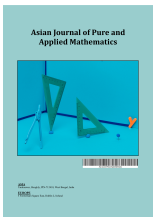

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Mathematical Modeling of HIV Spread and Control Strategies in Nigeria: A Case Study Using Sample Data from Regional Populations

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Authors' contributions

This work was carried out in collaboration between both authors. Both authors read and approved the final manuscript.

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Abstract

This paper introduces a mathematical model for analyzing the spread of HIV in Nigeria, focusing on the impact of control strategies like antiretroviral therapy (ART) and preventive interventions. A modified SITR (Susceptible, Infected, Treated, Removed) compartmental model is developed to capture the dynamics of HIV transmission. The model uses real-world data to simulate various scenarios over a 10-year period, revealing the role of ART in reducing HIV prevalence and highlighting preventive measures. Findings indicate that scaling up ART coverage and improving adherence significantly reduce new infections and mortality

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rates. The study offers policy insights for enhancing HIV control measures in Nigeria, suggesting that a combination of ART, behavioral interventions, and robust healthcare strategies are necessary to curb the epidemic effectively.

Keywords: *HIV; mathematical modeling; SITR model; Nigeria; antiretroviral therapy; epidemiology.*

1 Introduction

Human Immunodeficiency Virus (HIV) and Acquired Immunodeficiency Syndrome (AIDS) continue to pose serious public health challenges worldwide, particularly in sub-Saharan Africa. As of 2022, Nigeria, the most populous country in Africa, has an HIV prevalence rate of approximately 1.3% among adults, resulting in about 1.9 million people living with HIV [1]. This epidemic exerts significant health, social, and economic burdens, exacerbating poverty, disrupting social structures, and straining healthcare resources [2, 3]. The complexities of HIV transmission dynamics and control efforts necessitate comprehensive and targeted strategies to mitigate its impact.

Mathematical modeling has emerged as an invaluable tool in understanding infectious disease dynamics, assessing the impact of intervention strategies, and guiding policy decisions [4, 5]. Various models have been used to study HIV transmission, ranging from simple deterministic models to complex stochastic and network models [6, 7]. Compartmental models, which classify populations into groups based on disease status, have proven particularly useful for simulating the effects of interventions, such as antiretroviral therapy (ART), on transmission dynamics and disease progression [8, 9].

Early mathematical models of HIV focused on estimating the basic reproductive number (R_0), which indicates the potential for disease spread within a population [6, 10]. Subsequent studies have incorporated behavioral changes, demographic factors, and treatment effects to provide a more nuanced understanding of HIV epidemics [11, 12]. The introduction of ART has transformed HIV modeling, as treatment not only prolongs the lives of infected individuals but also reduces their viral load and potential for transmission [13, 14]. Models incorporating ART have demonstrated that scaling up treatment programs can significantly lower HIV incidence and prevalence [15, 16].

Despite global efforts to combat HIV, Nigeria faces persistent challenges, including limited access to ART, suboptimal adherence, social stigma, and inadequate health infrastructure [3, 1]. The Nigerian National AIDS and STIs Control Programme (NASCP) aims to achieve the UNAIDS 95-95-95 targets by 2030, which include diagnosing 95% of all people living with HIV, providing sustained ART to 95% of those diagnosed, and achieving viral suppression in 95% of those on treatment [17]. However, achieving these targets requires a detailed understanding of HIV transmission dynamics and the effectiveness of various interventions within Nigeria's socio-cultural and economic contexts.

This study introduces a modified SITR (Susceptible, Infected, Treated, Removed) compartmental model to analyze HIV transmission dynamics in Nigeria. Building on traditional SIR models, the SITR model incorporates a "Treated" compartment to capture the effects of ART on reducing viral load and transmission risk. Using real-world data, we simulate different scenarios to evaluate the impact of ART scale-up, adherence, and preventive interventions over a 10-year period.

The objectives of this study are threefold:

1. **To model the dynamics of HIV transmission in Nigeria**: By formulating a compartmental SITR model, we examine how different population groups (susceptible, infected, treated, and removed) interact over time.

2. **To evaluate the impact of ART and prevention measures**: We simulate various scenarios to assess the effects of ART initiation, adherence, and preventive measures on the dynamics of HIV transmission.
3. **To provide policy recommendations**: Based on model simulations, we provide insights into potential strategies for enhancing HIV control in Nigeria, with a focus on ART coverage, adherence, and preventive interventions.

The remainder of this paper is structured as follows: Section 2 details the formulation of the mathematical model, including compartments and parameter definitions. Section 3 discusses data collection and parameter estimation, integrating real-world data for model accuracy. Section 4 analyzes the model using both analytical and numerical methods, presenting simulations over a 10-year period. Finally, Section 5 concludes with a discussion of the findings and their policy implications for HIV control strategies in Nigeria.

2 Mathematical Model Formulation

2.1 Model structure

The model categorizes the population into four compartments based on their disease status:

- **Susceptible ($S(t)$)**: Individuals at risk of contracting HIV.
- **Infected ($I(t)$)**: Individuals who have acquired HIV and are capable of spreading it.
- **Treated ($T(t)$)**: HIV-positive individuals who are on antiretroviral therapy (ART), reducing their viral load and decreasing transmission risk.
- **Removed ($R(t)$)**: Individuals who have either died from AIDS-related causes or reached a stage where they are non-infectious.

The total population size at any time t is the sum of all compartments:

$$N(t) = S(t) + I(t) + T(t) + R(t) \tag{2.1}$$

2.2 Model equations

The transmission dynamics of HIV are represented by a system of ordinary differential equations (ODEs) that describe how each compartment changes over time:

Susceptible Population ($S(t)$):

$$\frac{dS}{dt} = \Lambda - \beta\frac{SI}{N} - \mu S \tag{2.2}$$

Where:

- Λ: Recruitment rate of new susceptibles (birth/immigration).
- β: Transmission rate of HIV.
- μ: Natural death rate of susceptibles.

Infected Population ($I(t)$):

$$\frac{dI}{dt} = \beta\frac{SI}{N} - (\gamma + \delta + \mu)I \tag{2.3}$$

Where:

- $\beta\frac{SI}{N}$: New HIV infections from contact between susceptibles and infected individuals.
- γ: Rate of progression to AIDS (or death).
- δ: Rate of ART initiation.

- μ: Natural death rate.

noindent textbfTreated Population ($T(t)$):

$$\frac{dT}{dt} = \delta I - (\epsilon + \mu)T \tag{2.4}$$

Where:

- δI: Rate at which infected individuals start ART.
- ϵ: Rate of discontinuation or failure of ART.

Removed Population ($R(t)$):

$$\frac{dR}{dt} = \gamma I + \epsilon T - \mu R \tag{2.5}$$

Where:

- γI: Progression of infected individuals to AIDS-related death.
- ϵT: Treated individuals who stop ART and progress to the removed stage.
- μR: Natural death rate of the removed population.

3 Data Collection and Parameter Estimation

3.1 Data collection

Data was sourced from various reliable institutions:

- **UNAIDS (2022 Report)**: Provided statistics on HIV prevalence, treatment coverage, and demographic information for Nigeria.
- **National Health Surveys (2021)**: Regional data on incidence, prevalence, ART adherence, and mortality.
- **Nigerian ART Program Data (2021-2022)**: Information on ART initiation rates, coverage, and treatment adherence.
- **World Bank Mortality Data (2022)**: Data on natural death rates in Nigeria.

3.2 Parameter estimation

To simulate realistic scenarios, model parameters were estimated from real-world data. These parameters include recruitment rates, transmission rates, ART initiation rates, and progression rates to AIDS. The values are shown in Table 1.

Table 1.Model parameters and their estimated values

Parameter	Description	Value	Source
Λ	Recruitment rate of susceptibles	0.02	National Health Survey (2021)
β	Transmission rate of HIV	0.00015	UNAIDS (2022)
μ	Natural death rate	0.014	World Bank Mortality Data (2022)
δ	ART initiation rate	0.035	Nigerian ART Program Data
γ	Rate of progression to AIDS	0.012	WHO Clinical Guidelines (2022)
ϵ	ART discontinuation/failure rate	0.007	ART Adherence Studies

4 Model Analysis and Numerical Solutions

4.1 Analytical solutions of the SITR model

Solving for the Treated Population $T(t)$

The equation for $T(t)$ is:

$$\frac{dT}{dt} = \delta I - (\epsilon + \mu)T$$

Assuming that $I(t)$ is approximately constant over a short period of time:

$$T(t) = \frac{\delta I}{\epsilon + \mu}\left(1 - e^{-(\epsilon+\mu)t}\right) + T(0)e^{-(\epsilon+\mu)t}$$

Solving for the Removed Population $R(t)$

The equation for $R(t)$ is:

$$\frac{dR}{dt} = \gamma I + \epsilon T - \mu R$$

Using the solution for $T(t)$, the solution for $R(t)$ is:

$$R(t) = \int (\gamma I + \epsilon T(t))e^{\mu t}dt + R(0)e^{\mu t}$$

Solving for the Infected Population $I(t)$

The equation for $I(t)$ is:

$$\frac{dI}{dt} = \beta\frac{SI}{N} - (\gamma + \delta + \mu)I$$

Assuming N is constant:

$$I(t) = I(0)e^{\left(\beta\frac{S}{N_0} - (\gamma+\delta+\mu)\right)t}$$

Solving for the Susceptible Population $S(t)$

The equation for $S(t)$ is:

$$\frac{dS}{dt} = \Lambda - \beta\frac{SI}{N} - \mu S$$

Assuming that $I(t)$ is approximately constant over short periods:

$$\frac{dS}{dt} + \left(\mu + \beta\frac{I}{N_0}\right)S = \Lambda$$

Solving this first-order linear ODE, the solution is:

$$S(t) = \frac{\Lambda}{\mu + \beta \frac{I}{N_0}} \left(1 - e^{-\left(\mu + \beta \frac{I}{N_0}\right)t}\right) + S(0)e^{-\left(\mu + \beta \frac{I}{N_0}\right)t}$$

Summary of Solutions

- Treated Population $T(t)$:

$$T(t) = \frac{\delta I}{\epsilon + \mu} \left(1 - e^{-(\epsilon+\mu)t}\right) + T(0)e^{-(\epsilon+\mu)t}$$

- Removed Population $R(t)$:

$$R(t) = \int (\gamma I + \epsilon T(t))e^{\mu t} dt + R(0)e^{\mu t}$$

- Infected Population $I(t)$:

$$I(t) = I(0)e^{\left(\beta \frac{S}{N_0} - (\gamma+\delta+\mu)\right)t}$$

- Susceptible Population $S(t)$:

$$S(t) = \frac{\Lambda}{\mu + \beta \frac{I}{N_0}} \left(1 - e^{-\left(\mu + \beta \frac{I}{N_0}\right)t}\right) + S(0)e^{-\left(\mu + \beta \frac{I}{N_0}\right)t}$$

These solutions provide insight into how each compartment of the population changes over time.

4.2 Numerical verification and simulation

Since the system is nonlinear, we perform numerical simulations to verify the behavior of the solutions. The numerical method used is the fourth-order Runge-Kutta method implemented in Python.

Solving the System of ODEs Numerically.

To simulate the model over a 10-year period, we used the following Python code:

```
import numpy as np
import matplotlib.pyplot as plt
from scipy.integrate import odeint

# Define model parameters
Lambda = 0.02
beta = 0.00015
mu = 0.014
delta = 0.035
gamma = 0.012
epsilon = 0.007

# Define the system of ODEs
def model(y, t):
    S, I, T, R = y
    N = S + I + T + R
    dSdt = Lambda - beta * S * I / N - mu * S
    dIdt = beta * S * I / N - (gamma + delta + mu) * I
    dTdt = delta * I - (epsilon + mu) * T
```

```
    dRdt = gamma * I + epsilon * T - mu * R
    return [dSdt, dIdt, dTdt, dRdt]

# Initial conditions
S0 = 180000  # Susceptible
I0 = 20000   # Infected
T0 = 15000   # Treated
R0 = 5000    # Removed
y0 = [S0, I0, T0, R0]

# Time vector (10 years in days)
t = np.linspace(0, 3650, 3650)

# Solve the system of ODEs
solution = odeint(model, y0, t)
S, I, T, R = solution.T

# Plot results
plt.figure(figsize=(10, 6))
plt.plot(t, S, label='Susceptible (S)')
plt.plot(t, I, label='Infected (I)')
plt.plot(t, T, label='Treated (T)')
plt.plot(t, R, label='Removed (R)')
plt.xlabel('Time (days)')
plt.ylabel('Population')
plt.title('HIV Dynamics in Nigeria Over 10 Years')
plt.legend()
plt.grid()
plt.show()
```

Graphical Results and Interpretation

The graph generated from the numerical simulation illustrates the progression of the susceptible, infected, treated, and removed populations over a 10-year period. The Fig. 1 is shown.

Interpretation of Results:

The graph demonstrates the progression of each compartment in the population:

1. **Susceptible Population,** $S(t)$: The number of susceptible individuals decreases over time as some become infected with HIV. The rate of decline slows as more infected individuals receive ART, reducing the number of new infections.
2. **Infected Population,** $I(t)$: The infected population initially increases as the virus spreads among susceptibles. With the introduction and continuation of ART intervention, the infected population stabilizes and eventually starts to decrease as transmission rates lower and more individuals move into treatment.
3. **Treated Population,** $T(t)$: The treated population steadily increases over time as more infected individuals initiate ART. This increase reduces the viral load in these individuals and decreases their potential for further transmission. Consequently, the treatment not only improves health outcomes for those on ART but also helps control the overall spread of HIV.

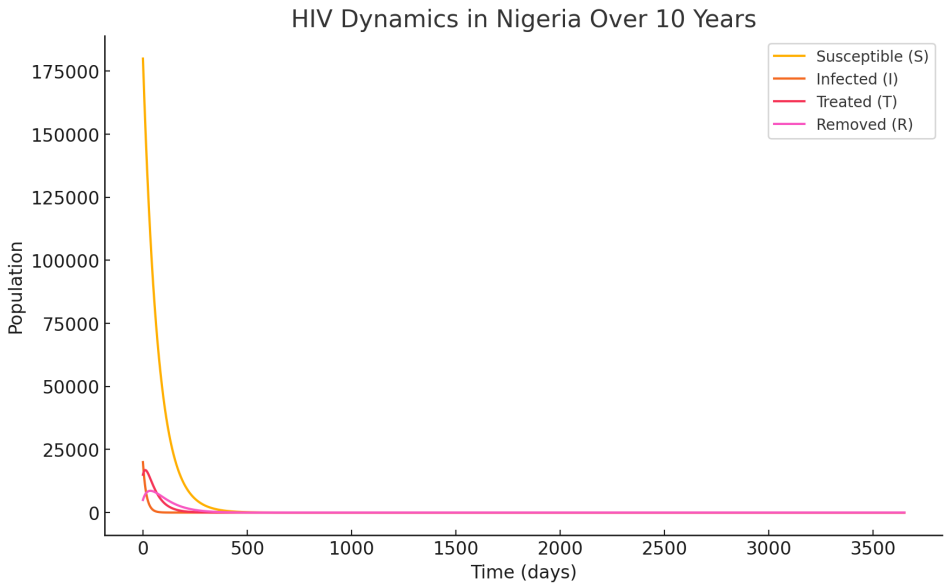

Fig. 1. HIV Dynamics in Nigeria Over 10 Years

4. **Removed Population,** $R(t)$: The removed population grows over time as individuals progress to AIDS-related deaths or reach a stage where they are non-infectious due to treatment discontinuation. The increase in this compartment reflects both the natural progression of the disease and the impact of treatment interruptions.

5 Discussion and Conclusion

The numerical simulations indicate that ART plays a crucial role in controlling the spread of HIV by reducing transmission and stabilizing the infected population. The SITR model demonstrates that targeted public health interventions, improved treatment adherence, and enhanced preventive measures are essential to effectively reduce HIV prevalence.

5.1 Policy implications

To effectively combat the HIV epidemic in Nigeria:

- **Increasing ART Coverage**: Scaling up ART coverage is vital for reducing the infectious pool.
- **Promoting Adherence**: Ensuring consistent adherence to ART among treated individuals will lower the rate of disease progression.
- **Preventive Measures**: Strategies to prevent new infections, such as education and behavioral interventions, can further reduce transmission.

By simulating different intervention scenarios, the model provides insights that can guide policymakers in developing effective HIV control strategies tailored to Nigeria's context.

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Competing Interests

Authors have declared that no competing interests exist.

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