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Resource Allocation for OFDMA-MIMO Relay Systems with Proportional Fairness Constraints

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ABSTRACT

In this paper, we study resource allocation problem in orthogonal frequency division multiple access multiple-input multiple-output (OFDMA-MIMO) relay systems and formulate the optimal instantaneous resource allocation problem including subcarrier assignment, relay selection and power allocation to maximize system capacity. Based on the assumption that the availability of perfect channel state information (CSI) is known at the resource allocation controller, we propose a new resource allocation algorithm which can guarantee proportional fairness among users. In the proposed algorithm, a two-step suboptimal method is taken into account. Firstly, we assume equal power allocation for each user to linearize the problem and propose the subcarrier assignment and relay selection scheme based on equivalent channel gain. Secondly, we derive the closed-form expressions for power allocation through relaxing the proportional fairness constraints. Numerical simulations show that the proposed algorithm performs well in terms of satisfying proportional fairness among users in strict sense and achieving improvement on system total capacity.

Keywords: Relay Network; OFDMA; MIMO; Proportional Fairness; Resource Allocation

1. Introduction

The Orthogonal Frequency Division Multiple Access (OFDMA) is regarded as a leading candidate for the fourth generation (4G) wireless communication system because of its high spectral efficiency, flexible resource allocation and inherent robustness against frequency-selective fading. Furthermore, Multiple-Input Multiple-Output (MIMO) system has been extensively studied in recent years. Since the multiple antenna technology provides extra spatial degrees of freedom, an OFDMA-MIMO system can improve transmission reliability and capacity without the need of increasing power or bandwidth. On the other hand, due to its potential benefits of enlarging the coverage of communication systems, increasing the capacity and enhancing the link reliability, cooperative relaying has attracted significant attention of many researchers.

Recently, there have been many research efforts on improving the system throughput by resource allocation in OFDMA-MIMO relay system. Resource allocations of Amplify-and-Forward (AF) and Decode-and-Forward (DF) MIMO-OFDM relay systems are proposed in [1,2] respectively. But, both [1] and [2] only focus on a single user scenario.

A number of results have been published on resource allocation for multi-user MIMO-OFDMA relay systems [3-5]. The optimal instantaneous resource allocation problem, including path selection, power allocation and sub-channel scheduling, is formulated in [3]. Optimal and suboptimal resource allocation algorithms for weighted sum rate maximization are presented in [4]. In [5], heterogeneous data rate requirements for delay sensitive and non-delay sensitive users are taken into consideration; the authors derive a distributed iterative resource allocation and scheduling algorithm with closed-form power and subcarrier allocation via employing dual decomposition.

However, most existing research resource allocation in OFDMA-MIMO relay systems focus on maximizing the system capacity, fairness among multiple users has not received much attention. In most practical communication system, the different business types of users have different rate, are endowed with different resource allocation priority. Therefore, to study the resource allocation problem with proportional fairness constraint is essential for OFDMA-MIMO relay systems.

In this paper, we investigate subcarrier assignment, relay selection and power allocation problem with proportional fairness constraint for OFDMA-MIMO relay assisted cellular system, and formulate the problem as a joint optimization problem. Since the problem is a NP-hard combination optimization problem with non-linear constraints, we use a two-step suboptimal method to solve it. Firstly, we assume equal power allocation for each user to linearize the problem and propose the subcarrier assignment and relay selection scheme based on equivalent channel gain. Secondly, we derive the closed-form expressions for power allocation through relaxing the proportional fairness constraints. Simulation results show that the proposed algorithm performs well in terms of satisfying proportional date rate constrains in strict sense and achieving improvement on system total rate.

The rest of this paper is organized as follows. The OFDMA-MIMO relay assisted cellular communication system model and optimization problem formulation is described in Section Ⅱ. The proposed resource allocation solution is presented in Section Ⅲ. In Section Ⅳ, simulations results are given and discussed. Finally, conclusions are drawn in section Ⅴ.

2. System Model

We consider a OFDMA-MIMO relay assisted single cellular communication system as shown in **Figure 1**. In the adopted system, the overall bandwidth is *B* which is divided into *N* orthogonal subcarriers. A base station (BS) is located at the center, while *K* relay stations (RS) at the inner boundary, operating in decode-and-forward (DF) mode. *M* mobile stations (MS) are separated between the inner and the outer boundaries. The BS and the RSs are equipped with N_{BS}, N_{RS} antennas, respectively. While, each MS is equipped with only one antenna. In order to guarantee all users can receive signals from the base station, the system operates in a time division duplex (TDD) mode, where transmission takes place in two time slots. In the first time slot, each RS receives and decodes the signal from BS, then re-enc odes and forwards the signal to the relevant MSs during the second time slot. For simplicity, we assume that the subcarriers used in the first time slot and the second time slot are the same. It is further assumed that each subcarrier can be assigned to only link in each slot [6].

Let $H_{BS,k,n}, H_{k,m,n}$ represent the channel gain matrix of the link from BS to relay *k* on the subcarrier *n* and the link from relay *k* to user *m* on the subcarrier *n*, respectively. And the corresponding transmission power is $p_{BS,k,n}, p_{k,m,n}$.

$$H_{BS,k,n} = \begin{bmatrix} h_{1,1}^{BS,k,n} & h_{1,2}^{BS,k,n} & \cdots & h_{1,N_{BS}}^{BS,k,n} \\ h_{2,1}^{BS,k,n} & h_{2,2}^{BS,k,n} & \cdots & h_{2,N_{BS}}^{BS,k,n} \\ \vdots & \vdots & \ddots & \vdots \\ h_{N_{RS},1}^{BS,k,n} & h_{N_{RS},2}^{BS,k,n} & \cdots & h_{N_{RS},N_{BS}}^{BS,k,n} \end{bmatrix}$$

$$H_{k,m,n} = \begin{bmatrix} h_{1,1}^{k,m,n} & h_{1,2}^{k,m,n} & \cdots & h_{1,N_{RS}}^{k,m,n} \end{bmatrix}$$

The singular value decomposition of $H_{BS,k,n}$ and $H_{k,m,n}$ are given by

$$H_{BS,k,n} = \sum_{i=1}^{N'} u_{BS,k,n}^{i} \sigma_{BS,k,n}^{i} \left(v_{BS,k,n}^{i}\right)^{H}$$

$$H_{k,m,n} = u_{k,m,n} \sigma_{k,m,n} v_{k,m,n}^{H}$$

where $\sigma_{BS,k,n}^{i} (i = 1, 2, \ldots N')$ and $\sigma_{k,m,n}$ are the singular value of the matrix $H_{BS,k,n}$, $H_{k,m,n}$, respectively. $N' = \{N_{BS}, N_{RS}\}$ is the rank of the matrix $H_{BS,k,n} H_{BS,k,n}^{H}$.

The instantaneous rate between BS and relay *k* on the subcarrier *n* can be written:

$$R_{BS,k,n} = \frac{B}{2N} \log_2 \left(1 + p_{BS,k,n} g_{BS,k,n}\right)$$

Similarly, the instantaneous rate between relay *k* and user *m* on the subcarrier *n* in the second time slot is given by

$$R_{k,m,n} = \frac{B}{2N} \log_2 \left(1 + p_{k,m,n} g_{k,m,n}\right)$$

The achievable rate of user *m* assisted by the relay *k* on the subcarrier *n* is the minimum rate of $R_{BS,k,n}$ and $R_{k,m,n}$ [7]

$$R_{BS,k,m}^{n} = \min\left(R_{BS,k,n}, R_{k,m,n}\right) \tag{1}$$

where

$$g_{BS,k,n} = \frac{\left(\sigma_{BS,k,n}^{1}\right)^2}{\Gamma N_0 B / N} \quad \text{and} \quad g_{k,m,n} = \frac{\left(\sigma_{k,m,n}\right)^2}{\Gamma N_0 B / N}$$

are the main characteristic sub-channel gain of the link from BS to relay *k* on the subcarrier *n* and the link from relay *k* to user *m* on the subcarrier *n*, respectively. Γ is the SNR gap related to BER, $\Gamma = \ln(-5BER)/1.5$ and N_0 denotes the power spectral density of the Gaussian white noise.

Define $\rho_{k,m,n} \in \{0,1\}$ as the joint user selection, path selection and subcarrier allocation indicator. $\rho_{k,m,n} = 1$ if and only if the subcarrier *n* is assigned to the user *m* and relayed by relay *k*. Thus the achievable data rate of the user *m* is $R_m = \sum_{k=1}^{K} \sum_{n=1}^{N} \rho_{k,m,n} R_{BS,k,m}^{n}$. Therefore, the adaptive optimal resource allocation problem subject to total transmit power constraint and guaranteeing proportional fairness among users is expressed as follows

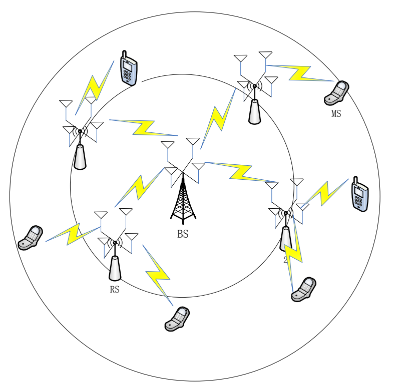

Figure 1. Single cellular OFDMA-MIMO relay system model.

$$\max \sum_{k=1}^{K}\sum_{m=1}^{M}\sum_{n=1}^{N}\rho_{k,m,n}R^{n}_{BS,k,m}$$
$$subject \quad to:$$
$$A1:\rho_{k,m,n}\in\{0,1\}$$
$$A2:\sum_{k=1}^{K}\sum_{m=1}^{M}\rho_{k,m,n}=1 \qquad (2)$$
$$A3:\sum_{k=1}^{K}\sum_{n=1}^{N}p_{BS,k,n}\le P_T$$
$$A4:\sum_{m=1}^{M}\sum_{n=1}^{N}p_{k,m,n}\le P_{k,T}$$
$$A5:R_1:R_2:\ldots:R_M=\tau_1:\tau_2:\ldots:\tau_M$$

where $A1$ and $A2$ emphasize that every subcarrier can be allocated at most one link in each slot. $A3$ and $A4$ are the total transmit power constraints for BS and RS, respectively. $A5$ denotes the proportional data rate constraint.

3. Proposed Resource Allocation Algorithm

It is difficult to solve the optimization problem in (2), because it includes both integer and continuous variables. In this section, we use a two-step suboptimal algorithm to reduce the computational complexity. In step one, the subcarrier allocation and relay selection under equal power allocation are discussed. In step two, the problem of power allocation is solved.

3.1. Subcarrier Allocation and Relay Selection Scheme

In order to approaches the maximum rate in (1), the following equation should be satisfied

$$p_{BS,k,n}g_{BS,k,n}=p_{k,m,n}g_{k,m,n}$$

So we obtain

$$p_{k,m,n}=\frac{p_{BS,k,n}g_{BS,k,n}}{g_{k,m,n}} \qquad (3)$$

and the equivalent channel gain of link [8],

$$g^{n}_{BS,k,m}=\frac{g_{BS,k,n}g_{k,m,n}}{g_{BS,k,n}+g_{k,m,n}}$$

Thus (1) can be rewritten as

$$\begin{aligned}R^{n}_{BS,k,m}&=\frac{B}{2N}\log_2\left(1+p_{BS,k,n}g_{BS,k,n}\right)\\&=\frac{B}{2N}\log_2\left(1+p_{k,m,n}g_{k,m,n}\right)\end{aligned} \qquad (4)$$

The subcarrier allocation and relay selection algorithm is described as follows

A) Determine the minimum number of subcarriers assigned for each user

$$N_m=\left\lfloor \tau_m N\Big/\left(\sum_{m=1}^{M}\tau_m\right)\right\rfloor, 1\le m\le M$$

B) Implement the subcarrier allocation and relay selection for each user

a) Initialization

$$\Omega_K=\{1,2,\cdots,K\},\Omega_N=\{1,2,\cdots,N\},$$
$$\Omega_M=\{1,2,\cdots,M\},R_m=0,p=\frac{P_T}{N}$$

b) While $N_m\ne 0$

1) Find m satisfying $\arg\min_{m\in\Omega_M}\left(R_m/\gamma_m\right)$

2) For the found m, find n^* and k^* satisfying

$$(n^*,k^*)=\arg\max_{n^*\in\Omega_N,k^*\in\Omega_K}\left(g^{n}_{BS,k,m}\right)$$

3) Let $\rho_{k^*,m,n^*}=1,\Omega_N=\Omega_N/\{n^*\}$, update $R^{n^*}_{BS,k^*,m}$ according to (3).

C) The remaining subcarrier allocation

a) While $\Omega_N\ne 0$, for $n=1$ to N and n doesn't be used, find the user m^* and relay k^* satisfying

$$(m^*,k^*)=\arg\max_{m^*\in\Omega_M,k^*\in\Omega_K}\left(g^{n}_{BS,k,m}\right)$$

b) Let $\rho_{k^*,m^*,n}=1,\Omega_N=\Omega_N/\{n\}$ and update R^{n}_{BS,k^*,m^*} according to (3).

3.2. Proposed Power Allocation Scheme

After the subcarrier allocation and relay selection, the next is to assign the available power on subcarriers. The optimization problem can be reformed as

$$\max\sum_{k=1}^{K}\sum_{n\in C_k}\frac{B}{2N}\log_2(1+p_{BS,k,n}H_{BS,k,n})$$
$$subject \quad to: \qquad (5)$$
$$B1:\sum_{k=1}^{K}\sum_{n=1}^{N}p_{BS,k,n}\le P_T$$
$$B2:R_1:R_2:\ldots:R_K=N_1:N_2:\ldots:N_K$$

Formulate Lagrangian problem as following

$$\begin{aligned}&L(p_{BS,k,n},\beta,\mu_k)\\&=\sum_{k=1}^{K}\sum_{n=1}^{N}\frac{B}{2N}\log_2(1+p_{BS,k,n}H_{BS,k,n})\\&\quad+\beta\Big(\sum_{k=1}^{K}\sum_{n=1}^{N}p_{BS,k,n}-P\Big)\\&\quad+\sum_{\substack{k=1\\k\ne k_0}}^{K}\mu_k\Big[\sum_{n\in C_{k_0}}\frac{B}{2N}\log_2(1+p_{BS,k_0,n}H_{BS,k_0,n})\\&\quad-\frac{N_{k_0}}{N_k}\sum_{n\in C_k}\frac{B}{2N}\log_2(1+p_{BS,k,n}H_{BS,k,n})\Big]\end{aligned}$$

Then differentiate $L\left(p_{BS,k,n},\beta,\mu_k\right)$ with respect to

$$\begin{aligned}&\frac{\partial L\left(p_{BS,k,n},\beta,\mu_k\right)}{\partial p_{BS,k,n}}\\&=\frac{BH_{BS,k,n}}{\left(1+p_{BS,k,n}H_{BS,k,n}\right)2N\ln 2}+\beta\\&-\mu_k\frac{N_{k_0}}{N_k}\frac{BH_{BS,k,n}}{\left(1+p_{BS,k,n}H_{BS,k,n}\right)2N\ln 2}\\&=0\end{aligned} \tag{6}$$

From (6) we have

$$\frac{H_{BS,k,n'}}{1+p_{BS,k,n'}H_{BS,k,n'}}=\frac{H_{BS,k,n}}{1+p_{BS,k,n}H_{BS,k,n}} \tag{7}$$

Then equation (7) can be reformed as

$$p_{BS,k,n}=p_{BS,k,1}+\frac{1}{H_{\mathbf{BS,k,1}}}-\frac{1}{H_{\mathbf{BS,k,}n}} \tag{8}$$

The rate of link k

$$R_k=\frac{B}{2N}\left[N_k\log_2\left(p_{BS,k,1}+\frac{1}{H_{BS,k,1}}\right)+\sum_{n\in C_k}\log_2 H_{BS,k,n}\right]$$

According to the last constraint in (5), we can obtain:

$$\begin{aligned}&\left[N_{k_0}\log_2\left(p_{BS,k_0,1}+\frac{1}{H_{BS,k_0,1}}\right)+w_{k_0}\right]\\&=\left[N_k\log_2\left(p_{BS,k,1}+\frac{1}{H_{BS,k,1}}\right)+w_k\right]\end{aligned} \tag{9}$$

where $w_k=\sum_{n\in C_k}\log_2 H_{BS,k,n}$, k_0 is a reference relay.

From (9), we can derive

$$p_{BS,k,1}=a_k\,p_{BS,k_0,1}+b_k \tag{10}$$

where $a_k=2^{\frac{N_k w_{k_0}-N_{k_0}w_k}{N_k N_{k_0}}}$, $b_k=\frac{a_k}{H_{BS,k_0,1}}-\frac{1}{H_{BS,k,1}}$

Therefore,

$$p_{BS,k_0,1}=\frac{P_T-\sum_{k=1}^{K}\left(N_k b_k+e_k\right)}{\sum_{k=1}^{K}N_k a_k} \tag{11}$$

According to (8) and (11), we have

$$\begin{cases}p_{BS,k,n}=a_k\,p_{BS,k_0,1}+b_k+\dfrac{1}{H_{BS,k,1}}-\dfrac{1}{H_{BS,k,n}}\\[2ex] p_{k,m,n}=\dfrac{p_{BS,k,n}g_{BS,k,n}}{g_{k,m,n}}\end{cases} \tag{12}$$

4. Numerical Results and Analysis

4.1. Simulation Parameters

In this section, we consider a single cellular OFDMA-MIMO relay communication system with a BS located in the center, RSs equally distributed at the inner boundary and MSs separated between the inner and the outer boundaries. Both BS and RS are equipped with multiple antennas, each MS is equipped with one antenna. We adopt the channel for simulation consists of six-path Rayleigh fading and the maximum doppler shift is 30Hz. The total bandwidth is set to be 1MHz. Assume that Gaussian white noise single PSD is 10^{-8} and BER is 10^{-3}.

4.2. Results and Analysis

In our simulation, a heuristic algorithm, PAARS+EquPo and a static algorithm, Static+ EquPo are compared with the proposed algorithm. PAARS+EquPo means the subcarriers allocation and relay selection according to the proposed scheme, equal power allocation for each subcarrier, which is adopted in [6]. Static+ EquPo means each user is assigned fixed subcarriers and the power equally allocated to subcarriers. In the following, the performance of proposed algorithm is evaluated from two aspects: user fairness and system capacity.

To evaluate user fairness, we first define the normalized capacity

$$\text{The normalized capacity} = \frac{R_m}{M\sum_{i=m}^{M}R_m}$$

Figure 2 and **Figure 3** depict the normalized capacity of each user with different subcarrier numbers, respectively. It is shown that, the normalized capacity of proposed algorithm is close to the original normalized fairness constraint. PAARS+EquPo algorithm can achieve better fairness among users when the number of subcarriers is large. However, Static+ EquPo algorithm can't obtain proportional fairness among users.

Figure 4 shows the performance of system capacity with respect to the number of users. As shown in this figure, the capacity increases as the number of users in the system increases. This is due to multiuser diversity gain, which is more prominent in systems with larger number of users. Furthermore, the capacity using PAARS + EquPo algorithm is higher than that adopting the proposed algorithm, because in the proposed algorithm, we consider proportional fairness among users in subcarrier allocation, relay selection scheme, and power allocation scheme. The proposed algorithm outperforms Static+EquPo algorithm, because the proposed algorithm is an adaptive resource allocation algorithm which can adapt to different channel information.

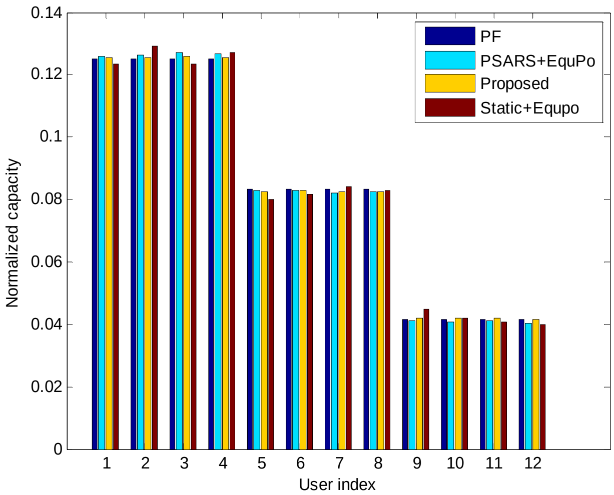

Figure 2. Normalized capacity versus user index after 50 iterations, the number of subcarriers is 1024.

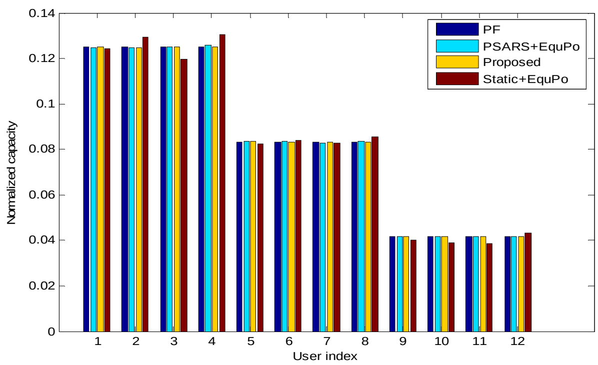

Figure 3. Normalized capacity versus user index after 50 iterations, the number of subcarriers is 2048.

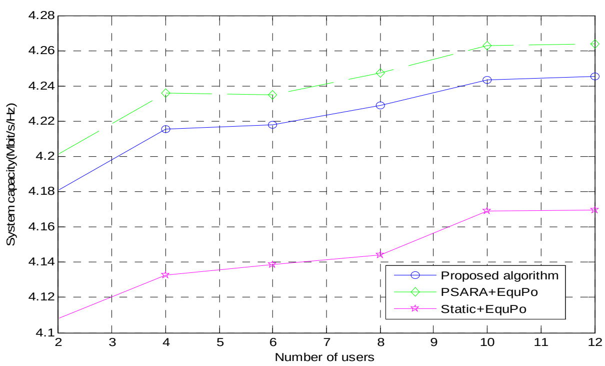

Figure 4. System capacity versus number of users after 50 iterations.

5. Conclusions

In this paper, we propose a new resource allocation algorithm for OFDMA-MIMO relay assisted single cellular communication system, which consider proportional fairness among users. The performance of the proposed algorithm is compared with other algorithms and simulation results demonstrate that he proposed algorithm performs well in terms of satisfying proportional fairness among users in strict sense and achieving improvement on system total capacity.

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