1+\delta w)^2}$ represent the distribution variations of the directed species dispersals [12]. Obviously, they depend on the density of the prey population. All parameters exhibited in the system (1) are set to be positive.

For system (1), define

$$f(u,v,w) = \frac{\beta_1 uw}{\alpha_1 + w + su} - \frac{\rho_1 uw}{\alpha_1 + w + su} - \delta_1 u,$$

$$g(u,v,w)=\frac{\beta_2 vw}{\alpha_2+w}-\frac{\rho_2 vw}{\alpha_2+w}-\delta_2 v,$$

and

$$h(u,v,w)=rw\left(1-\frac{w}{K}\right)-\frac{\mu_1 uw}{\alpha_1+w+su}-\frac{\mu_2 vw}{\alpha_2+w},$$

as well as some assumptions

(H1) $\beta_1-\rho_1-\delta_1>0,\ \hat{w}>\frac{\alpha_1\delta_1}{\beta_1-\rho_1-\delta_1}$ with $\hat{w}=\frac{-\gamma_2+\sqrt{\gamma_2^2-4\gamma_1\gamma_3}}{2\gamma_1}$, $\gamma_1=rs(\beta_1-\rho_1),\gamma_2=\mu_1(\beta_1-\rho_1-\delta_1)K-rs(\beta_1-\rho_1)K,\gamma_3=-\mu_1\alpha_1\delta_1K.$

(H2) $\beta_2-\rho_2-\delta_2>0,(\beta_2-\rho_2)K-(\alpha_2+K)\delta_2>0.$

(H3) $\beta_2-\rho_2-\delta_2>0,\alpha_2\delta_2(\beta_1-\rho_1-\delta_1)-\alpha_1\delta_1(\beta_2-\rho_2-\delta_2)>0,$ $(r-\mu_1)(\beta_2-\rho_2-\delta_2)K-r\alpha_2\delta_2>0.$

As a result, we can conclude the classifications of the equilibria of system (1). 1) system (1) has a trivial equilibrium $E_0=(0,0,0)$ and a semi-trivial equilibrium $E_1=(0,0,K)$; 2) if (H1) holds, system (1) has a boundary equilibrium $E_2=(\hat{u},0,\hat{w})$, where

$$\hat{w}=\frac{-\gamma_2+\sqrt{\gamma_2^2-4\gamma_1\gamma_3}}{2\gamma_1},\ \hat{u}=\frac{(\beta_1-\rho_1-\delta_1)\hat{w}-\alpha_1\delta_1}{s\delta_1};$$

3) if (H2) is valid, system (1) has a boundary equilibrium $E_3=(0,\tilde{v},\tilde{w})$, where

$$\tilde{w}=\frac{\alpha_2\delta_2}{\beta_2-\rho_2-\delta_2},\ \tilde{v}=\frac{r\alpha_2(\beta_2-\rho_2)\left[(\beta_2-\rho_2)K-(\alpha_2+K)\delta_2\right]}{\mu_2(\beta_2-\rho_2-\delta_2)K};$$

4) if (H3) is satisfied, system (1) has a unique positive equilibrium $E_*=(u^*,v^*,w^*)$, where

$$u^*=\frac{\alpha_2\delta_2(\beta_1-\rho_1-\delta_1)-\alpha_1\delta_1(\beta_2-\rho_2-\delta_2)}{s\delta_1(\beta_2-\rho_2-\delta_2)},$$

$$v^*=\frac{(\alpha_1+w^*)\gamma_4}{\delta_2\mu_2\alpha_2K(\beta_1-\rho_1)(\beta_2-\rho_2-\delta_2)},$$

and

$$w^*=\frac{\alpha_2\delta_2}{\beta_2-\rho_2-\delta_2},$$

with

$$\gamma_4=\alpha_2\delta_2(\beta_1-\rho_1)\left[(r-\mu_1)(\beta_2-\rho_2-\delta_2)K-r\alpha_2\delta_2\right]+\mu_1\delta_1K(\beta_2-\rho_2-\delta_2)\left[\alpha_2\delta_2+\alpha_1(\beta_2-\rho_2-\delta_2)\right].$$

In this present paper, we will establish the global asymptotic stabilities of semi-trivial equilibrium $E_1=(0,0,K)$, boundary equilibria $E_2=(\hat{u},0,\hat{w}),E_3=(0,\tilde{v},\tilde{w})$ and the unique positive equilibrium $E_*=(u^*,v^*,w^*)$ by constructing some suitable time evolution Lyapunov functions, respectively.

This paper is structured as follows. In Section 2, we perform the main results of the present paper. In Section 3, the local-in-time existence of the classical solution of the model is given. In Section 4, the proofs of the main results are displayed. Finally, some conclusions are made in Section 5.

2. Main Results

Theorem 2.1 *Let* $\Omega \subset \mathbb{R}^N$ *be a bounded domain with the smooth boundary* $\partial\Omega$. *Suppose* (u,v,w) *is a classical solution of system* (1) *with the initial conditions* $(u_0(x), v_0(x), w_0(x)) \in \left[W^{1,p}(\Omega)\right]^3$ *and* $u_0(x) \ge 0, v_0(x), w_0(x) \ge 0$ *for* $x \in \bar{\Omega}$. *We have the following global asymptotic stability results.*

1) *For any* $\chi, \xi > 0$ *and*

$$0 < \beta_1 \le \rho_1 + \mu_1,\ 0 < \beta_2 \le \rho_2 + \mu_2,\ 0 < K \le \min\left\{\frac{\alpha_1\delta_1}{\mu_1}, \frac{\alpha_2\delta_2}{\mu_2}\right\}, \quad (2)$$

then $E_1 = (0,0,K)$ *is globally asymptotically stable.*

2) *If the condition* $(H1)$ *holds and*

$$\beta_2 \le \mu_2 \le \frac{\delta_2\alpha_2}{\hat{w}},\ \mu_1 = \frac{(\beta_1 - \rho_1)(\alpha_1 + s\hat{u})}{\alpha_1 + \hat{w}},\ 0 < K \le \frac{r\alpha_1^2}{\mu_1\hat{u}},\ 0 < \chi^2 \le \frac{4d\hat{w}}{C^2\hat{u}}, \quad (3)$$

then boundary equilibrium $E_2 = (\hat{u}, 0, \hat{w})$ *is globally asymptotically stable for any* $\xi > 0$.

3) *If the condition* $(H2)$ *is valid and*

$$\beta_1 \le \mu_1 \le \frac{\delta_1\alpha_1}{\tilde{w}},\ \mu_2 = \frac{\alpha_2(\beta_2 - \rho_2)}{\alpha_2 + \tilde{w}},\ 0 < K \le \frac{r\alpha_2^2}{\mu_2\tilde{v}},\ 0 < \xi^2 \le \frac{4d\tilde{w}}{C^2\tilde{v}}, \quad (4)$$

then $E_3 = (0, \tilde{v}, \tilde{w})$ *is globally asymptotically stable for any* $\chi > 0$.

4) *If the condition* $(H3)$ *holds and*

$$\mu_1 = \frac{(\beta_1 - \rho_1)(\alpha_1 + su^*)}{\alpha_1 + w^*},\ \mu_2 = \frac{\alpha_2(\beta_2 - \rho_2)}{\alpha_2 + w^*},\ 0 < K \le \frac{r\alpha_1^2}{\mu_1 u^*} + \frac{r\alpha_2^2}{\mu_2 v^*}, \quad (5)$$

as well as

$$0 < \chi^2 + \xi^2 \le \frac{4dw^*}{\max\{u^*, v^*\}C^2}, \quad (6)$$

then $E_* = (u^*, v^*, w^*)$ *is globally asymptotically stable, where* $C = \max\left\{\|w_0(x)\|_{L^\infty(\Omega)}, K\right\}$.

Remark 2.1 *From Theorem* 2.1, *we can find that the global asymptotic stability of the semi-trivial equilibrium* $E_1 = (0,0,K)$ *does not involve the prey-taxis coefficients* χ *and* ξ. *The global asymptotic stabilities of the boundary equilibria* $E_2 = (\hat{u}, 0, \hat{w})$ *and* $E_3 = (0, \tilde{v}, \tilde{w})$ *only depend on prey-taxis coefficient* χ *and* ξ, *respectively. However, the global asymptotic stability of the unique positive equilibrium* $E_* = (u^*, v^*, w^*)$ *depends on prey-taxis coefficients* χ *and* ξ.

Remark 2.2 *The control conditions* (3), (4) *and* (6) *of the global asymptotic* *stabilities of the equilibria only involve the initial spatial density* $w_0(x)$ *of prey species but are independent of the initial spatial densities* $u_0(x)$ *and* $v_0(x)$ *of the predators.*

The following conclusion is helpful to obtain the desired results.

3. Existence

Lemma 1 *Suppose that* $\Omega \subset \mathbb{R}^N$ *with the smooth boundary* $\partial\Omega$. *For any initial conditions* $(u_0(x), v_0(x), w_0(x)) \in [W^{1,p}(\Omega)]^3$ *satisfies* $u_0(x) \ge 0, v_0(x), w_0(x) \ge 0$ *for* $x \in \bar{\Omega}$. *Then there is a maximal existence time* $T_{\max} > 0$ *such that system* (1) *has a unique local non-negative classical solution* $(u(x,t), v(x,t), w(x,t)) \in \left[C([0,T_{\max}); W^{1,p}(\Omega)) \cap C^{2,1}(\bar{\Omega} \times (0,T_{\max}))\right]^3$. *Moreover, we have* $u(x,t), v(x,t) > 0, w(x,t) \le C$ *for* $x \in \bar{\Omega}$, $t \in [0, T_{\max})$, *where* $C = \max\left\{\|w_0(x)\|_{L^\infty(\Omega)}, K\right\}$.

Proof. Denote by $\psi(x,t) = (u(x,t), v(x,t), w(x,t))$. Then system (1) takes the form

$$\begin{cases} \dfrac{\partial \psi}{\partial t} = \partial_x \cdot (z(\psi)\partial_x \psi) + \Psi(\psi), & x \in \Omega,\ t > 0, \\ \dfrac{\partial \psi}{\partial \nu} = 0, & x \in \partial\Omega,\ t \ge 0, \\ \psi(\cdot, 0) = (u_0(x), v_0(x), w_0(x)), & x \in \Omega. \end{cases}$$

where

$$z(\psi) = \begin{pmatrix} 1 & 0 & -\chi \dfrac{u}{(1+\delta w)^2} \\ 0 & 1 & -\xi \dfrac{u}{(1+\delta w)^2} \\ 0 & 0 & d \end{pmatrix},\quad \Psi(\psi) = \begin{pmatrix} \dfrac{\beta_1 uw}{\alpha_1 + w + su} - \dfrac{\rho_1 uw}{\alpha_1 + w + su} - \delta_1 u \\ \dfrac{\beta_2 vw}{\alpha_2 + w} - \dfrac{\rho_2 vw}{\alpha_2 + w} - \delta_2 v \\ rw\left(1 - \dfrac{w}{K}\right) - \dfrac{\mu_1 uw}{\alpha_1 + w + su} - \dfrac{\mu_2 vw}{\alpha_2 + w} \end{pmatrix}.$$

Obviously, $z(\psi)$ is an upper-triangular matrix and is positive definite since $d > 0$ is valid. Therefore, the local existence can be checked by Amman's fixed point argument [13]. Now rewrite the first equation of system (1) as follows.

$$\begin{cases} \partial_t u = \partial_{xx} u - \chi \dfrac{\partial_x u \cdot \partial_x w}{(1+\delta w)^2} + \dfrac{2\chi\delta u}{(1+\delta w)^3} \partial_{xx} w + u f_1(u,v,w), & x \in \Omega,\ t \in (0, T_{\max}), \\ \partial_\nu u = 0, & x \in \partial\Omega,\ t \in (0, T_{\max}), \\ u(x,0) = u_0(x) \ge 0, & x \in \Omega, \end{cases} \tag{7}$$

where $f_1(u,v,w) = \dfrac{\beta_1 w}{\alpha_1 + w + su} - \dfrac{\rho_1 w}{\alpha_1 + w + su} - \delta_1$. Obviously, 0 is a lower solution of (7). Therefore, the maximum principle shows $u(x,t) \ge 0$ for all $(x,t) \in \Omega \times (0, T_{\max})$. Combine $u_0(x) \ge 0 (\not\equiv 0)$ with the strong maximum principle, $u(x,t) > 0$ is valid. By the same way, we have $v(x,t), w(x,t) > 0$ for all $(x,t) \in \Omega \times (0, T_{\max})$. Finally, the maximum principle ensures that $w(x,t) \le C$

for $(x,t)\in\Omega\times(0,T_{\max})$. This ends the proof.

In the sequel, we shall give proof of Theorem 2.1 by constructing some suitable time evolution Lyapunov functions.

4. Proof of Theorem 2.1

1) For $E_1=(0,0,K)$, define the following Lyapunov function

$$V_1(t)=\int_\Omega u(\cdot,t)\mathrm{d}x+\int_\Omega v(\cdot,t)\mathrm{d}x+\int_\Omega\int_K^w\frac{w(\cdot,t)-K}{w(\cdot,t)}\mathrm{d}w\mathrm{d}x. \tag{8}$$

Then we deduce

$$\begin{aligned}\dot{V}_1(t)=&\int_\Omega\left(\frac{\beta_1uw}{\alpha_1+w+su}-\frac{\rho_1uw}{\alpha_1+w+su}-\delta_1u\right)\mathrm{d}x\\&+\int_\Omega\left(\frac{\beta_2vw}{\alpha_2+w}-\frac{\rho_2vw}{\alpha_2+w}-\delta_2v\right)\mathrm{d}x-\int_\Omega\frac{dK|\partial_xw|^2}{w^2}\mathrm{d}x\\&+\int_\Omega(w-K)\left[r\left(1-\frac{w}{K}\right)-\frac{\mu_1u}{\alpha_1+w+su}-\frac{\mu_2v}{\alpha_2+w}\right]\\\le&\int_\Omega\left(\frac{(\beta_1-\rho_1-\mu_1)uw}{\alpha_1+w+su}-\delta_1u\right)\mathrm{d}x+\int_\Omega\frac{\mu_1Ku}{\alpha_1+w+su}\mathrm{d}x\\&+\int_\Omega\frac{\mu_2Kv}{\alpha_2+w}\mathrm{d}x+\int_\Omega\left(\frac{(\beta_2-\rho_2-\mu_2)vw}{\alpha_2+w}-\delta_2v\right)\mathrm{d}x\\&-\int_\Omega\frac{dK|\partial_xw|^2}{w^2}\mathrm{d}x+\int_\Omega r(w-K)\left(1-\frac{w}{K}\right)\mathrm{d}x\\\le&\int_\Omega\frac{(\beta_1-\rho_1-\mu_1)uw}{\alpha_1+w+su}\mathrm{d}x+\int_\Omega\left(\frac{\mu_1K}{\alpha_1}-\delta_1\right)u\mathrm{d}x+\int_\Omega\frac{(\beta_2-\rho_2-\mu_2)vw}{\alpha_2+w}\mathrm{d}x\\&+\int_\Omega\left(\frac{\mu_2K}{\alpha_2}-\delta_2\right)v\mathrm{d}x-\int_\Omega\frac{dK|\partial_xw|^2}{w^2}\mathrm{d}x-\frac{r}{K}\int_\Omega(w-K)^2\,\mathrm{d}x.\end{aligned}$$

Consequently, $\dot{V}_1(t)\le0$ and $E_1=(0,0,K)$ is globally asymptotically stable if (2) holds.

2) Define

$$V_2(t)=\int_\Omega\int_{\hat{u}}^u\frac{u(\cdot,t)-\hat{u}}{u(\cdot,t)}\mathrm{d}u\mathrm{d}x+\int_\Omega v(\cdot,t)\mathrm{d}x+\int_\Omega\int_{\hat{w}}^w\frac{w(\cdot,t)-\hat{w}}{w(\cdot,t)}\mathrm{d}w\mathrm{d}x. \tag{9}$$

As a result, one deduces

$$\begin{aligned}\dot{V}_2(t)=&\int_\Omega\left(1-\frac{\hat{u}}{u}\right)\partial_tu\mathrm{d}x+\int_\Omega\partial_tv\mathrm{d}x+\int_\Omega\left(1-\frac{\hat{w}}{w}\right)\partial_tw\mathrm{d}x\\=&\int_\Omega(u-\hat{u})\left(\frac{\beta_1w}{\alpha_1+w+su}-\frac{\rho_1w}{\alpha_1+w+su}-\delta_1\right)\mathrm{d}x-\int_\Omega\frac{\hat{u}|\partial_xu|^2}{u^2}\mathrm{d}x\\&+\int_\Omega\frac{\chi\hat{u}\partial_xw\cdot\partial_xu}{u(1+\delta w)^2}\mathrm{d}x+\int_\Omega\left(\frac{\beta_2vw}{\alpha_2+w}-\frac{\rho_2vw}{\alpha_2+w}-\delta_2v\right)\mathrm{d}x\\&-\int_\Omega\frac{d\hat{w}|\partial_xw|^2}{w^2}\mathrm{d}x+\int_\Omega(w-\hat{w})\left[r\left(1-\frac{w}{K}\right)-\frac{\mu_1u}{\alpha_1+w+su}-\frac{\mu_2v}{\alpha_2+w}\right]\\=&\hat{J}_1(t)+\hat{J}_2(t),\end{aligned}$$

where

$$\hat{J}_1(t)=\int_\Omega(u-\hat{u})\left(\frac{\beta_1 w}{\alpha_1+w+su}-\frac{\rho_1 w}{\alpha_1+w+su}-\delta_1\right)\mathrm{d}x$$
$$+\int_\Omega\left(\frac{\beta_2 vw}{\alpha_2+w}-\frac{\rho_2 vw}{\alpha_2+w}-\delta_2 v\right)\mathrm{d}x$$
$$+\int_\Omega(w-\hat{w})\left[r\left(1-\frac{w}{K}\right)-\frac{\mu_1 u}{\alpha_1+w+su}-\frac{\mu_2 v}{\alpha_2+w}\right],$$

and

$$\hat{J}_2(t)=-\int_\Omega\frac{\hat{u}|\partial_x u|^2}{u^2}\mathrm{d}x+\int_\Omega\frac{\chi\hat{u}\partial_x w\cdot\partial_x u}{u(1+\delta w)^2}\mathrm{d}x-\int_\Omega\frac{d\hat{w}|\partial_x w|^2}{w^2}\mathrm{d}x.$$

By using

$$\delta_1=\frac{\beta_1\hat{w}}{\alpha_1+\hat{w}+s\hat{u}}+\frac{\rho_1\hat{w}}{\alpha_1+\hat{w}+s\hat{u}},\quad r=\frac{r\hat{w}}{K}+\frac{\mu_1\hat{u}}{\alpha_1+\hat{w}+s\hat{u}},$$

one yields

$$\begin{aligned}
\hat{J}_1(t)&=\int_\Omega(u-\hat{u})\left(\frac{\beta_1 w}{\alpha_1+w+su}-\frac{\rho_1 w}{\alpha_1+w+su}-\delta_1\right)\mathrm{d}x\\
&\quad+\int_\Omega\left(\frac{\beta_2 vw}{\alpha_2+w}-\frac{\rho_2 vw}{\alpha_2+w}-\delta_2 v\right)\mathrm{d}x\\
&\quad+\int_\Omega(w-\hat{w})\left[r\left(1-\frac{w}{K}\right)-\frac{\mu_1 u}{\alpha_1+w+su}-\frac{\mu_2 v}{\alpha_2+w}\right]\\
&=\int_\Omega\frac{(\beta_1-\rho_1)(\alpha_1+s\hat{u})(u-\hat{u})(w-\hat{w})}{(\alpha_1+w+su)(\alpha_1+\hat{w}+s\hat{u})}\mathrm{d}x\\
&\quad-\int_\Omega\frac{s\hat{w}(\beta_1-\rho_1)(u-\hat{u})^2}{(\alpha_1+w+su)(\alpha_1+\hat{w}+s\hat{u})}\mathrm{d}x+\int_\Omega\left(\frac{\beta_2 vw}{\alpha_2+w}-\frac{\rho_2 vw}{\alpha_2+w}-\delta_2 v\right)\mathrm{d}x\\
&\quad+\int_\Omega\frac{\mu_1\hat{u}(w-\hat{w})^2}{(\alpha_1+w+su)(\alpha_1+\hat{w}+s\hat{u})}\mathrm{d}x-\int_\Omega\frac{\mu_1(\alpha_1+\hat{w})(u-\hat{u})(w-\hat{w})}{(\alpha_1+w+su)(\alpha_1+\hat{w}+s\hat{u})}\mathrm{d}x\\
&\quad-\frac{r}{K}\int_\Omega(w-\hat{w})^2\,\mathrm{d}x-\int_\Omega\frac{\mu_2 v(w-\hat{w})}{\alpha_2+w}\mathrm{d}x\\
&\le\int_\Omega\frac{\left[(\beta_1-\rho_1)(\alpha_1+s\hat{u})-\mu_1(\alpha_1+\hat{w})\right](u-\hat{u})(w-\hat{w})}{(\alpha_1+w+su)(\alpha_1+\hat{w}+s\hat{u})}\mathrm{d}x-\int_\Omega\frac{\rho_2 vw}{\alpha_2+w}\mathrm{d}x\\
&\quad+\int_\Omega\frac{\mu_1\hat{u}(w-\hat{w})^2}{(\alpha_1+w+su)(\alpha_1+\hat{w}+s\hat{u})}\mathrm{d}x-\frac{r}{K}\int_\Omega(w-\hat{w})^2\,\mathrm{d}x+\int_\Omega\frac{(\beta_2-\mu_2)vw}{\alpha_2+w}\mathrm{d}x\\
&\quad+\int_\Omega\left(\frac{\mu_2\hat{w}}{\alpha_2+w}-\delta_2\right)v\mathrm{d}x-\int_\Omega\frac{s\hat{w}(\beta_1-\rho_1)(u-\hat{u})^2}{(\alpha_1+w+su)(\alpha_1+\hat{w}+s\hat{u})}\mathrm{d}x\\
&\le\int_\Omega\frac{\left[(\beta_1-\rho_1)(\alpha_1+s\hat{u})-\mu_1(\alpha_1+\hat{w})\right](u-\hat{u})(w-\hat{w})}{(\alpha_1+w+su)(\alpha_1+\hat{w}+s\hat{u})}\mathrm{d}x\\
&\quad+\int_\Omega\left(\frac{\mu_1 s\hat{u}}{\alpha_1^2}-\frac{r}{K}\right)(w-\hat{w})^2\,\mathrm{d}x\\
&\quad+\int_\Omega\frac{(\beta_2-\mu_2)vw}{\alpha_2+w}\mathrm{d}x+\int_\Omega\left(\frac{\mu_2\hat{w}}{\alpha_2}-\delta_2\right)v\mathrm{d}x\\
&\le 0,
\end{aligned}$$

due to (3) is valid. For $\hat{J}_2(t)$, we have

$$\begin{aligned}\hat{J}_2(t) &= -\int_\Omega \frac{\hat{u}|\partial_x u|^2}{u^2}\mathrm{d}x + \int_\Omega \frac{\chi\hat{u}\partial_x w\cdot\partial_x u}{u(1+\delta w)^2}\mathrm{d}x - \int_\Omega \frac{d\hat{w}|\partial_x w|^2}{w^2}\mathrm{d}x \\ &\le -\int_\Omega \frac{\hat{u}|\partial_x u|^2}{u^2}\mathrm{d}x + \int_\Omega \frac{\chi\hat{u}|\partial_x w|\cdot|\partial_x u|}{u}\mathrm{d}x - \int_\Omega \frac{d\hat{w}|\partial_x w|^2}{w^2}\mathrm{d}x \\ &= -\int_\Omega X_1 Q_1 X_1^{\mathrm{T}}\mathrm{d}x,\end{aligned}$$

where we define $X_1(x,t) = \left(|\partial_x u(x,t)|, |\partial_x w(x,t)|\right)$ in $\Omega\times(0,\infty)$, and the matrix Q_1 is

$$Q_1 = \begin{pmatrix} \frac{\hat{u}}{u^2} & -\frac{\chi\hat{u}}{2u} \\ -\frac{\chi\hat{u}}{2u} & \frac{d\hat{w}}{w^2} \end{pmatrix}.$$

Accordingly, we have $\frac{\hat{u}}{u^2} > 0$ and $\frac{\hat{u}}{u^2}\left(\frac{d\hat{w}}{w^2} - \frac{\chi^2\hat{u}}{4}\right) \ge 0$ as (3) holds. These imply $\dot{V}_2(t) = \hat{J}_1(t) + \hat{J}_2(t) \le 0$ and $E_2 = (\hat{u}, 0, \hat{w})$ is globally asymptotically stable.

3) Consider the following function

$$V_3(t) = \int_\Omega u(\cdot,t)\mathrm{d}x + \int_\Omega\int_{\tilde{v}}^{v} \frac{v(\cdot,t)-\tilde{v}}{v(\cdot,t)}\mathrm{d}v\mathrm{d}x + \int_\Omega\int_{\tilde{w}}^{w} \frac{w(\cdot,t)-\tilde{w}}{w(\cdot,t)}\mathrm{d}w\mathrm{d}x. \tag{10}$$

Straightforward computation showing

$$\begin{aligned}\dot{V}_3(t) &= \int_\Omega \partial_t u\mathrm{d}x + \int_\Omega\left(1-\frac{\tilde{v}}{v}\right)\partial_t v\mathrm{d}x + \int_\Omega\left(1-\frac{\tilde{w}}{w}\right)\partial_t w\mathrm{d}x \\ &= \int_\Omega\left(\frac{\beta_1 uw}{\alpha_1+w+su} - \frac{\rho_1 uw}{\alpha_1+w+su} - \delta_1 u\right)\mathrm{d}x - \int_\Omega \frac{\tilde{v}|\partial_x v|^2}{v^2}\mathrm{d}x \\ &\quad + \int_\Omega \frac{\xi\tilde{v}\partial_x w\cdot\partial_x v}{v(1+\delta w)^2}\mathrm{d}x + \int_\Omega (v-\tilde{v})\left(\frac{\beta_2 w}{\alpha_2+w} - \frac{\rho_2 w}{\alpha_2+w} - \delta_2\right)\mathrm{d}x \\ &\quad - \int_\Omega \frac{d\tilde{w}|\partial_x w|^2}{w^2}\mathrm{d}x + \int_\Omega (w-\tilde{w})\left[r\left(1-\frac{w}{K}\right) - \frac{\mu_1 u}{\alpha_1+w+su} - \frac{\mu_2 v}{\alpha_2+w}\right] \\ &= \tilde{J}_1(t) + \tilde{J}_2(t),\end{aligned}$$

where

$$\begin{aligned}\tilde{J}_1(t) &= \int_\Omega\left(\frac{\beta_1 uw}{\alpha_1+w+su} - \frac{\rho_1 uw}{\alpha_1+w+su} - \delta_1 u\right)\mathrm{d}x \\ &\quad + \int_\Omega (v-\tilde{v})\left(\frac{\beta_2 w}{\alpha_2+w} - \frac{\rho_2 w}{\alpha_2+w} - \delta_2\right)\mathrm{d}x \\ &\quad + \int_\Omega (w-\tilde{w})\left[r\left(1-\frac{w}{K}\right) - \frac{\mu_1 u}{\alpha_1+w+su} - \frac{\mu_2 v}{\alpha_2+w}\right],\end{aligned}$$

and

$$\tilde{J}_2(t) = -\int_\Omega \frac{\tilde{v}|\partial_x v|^2}{v^2}\mathrm{d}x + \int_\Omega \frac{\xi\tilde{v}\partial_x w\cdot\partial_x v}{v(1+\delta w)^2}\mathrm{d}x - \int_\Omega \frac{d\tilde{w}|\partial_x w|^2}{w^2}\mathrm{d}x.$$

Note that

$$\delta_2 = \frac{\beta_2 \tilde{w}}{\alpha_2 + \tilde{w}} + \frac{\rho_2 \tilde{w}}{\alpha_2 + \tilde{w}},\ r = \frac{r\tilde{w}}{K} + \frac{\mu_2 v}{\alpha_2 + \tilde{w}},$$

we get

$$\begin{aligned}
\tilde{J}_1(t) &= \int_\Omega \left(\frac{\beta_1 uw}{\alpha_1 + w + su} - \frac{\rho_1 uw}{\alpha_1 + w + su} - \delta_1 u \right) \mathrm{d}x \\
&\quad + \int_\Omega (v - \tilde{v}) \left(\frac{\beta_2 w}{\alpha_2 + w} - \frac{\rho_2 w}{\alpha_2 + w} - \delta_2 \right) \mathrm{d}x \\
&\quad + \int_\Omega (w - \tilde{w}) \left[r\left(1 - \frac{w}{K}\right) - \frac{\mu_1 u}{\alpha_1 + w + su} - \frac{\mu_2 v}{\alpha_2 + w} \right] \\
&= \int_\Omega \left(\frac{\beta_1 uw}{\alpha_1 + w + su} - \frac{\rho_1 uw}{\alpha_1 + w + su} - \delta_1 u \right) \mathrm{d}x \\
&\quad + \int_\Omega \frac{\alpha_2 (\beta_2 - \rho_2)(v - \tilde{v})(w - \tilde{w})}{(\alpha_2 + w)(\alpha_2 + \tilde{w})} \mathrm{d}x \\
&\quad + \int_\Omega \frac{\mu_2 \tilde{v} (w - \tilde{w})^2}{(\alpha_2 + w)(\alpha_2 + \tilde{w})} \mathrm{d}x - \int_\Omega \frac{\mu_2 (\alpha_2 + \tilde{w})(v - \tilde{v})(w - \tilde{w})}{(\alpha_2 + w)(\alpha_2 + \tilde{w})} \mathrm{d}x \\
&\quad - \frac{r}{K} \int_\Omega (w - \tilde{w})^2 \mathrm{d}x - \int_\Omega \frac{\mu_1 u (w - \tilde{w})}{\alpha_1 + w + su} \mathrm{d}x \\
&= -\int_\Omega \frac{\rho_1 uw}{\alpha_1 + w + su} \mathrm{d}x + \int_\Omega \frac{[\alpha_2 (\beta_2 - \rho_2) - \mu_2 (\alpha_2 + \tilde{w})](v - \tilde{v})(w - \tilde{w})}{(\alpha_2 + w)(\alpha_2 + \tilde{w})} \mathrm{d}x \\
&\quad + \int_\Omega \left(\frac{\mu_2 \tilde{v}}{(\alpha_2 + w)(\alpha_2 + \tilde{w})} - \frac{r}{K} \right) (w - \tilde{w})^2 \mathrm{d}x \\
&\quad + \int_\Omega \frac{(\beta_1 - \mu_1) uw}{\alpha_1 + w + su} \mathrm{d}x + \int_\Omega \left(\frac{\mu_1 \tilde{w}}{\alpha_1 + w + su} - \delta_1 \right) u \mathrm{d}x \\
&\le \int_\Omega \frac{[\alpha_2 (\beta_2 - \rho_2) - \mu_2 (\alpha_2 + \tilde{w})](v - \tilde{v})(w - \tilde{w})}{(\alpha_2 + w)(\alpha_2 + \tilde{w})} \mathrm{d}x \\
&\quad + \int_\Omega \left(\frac{\mu_2 \tilde{v}}{\alpha_2^2} - \frac{r}{K} \right) (w - \tilde{w})^2 \mathrm{d}x + \int_\Omega \frac{(\beta_1 - \mu_1) uw}{\alpha_1 + w + su} \mathrm{d}x \\
&\quad + \int_\Omega \left(\frac{\mu_1 \tilde{w}}{\alpha_1} - \delta_1 \right) u \mathrm{d}x \\
&\le 0,
\end{aligned}$$

if (4) is satisfied. For $\tilde{J}_2(t)$, we have

$$\begin{aligned}
\tilde{J}_2(t) &= -\int_\Omega \frac{\tilde{v} |\partial_x v|^2}{v^2} \mathrm{d}x + \int_\Omega \frac{\xi \tilde{v} \partial_x w \cdot \partial_x v}{v (1 + \delta w)^2} \mathrm{d}x - \int_\Omega \frac{d\tilde{w} |\partial_x w|^2}{w^2} \mathrm{d}x \\
&\le -\int_\Omega \frac{\tilde{v} |\partial_x v|^2}{v^2} \mathrm{d}x + \int_\Omega \frac{\xi \tilde{v} |\partial_x w| \cdot |\partial_x v|}{v} \mathrm{d}x - \int_\Omega \frac{d\tilde{w} |\partial_x w|^2}{w^2} \mathrm{d}x \\
&= -\int_\Omega X_2 Q_2 X_2^{\mathrm{T}} \mathrm{d}x,
\end{aligned}$$

where denote by $X_2(x,t) = \left(|\partial_x v(x,t)|, |\partial_x w(x,t)| \right)$ in $\Omega \times (0, \infty)$ and

$$Q_2 = \begin{pmatrix} \dfrac{\tilde{v}}{v^2} & -\dfrac{\xi\tilde{v}}{2v} \\ -\dfrac{\xi\tilde{v}}{2v} & \dfrac{d\tilde{w}}{w^2} \end{pmatrix}.$$

It is clear that $\frac{\tilde{v}}{v^2} > 0$ and $|Q_2| = \frac{\tilde{v}}{v^2}\left(\frac{d\tilde{w}}{w^2} - \frac{\xi^2\tilde{v}}{4}\right) \ge 0$ since $0 < \xi^2 \le \frac{4d\tilde{w}}{C^2\tilde{v}}$. Consequently, $\dot{V}_3(t) = \tilde{J}_1(t) + \tilde{J}_2(t) \le 0$ and thus $E_3 = (0, \tilde{v}, \tilde{w})$ is globally asymptotically stable.

4) Introducing the following time evolution Lyapunov function

$$\begin{aligned} V_4(t) = {} & \int_\Omega \int_{u^*}^{u} \frac{u(\cdot,t) - u^*}{u(\cdot,t)} \mathrm{d}u\mathrm{d}x + \int_\Omega \int_{v^*}^{v} \frac{v(\cdot,t) - v^*}{v(\cdot,t)} \mathrm{d}v\mathrm{d}x \\ & + \int_\Omega \int_{w^*}^{w} \frac{w(\cdot,t) - w^*}{w(\cdot,t)} \mathrm{d}w\mathrm{d}x. \end{aligned} \tag{11}$$

Direct computations illustrate that

$$\begin{aligned} \dot{V}_4(t) = {} & \int_\Omega \left(1 - \frac{u^*}{u}\right)\partial_t u\mathrm{d}x + \int_\Omega \left(1 - \frac{v^*}{v}\right)\partial_t v\mathrm{d}x + \int_\Omega \left(1 - \frac{w^*}{w}\right)\partial_t w\mathrm{d}x \\ = {} & \int_\Omega (u - u^*)\left(\frac{\beta_1 w}{\alpha_1 + w + su} - \frac{\rho_1 w}{\alpha_1 + w + su} - \delta_1\right)\mathrm{d}x \\ & + \int_\Omega (v - v^*)\left(\frac{\beta_2 w}{\alpha_2 + w} - \frac{\rho_2 w}{\alpha_2 + w} - \delta_2\right)\mathrm{d}x \\ & + \int_\Omega (w - w^*)\left[r\left(1 - \frac{w}{K}\right) - \frac{\mu_1 u}{\alpha_1 + w + su} - \frac{\mu_2 v}{\alpha_2 + w}\right] \\ & - \int_\Omega \frac{u^*|\partial_x u|^2}{u^2}\mathrm{d}x - \int_\Omega \frac{v^*|\partial_x v|^2}{v^2}\mathrm{d}x - \int_\Omega \frac{dw^*|\partial_x w|^2}{w^2}\mathrm{d}x \\ & + \int_\Omega \frac{\chi u^*\partial_x w \cdot \partial_x u}{u(1+\delta w)^2}\mathrm{d}x + \int_\Omega \frac{\xi v^*\partial_x w \cdot \partial_x v}{v(1+\delta w)^2}\mathrm{d}x \\ = {} & J_1^*(t) + J_2^*(t), \end{aligned}$$

where

$$\begin{aligned} J_1^*(t) = {} & \int_\Omega (u - u^*)\left(\frac{\beta_1 w}{\alpha_1 + w + su} - \frac{\rho_1 w}{\alpha_1 + w + su} - \delta_1\right)\mathrm{d}x \\ & + \int_\Omega (v - v^*)\left(\frac{\beta_2 w}{\alpha_2 + w} - \frac{\rho_2 w}{\alpha_2 + w} - \delta_2\right)\mathrm{d}x \\ & + \int_\Omega (w - w^*)\left[r\left(1 - \frac{w}{K}\right) - \frac{\mu_1 u}{\alpha_1 + w + su} - \frac{\mu_2 v}{\alpha_2 + w}\right], \end{aligned}$$

and

$$\begin{aligned} J_2^*(t) = {} & -\int_\Omega \frac{u^*|\partial_x u|^2}{u^2}\mathrm{d}x - \int_\Omega \frac{v^*|\partial_x v|^2}{v^2}\mathrm{d}x - \int_\Omega \frac{dw^*|\partial_x w|^2}{w^2}\mathrm{d}x \\ & + \int_\Omega \frac{\chi u^*\partial_x w \cdot \partial_x u}{u(1+\delta w)^2}\mathrm{d}x + \int_\Omega \frac{\xi v^*\partial_x w \cdot \partial_x v}{v(1+\delta w)^2}\mathrm{d}x. \end{aligned}$$

By employing these facts

$$\delta_1=\frac{\beta_1 w^*}{\alpha_1+w^*+su^*}+\frac{\rho_1 w^*}{\alpha_1+w^*+su^*},\ \delta_2=\frac{\beta_2 w^*}{\alpha_2+w^*}+\frac{\rho_2 w^*}{\alpha_2+w^*},$$

$$r=\frac{w^*}{K}+\frac{\mu_1 u^*}{\alpha_1+w^*+su^*}+\frac{\mu_2 v^*}{\alpha_2+w^*},$$

we can obtain

$$\begin{aligned}
J_1^*(t)&=\int_\Omega \frac{(\beta_1-\rho_1)(\alpha_1+su^*)(u-u^*)(w-w^*)}{(\alpha_1+w+su)(\alpha_1+w^*+su^*)}\mathrm{d}x\\
&\quad-\int_\Omega \frac{sw^*(\beta_1-\rho_1)(u-u^*)^2}{(\alpha_1+w+su)(\alpha_1+w^*+su^*)}\mathrm{d}x\\
&\quad+\int_\Omega \frac{\alpha_2(\beta_2-\rho_2)(v-v^*)(w-w^*)}{(\alpha_2+w)(\alpha_2+w^*)}\mathrm{d}x\\
&\quad-\int_\Omega \frac{\mu_2(\alpha_2+w^*)(v-v^*)(w-w^*)}{(\alpha_2+w)(\alpha_2+w^*)}\mathrm{d}x\\
&\quad+\int_\Omega \frac{\mu_2 v^*(w-w^*)^2}{(\alpha_2+w)(\alpha_2+w^*)}\mathrm{d}x-\frac{r}{K}\int_\Omega (w-w^*)^2\,\mathrm{d}x\\
&\quad+\int_\Omega \frac{\mu_1 u^*(w-w^*)^2}{(\alpha_1+w+su)(\alpha_1+w^*+su^*)}\mathrm{d}x\\
&\quad-\int_\Omega \frac{\mu_1(\alpha_1+w^*)(u-u^*)(w-w^*)}{(\alpha_1+w+su)(\alpha_1+w^*+su^*)}\mathrm{d}x\\
&\le \int_\Omega \frac{\left[(\beta_1-\rho_1)(\alpha_1+su^*)-\mu_1(\alpha_1+w^*)\right](u-u^*)(w-w^*)}{(\alpha_1+w+su)(\alpha_1+w^*+su^*)}\mathrm{d}x\\
&\quad+\int_\Omega \frac{\left[\alpha_2(\beta_2-\rho_2)-\mu_2(\alpha_2+w^*)\right](v-v^*)(w-w^*)}{(\alpha_2+w)(\alpha_2+w^*)}\mathrm{d}x\\
&\quad+\int_\Omega \left(\frac{\mu_1 u^*}{\alpha_1^2}+\frac{\mu_2 v^*}{\alpha_2^2}-\frac{r}{K}\right)(w-w^*)^2\,\mathrm{d}x\\
&\le 0,
\end{aligned}$$

here we use the Condition (5). Moreover

$$\begin{aligned}
J_2^*(t)&=-\int_\Omega \frac{u^*|\partial_x u|^2}{u^2}\mathrm{d}x-\int_\Omega \frac{v^*|\partial_x v|^2}{v^2}\mathrm{d}x-\int_\Omega \frac{dw^*|\partial_x w|^2}{w^2}\mathrm{d}x\\
&\quad+\int_\Omega \frac{\chi u^*\partial_x w\cdot\partial_x u}{u(1+\delta w)^2}\mathrm{d}x+\int_\Omega \frac{\xi v^*\partial_x w\cdot\partial_x v}{v(1+\delta w)^2}\mathrm{d}x.\\
&\le -\int_\Omega \frac{u^*|\partial_x u|^2}{u^2}\mathrm{d}x-\int_\Omega \frac{v^*|\partial_x v|^2}{v^2}\mathrm{d}x-\int_\Omega \frac{dw^*|\partial_x w|^2}{w^2}\mathrm{d}x\\
&\quad+\int_\Omega \frac{\chi u^*|\partial_x w|\cdot|\partial_x u|}{u}\mathrm{d}x+\int_\Omega \frac{\xi v^*|\partial_x w|\cdot|\partial_x v|}{v}\mathrm{d}x\\
&=-\int_\Omega X_3 Q_3 X_3^{\mathrm{T}}\mathrm{d}x,
\end{aligned}$$

where the vector function $X_3(x,t)$ is given by $X_3(x,t)=\left(\left|\partial_x u(x,t)\right|,\left|\partial_x v(x,t)\right|,\left|\partial_x w(x,t)\right|\right)$ in $\Omega\times(0,\infty)$ and

$$Q_3=\begin{pmatrix} \frac{u^*}{u^2} & 0 & -\frac{\chi u^*}{2u} \\ 0 & \frac{v^*}{v^2} & -\frac{\xi v^*}{2v} \\ -\frac{\chi u^*}{2u} & -\frac{\xi v^*}{2v} & \frac{dw^*}{w^2} \end{pmatrix}.$$

We can obtain

$$\left|\frac{u^*}{u^2}\right|>0,\quad \begin{vmatrix} \frac{u^*}{u^2} & 0 \\ 0 & \frac{v^*}{v^2} \end{vmatrix}=\frac{u^*v^*}{u^2v^2}>0.$$

as well as

$$\begin{aligned}|Q_3|&=\frac{u^*}{u^2}\begin{vmatrix} \frac{v^*}{v^2} & -\frac{\xi v^*}{2v} \\ -\frac{\xi v^*}{2v} & \frac{dw^*}{w^2} \end{vmatrix}-\frac{\chi u^*}{2u}\begin{vmatrix} 0 & \frac{v^*}{v^2} \\ -\frac{\chi u^*}{2u} & -\frac{\xi v^*}{2v} \end{vmatrix}\\ &=\frac{u^*v^*}{u^2v^2}\left(\frac{dw^*}{w^2}-\frac{\xi^2v^*}{4}-\frac{\chi^2u^*}{4}\right)\\ &\ge 0\end{aligned}$$

if

$$0<\chi^2+\xi^2\le\frac{4dw^*}{\max\left\{u^*,v^*\right\}C^2}.$$

Thence A_3 is a nonnegative definite matrix, which gives $J_2^*(t)=-\int_\Omega X_3Q_3X_3^{\mathrm{T}}\mathrm{d}x\le 0$. We conclude that $E_*=\left(u^*,v^*,w^*\right)$ is globally asymptotically stable. These end the proof.

5. Conclusions

This present paper deals with the global asymptotic stability of a three-species predator-prey model with prey-taxis. This system possesses a semi-trivial equilibrium $E_1=(0,0,K)$, two boundary equilibria $E_2=(\hat{u},0,\hat{w})$ and $E_3=(0,\tilde{v},\tilde{w})$, and a unique positive equilibrium $E_*=\left(u^*,v^*,w^*\right)$. By constructing some suitable Lyapunov functions, we establish their global asymptotic stability, respectively. It is concluded that the prey-taxis coefficients χ,ξ can not influence the global asymptotic stability of the semi-trivial equilibrium $E_1=(0,0,K)$. Also, the global asymptotic stability of two boundary equilibria $E_2=(\hat{u},0,\hat{w})$ and $E_3=(0,\tilde{v},\tilde{w})$ rely on the single prey-taxis coefficient χ and ξ, respectively. However, the global asymptotic stability of the unique positive equilibrium $E_*=\left(u^*,v^*,w^*\right)$ is determined by prey-taxis coefficients χ and ξ. These phenomena suggest that the prey-taxis has an influence on the global asymptotic stability of the equilibria of the System (1). Consequently, we will continuously explore the complicated dynamics of the System (1) with prey-taxis effect in the future.

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Conflicts of Interest

The authors declare no conflicts of interest regarding the publication of this paper.

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