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Modified Lorentz Transformations and Space-Time Splitting According to the Inverse Relativity Model

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Abstract

Analysis of a four-dimensional displacement vector on the fabric of space-time in the special or general case into two Four-dimensional vectors, according to specific conditions leads to the splitting of the total fabric of space-time into a positive subspace-time that represents the space of causality and a negative subspace-time which represents a space without causality, thus, in the special case, we have new transformations for the coordinates of space and time modified from Lorentz transformations specific to each subspace, where the contraction of length disappears and the speed of light is no longer a universal constant. In the general case, we have new types of matric tensor, one for positive subspace-time and the other for negative subspace-time. We also find that the speed of the photon decreases in positive subspace-time until it reaches zero and increases in negative subspace-time until it reaches the speed of light when the photon reaches the Schwarzschild radius.

Keywords

Four-Dimensional Vector Analysis, Four-Dimensional Subspace, Causal Space, Analysis of the Speed of Light, Inverse Theory of Relativity

1. Introduction

Lorentz transformations are defined as a set of equations that describe the transformations of space and time coordinates from one inertial reference frame to another while assuming the speed of light to be constant for both observers in the reference frames. It was suggested by the scientist Hendrik Lorentz in 1904 [1] [2]. But in 1908, Hermann Minkowski provided a geometric interpretation of the Lorentz transformations; he considered it to be a process of rotation in a four-dimensional space known as Minkowski space-time [3]. Lorentz transformations are the mathematical and physical basis for the theory of special relativity; it also paved the way to general relativity. These transformations have also revealed to us the properties of space and time, such as length contraction, time dilation, and the relativity of simultaneity, time as a fourth dimension, the speed of light is the maximum causal speed between two events in space-time. Lorentz transformations preserve the symmetry of the laws of physics in both reference frames [4]. But are there other properties or a more complex structure of space-time that Lorentz's transformations did not reach? Are there other types of symmetry that Lorentz's transformations violated? Is Minkowski space the only four-dimensional space that expresses causality? Is the speed factor the only factor to describe causality in a four-dimensional space?

The new model is based on the analysis approach for a four-dimensional vector on the fabric of space-time, although this approach is followed in special relativity, where the four-dimensional vector can be analyzed into vectors with fewer dimensions or into their components and represented in subspaces with dimensions less than Minkowski space [5] [6]. But this type of analysis of the vector is in both reference frames or with respect to both observers, as the vectors resulting from the analysis or components of the vector are subject to Lorentz transformations as well and lead to the same results for these transformations. The purpose of this type of analysis is to understand the four-dimensional vectors better, the study of physical phenomena on the fabric of space-time, such as the movement of a particle in spatial space or the study of the movement of a particle in the dimension of time [7] [8]. But the analysis of the four-dimensional vector on the fabric of space-time according to the new model has different conditions in the analysis process, which are explained in the following points.

• The analysis of a four-dimensional vector on the fabric of space-time is done into two four-dimensional vectors through the analysis of the components of the vector, and not the analysis of the four-dimensional vector into vectors of lower dimensions or into the components of the original vector, and its representation is in four-dimensional subspaces.

The analysis of the four-dimensional vector is with respect to one of the observers or with respect to one of the reference frames and not in both together. Therefore, the transformation of each new four-dimensional vector from the reference frame (the frame that contains the analysis) to another reference frame (the frame without analysis) is not subject to Lorentz transformations, because the new vectors have different properties from the original vectors.

Analysis of the four-dimensional vector according to the previous condition gives us the possibility to impose specific properties or values for one of the new vectors or an arbitrary transformation that is modified from the Lorentz transformations. As for the second vector resulting from the analysis process, its transformations are related to the transformation of the first vector that we imposed and the Lorentz transformation of the original vector. The new model is called the inverse relativity model because some of its results in subspaces are opposite to the results of special and general relativity in total space-time.

2. Methods

2.1. Lorentz Transformations and Breaking the Symmetry of the Structure of Spatial Space

We assume the existence of two inertial reference frames S and S' Cartesian coordinate system, each reference frame has an observer at the origin point O and O'. We also assume that the frame S' is moving at a uniform velocity V_S with respect to the frame S in the positive direction of the x-axis [9], as is shown in **Figure 1**.

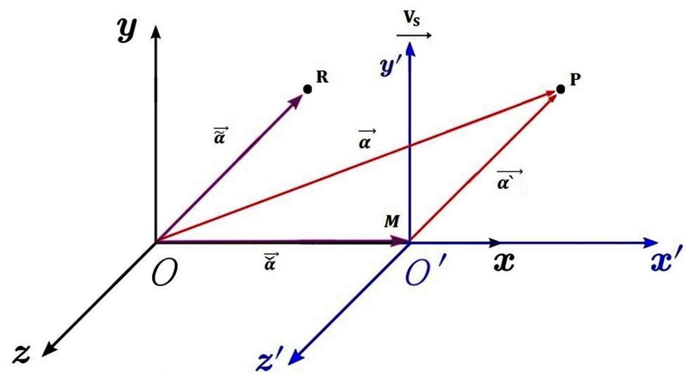


Figure 1. Shows the transformation of a 3D displacement vector from one reference frame to another with vector analysis in the reference frame S into two vectors.

$$S` \to x`\, y`\, z`\, t` \qquad S \to x\; y\; z\; t$$

While crossing the reference frame S' and at the moment when the frames S and S' coincide (that is, when O' coincides with O), where $x_0 = x`_0 = 0$ and the observers' clocks were also, $t_0 = t`_0 = 0$. An event began to occur in this frame, which is the emission of a photon from a light source located at the origin O',after a period of time Δt, the photon arrived at the point P in space, and the frame of reference S' arrived at the point M on the x-axis. Where each observer here observes the displacement vector of the photon with respect to his own reference frame, the observer O' observes the displacement vector with respect to the reference frame S' from the origin point O' to the point P, which is the three-dimensional position vector $\overrightarrow{\mathrm{O'P}}$ or $\overrightarrow{\alpha`}$ (See **Figure 1**), but by adding the time dimension, we get a new four-dimensional vector [10] [11], which is $A^{`\mu}\left(\overrightarrow{\alpha`}, ct`\right)$, where $\mu = 1,2,3,4$ and its components are $\left(A^{`1} = x`\right)$, $\left(A^{`2} = y`\right)$, $\left(A^{`3} = z`\right)$, $\left(A^{`4} = ct`\right)$, c is the speed of light in a vacuum, $t`$-the time taken by a photon along the vector, while the square magnitude of the vector according to Einstein's notation $s^{`2} = A^{`\mu} A^{`}_{\mu}$ and this is equal to

$$x^{`2} + y^{`2} + z^{`2} - c^2 t^{`2} = 0 \tag{1.1}$$

As for the observer O, he observes the displacement vector with respect to the reference frame S from the origin point O (where at the moment of emission, O' coincided with O) to the point P in space, which is the three-dimensional position vector $\overrightarrow{\mathrm{OP}}$ or $\vec{\alpha}$, by adding the time dimension here also, we obtain the four-dimensional vector $A^{\nu}\left(\vec{\alpha},ct\right)$ where $\nu=1,2,3,4$, and its components are $\left(A^1=x\right)$ $\left(A^2=y\right)$, $\left(A^3=z\right)\left(A^4=ct\right)$, *t*, the time taken by a photon along the vector. The square magnitude of the vector is $s^2=A^{\nu}A_{\nu}$ and is equal to

$$x^2+y^2+z^2-c^2t^2=0 \tag{2.1}$$

We note from Equations (1.1) and (2.1) that the speed of light is constant and does not change for the observers or for the two frames, which fulfills the second postulate of special relativity [12], therefore, the transformation between the previous two vectors is through Lorentz transformations. We also note that the two four-dimensional vectors $A^{\nu},A^{`\mu}$ are equal to the square of the magnitude, and therefore they are represented in a four-dimensional space known as Minkowski space-time, in which the square magnitude of the vector is a constant quantity between the reference frames, therefore, the transformation between the two vectors is through orthogonal transformation [13], so the Lorentz transformation is represented as a rotation of coordinates in a four-dimensional space through the Lorentz rotation matrix Λ^{μ}_{ν} [14].

$$A^{`\mu}=\Lambda^{\mu}_{\nu}A^{\nu} \tag{3.1}$$

where

$$\Lambda^{\mu}_{\nu}=\begin{bmatrix}\gamma & 0 & 0 & -\beta\gamma\\ 0 & 1 & 0 & 0\\ 0 & 0 & 1 & 0\\ -\beta\gamma & 0 & 0 & \gamma\end{bmatrix},\ \beta=\frac{V_s}{c},\ \gamma=\frac{1}{\sqrt{1-\frac{V_s^2}{c^2}}} \tag{4.1}$$

From the equality of vectors $A^{\nu},A^{`\mu}$ in the square magnitude, we obtain a transformation of the spatial and time periods that describe an event on Minkowski space-time.

$$\Delta x^2+\Delta y^2+\Delta z^2-c^2\Delta t^2=\Delta x^{`2}+\Delta y^{`2}+\Delta z^{`2}-c^2\Delta t^{`2} \tag{5.1}$$

From the inverse Lorentz transformation we also get, $\Delta y=\Delta y`$, $\Delta z=\Delta z`$

$$\Delta x^2-c^2\Delta t^2=\Delta x^{`2}-c^2\Delta t^{`2} \tag{6.1}$$

$$c^2\Delta t^2>c^2\Delta t^{`2} \tag{7.1}$$

$$\Delta x>\Delta x` \tag{8.1}$$

We conclude from this that although the spatial periods on the $yy`,zz`$ axes are constant, the difference in the time period on the time dimension from one frame of reference to another leads to a difference in the spatial period on the $xx`$ axis, even assuming $\Delta t`=0$, we find $\Delta x\neq\Delta x`$ due to length contraction, but in Galilean transformation, for example, we find $\Delta t=\Delta t`$, so when $\Delta t`=0$ we find that $\Delta x=\Delta x`$ according to the classical equation $\Delta x=\Delta x`+V_s\Delta t`$ That is, all the spatial and time periods of the event are constant under the transformation, even if the location of the event itself varies from one frame of reference to another. This means that the structure of the spatial space is symmetrical for both observers in the classical Galilean transformations, something that the Lorentz transformations violate in order to make the speed of light constant in both reference frames, as shown in Equation No. (5.1) [15].

2.2. Analysis of the Four-Dimensional Vector and Space-Time Splitting in the Special Case

To achieve symmetry in the structure of spatial space (*i.e.* all spatial periods are constant) when transferring from one inertial reference frame to another while maintaining the second postulate of special relativity. We analyze a four-dimensional vector to be the vector $\boldsymbol{A}^{\nu}$ in the reference frame S into two 4D vectors. So we can impose symmetry properties on one of them in spatial space, and this is done by analyzing the three-dimensional vector $\vec{\boldsymbol{\alpha}}$ in spatial space into two 3D vectors, which will result in analyzing the velocity on this vector as well. Thus, we analyze the time component *ct*, and thus we obtain a complete analysis of the four-dimensional vector, representing the new four-dimensional vectors in new spaces split from the Minkowski space. To distinguish each new space from the other and its transformations, we call the first the positive subspace-time and the second the negative subspace-time, this designation also has a connotation with the concept of causality. We will explain this in each naming item.

2.3. Modified Lorentz Transformations for Positive Subspace-Time

The first displacement vector resulting from the analysis is the three-dimensional position vector $\overrightarrow{\mathrm{OR}}$, to distinguish this component from the resultant vector, it is written in the form $\vec{\tilde{\alpha}}$ (See Figure 1). By adding the time dimension, we obtain the four-dimensional vector $\tilde{A}^{\epsilon}\left(\vec{\tilde{\alpha}},\tilde{V}\tilde{t}\right)$ where $\epsilon=1,2,3,4$ and its components $\left(\tilde{A}^{1}=\tilde{x}\right)$, $\left(\tilde{A}^{2}=\tilde{y}\right)$, $\left(\tilde{A}^{3}=\tilde{z}\right)$, $\left(\tilde{A}^{4}=\tilde{V}\tilde{t}\right)$, where $\tilde{V}$ is the velocity of the photon on the vector $\vec{\tilde{\alpha}}$ and $\tilde{t}$, the time taken by a photon along the vector, and the square magnitude of the vector is $\tilde{s}^{2}=\tilde{A}^{\epsilon}\tilde{A}_{\epsilon}$, and it is equal.

$$\tilde{x}^{2}+\tilde{y}^{2}+\tilde{z}^{2}-\tilde{V}^{2}\tilde{t}^{2}=0 \tag{9.1}$$

We find from Equations (1.1) and (9.1) that the four-dimensional vectors $\tilde{A}^{\epsilon}, A^{`\mu}$ are equal in the square of magnitude, and therefore they can be represented in a new four-dimensional space called positive space-time. It is subspace because it is derived from the total space-time, Minkowski space-time.

$$\tilde{x}^{2}+\tilde{y}^{2}+\tilde{z}^{2}-\tilde{V}^{2}\tilde{t}^{2}=x^{`2}+y^{`2}+z^{`2}-c^{2}t^{`2} \tag{10.1}$$

In order for the vector $\vec{\tilde{\alpha}}$ to fulfill the symmetry condition, we impose the following properties on it: it is equal in magnitude and parallel in direction to the vector $\overrightarrow{\alpha`}$ in spatial space, meaning that the vector $\vec{\tilde{\alpha}}$ is identical to $\overrightarrow{\alpha`}$. Therefore, the vector $\vec{\tilde{\alpha}}$ is called the identical vector. As a result of the properties that we imposed for this vector, the components of both vectors $\overrightarrow{\alpha`}$ and $\vec{\tilde{\alpha}}$ are equal in spatial space with the transformation from one frame of reference to another.

$$\tilde{x} = x` \tag{11.1}$$

$$\tilde{y} = y` \tag{12.1}$$

$$\tilde{z} = z` \tag{13.1}$$

As for the time transformation equation, we find in the inverse Lorentz transformations two forms [16] of the transformation equation.

$$t = \gamma\left(t` + \frac{V_s x`}{c^2}\right) \text{ or } t = \gamma t` \tag{14.1}$$

The first formula represents the time transformation for a particle moving along the $x`$-axis, and the second formula we obtain when $x` = 0$, it represents the time transformation for a particle moving along the $y`, z`$ axes. This means that the time transformation equation for a moving particle varies from one spatial dimension to another in special relativity. Because the spatial coordinate transformation equations that we imposed above have the same mathematical formula, therefore the time transformation equation $\tilde{t}$ must also have the same mathematical formula with respect to the dimensions of spatial space. In other words, the mathematical formula of the time transformation equation must not change from one spatial dimension to another, as in Lorentz transformations. So we assume the following transformation for it.

$$\tilde{t} = \gamma t` \tag{15.1}$$

By substituting from the set of Equations (11.1)-(13.1), in (10.1), we obtain the transformation of the time dimension. We conclude from this that the stability of the spatial components under transformation necessarily leads to the stability of the component of time.

$$\tilde{V}\tilde{t} = ct` \tag{16.1}$$

The set of Equations (11.1)-(13.1), (15.1) are called inverse modified Lorentz transformations for positive subspace-time. Since all components in positive subspace-time are constant, therefore we can express the transformation from the vector $A`^{\mu}$ to the vector $\tilde{A}^{\epsilon}$ in positive subspace-time through a neutral matrix, which is the unit matrix, that is, $\tilde{\Lambda}^{\epsilon}_{\mu} = \mathrm{I}_{4\times4}$

$$\tilde{A}^{\epsilon} = \tilde{\Lambda}^{\epsilon}_{\mu} A`^{\mu} \tag{17.1}$$

$$\tilde{\Lambda}^{\epsilon}_{\mu} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{18.1}$$

As for the modified Lorentz transformations for the same event in positive subspace-time, it is done by analyzing the three-dimensional vector $\overrightarrow{\alpha}`$ and the four-dimensional vector $A`^{\mu}$ with respect to the reference frame S' into two vectors in order to obtain the three-dimensional identical vector $\overrightarrow{\tilde{\alpha}}`$ and the four-dimensional $\tilde{A}`^{\delta}$ with respect to the reference frame S'. By following the same previous steps, the transformation from the vector A^{ν} to the vector $\tilde{A}`^{\delta}$ is done through the neutral transformation matrix $\tilde{\Lambda}^{\delta}_{\nu} = \mathrm{I}_{4\times4}$, and thus the transformations are written in the following two forms:

$$\tilde{x}` = x \qquad \Delta\tilde{x}` = \Delta x \tag{19.1}$$

$$\tilde{y}` = y \qquad \Delta\tilde{y}` = \Delta y \tag{20.1}$$

$$\tilde{z}` = z \qquad \Delta\tilde{z}` = \Delta z \tag{21.1}$$

$$\tilde{t}` = \gamma t \qquad \Delta\tilde{t}` = \gamma\Delta t \tag{22.1}$$

The modified and inverse modified Lorentz transformations are characterized by the symmetry of the structure of four-dimensional space (positive subspace-time) and three-dimensional space (positive spatial subspace) for both observers, we find that all spatial and time periods remain constant under transformation.It represents the transformation of a four-dimensional vector from one inertial reference frame to another while preserving the geometric properties of the vector such as length and direction in spatial space. What concerns us here is the symmetry of the structure of spatial space, as the symmetry of the structure of spatial space with the modified transformations leads to the symmetry of the laws of physics as well with these transformations, that is, we must use the same mathematical formulas for the laws of physics with the modified Lorentz transformation as well. This is a commitment to the principle of special relativity, but in positive subspace-time.

Substituting from (15.1) into (16.1)

$$\tilde{V} = c\gamma^{-1} \tag{23.1}$$

This means that the observer O observes the speed of light decreasing along the vector $\vec{\tilde{\alpha}}$ or in positive subspace-time with increasing V_s, even though both vectors $\vec{\alpha`}$ and $\vec{\tilde{\alpha}}$ have the same length (event path), or in a spatial space with symmetrical structure, But time dilation causes the speed of light to slow down along the path of the event. In other words, if the speed of light in Minkowski space-time is constant for all observers in exchange for the contraction of length and the dilation of time, then we find here in positive subspace-time that the structure of spatial space is symmetrical for all observers in exchange for the dilation of time and the reduction of the speed of light. That is, the speed of light is not a universal constant in positive subspace-time, which is an opposite result of special relativity.

To understand positive subspace-time more deeply and the importance of the symmetry of the structure of spatial space here, we assume the occurrence of an event other than the emission of a photon in the reference frame S', such as an elastic collision between a photon and an electron at the microscopic level, or between two particles of similar mass and speed at the macroscopic level. We represent this event through two vectors whose intersection point represents the collision point, as shown in **Figure 2**. This event expresses physical causation, where each particle causes a change in the direction of the other particle and also changes the physical quantities of the other body in the event that the particles before the collision were unequal in mass and speed. As a result of the symmetry of the structure of spatial space for all observers in positive space-time, this physical causality is also symmetry, In other words, any causality that occurs between two vectors in the observer's spatial space O' also occurs between their identical vectors in the positive spatial subspace of the observer O. Therefore, the physical quantities on this subspace-time are positively affected by the causality of the event or the behavior of the physical phenomenon, so we call it positive space-time.

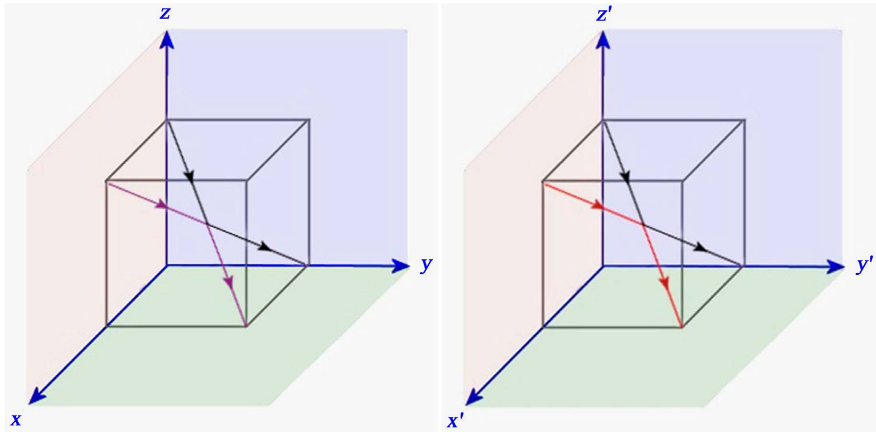


Figure 2. The right side shows a collision between two particles in the 3D space of observer O', and the left side shows the same collision in the positive 3D subspace of observer O.

Through this example, we can provide a mathematical, geometric, physical definition of positive space-time. Mathematically, it is a four-dimensional subspace consisting of three spatial dimensions and a fourth time dimension splintered from the total space-time. This space results from positive modified Lorentz transformations or positive modified inverse Lorentz transformations. Geometrically, it is the space of oblique vectors, or intersecting paths. Physically it is the space of causality in which the laws of physics appear in fixed or symmetrical mathematical formulas for all observers.

2.4. Modified Lorentz Transformations for Negative Subspace-Time

The second displacement vector resulting from the analysis process is the three-dimensional position vector $\overrightarrow{\mathrm{OM}}$, To distinguish this component also from the resultant vector, it is written in the formula $\breve{\vec{\alpha}}$ (See **Figure 1**), By adding the time dimension, we obtain the four-dimensional vector $\breve{A}^{\sigma}\left(\breve{\vec{\alpha}},\breve{V}\breve{t}\right)$, where $\sigma = 1,2,3,4$, and its components $\left(\breve{A}^1 = \breve{x}\right)$, $\left(\breve{A}^2 = \breve{y}\right)$, $\left(\breve{A}^3 = \breve{z}\right)$, $\left(\breve{A}^4 = \breve{V}\breve{t}\right)$, where $\breve{V}$ is the velocity of the photon on the vector $\breve{\vec{\alpha}}$ and $\breve{t}$, the time taken by a photon along the vector. The square magnitude of the vector is $\breve{s}^2 = \breve{A}^{\sigma}\breve{A}_{\sigma}$, and it is equal

$$\breve{x}^2 + \breve{y}^2 + \breve{z}^2 - \breve{V}^2\breve{t}^2 = 0 \tag{24.1}$$

From Equations (1.1) and (24.1), we find that the four-dimensional vectors $\breve{A}^{\sigma}, \grave{A}^{\mu}$ are also equal to the square of magnitude, and therefore they can be represented in another new four-dimensional space called negative subspace-time, which is also a subspace from Minkowski's total space-time.

$$\breve{x}^2+\breve{y}^2+\breve{z}^2-\breve{V}^2\breve{t}^2=x^{`2}+y^{`2}+z^{`2}-c^2t^{`2} \tag{25.1}$$

In order to obtain negative subspace-time coordinate transformations, we must obtain the transformation matrix from the vector $A^{`\mu}$ to the vector $\breve{A}^{\sigma}$. From the drawing we conclude that the vector $\vec{\breve{\alpha}}$ is the product of subtracting the vector $\vec{\tilde{\alpha}}$ from the vector $\vec{\alpha}$ in three-dimensional space, and by generalizing this relationship in four-dimensional space, we can write the vector $\breve{A}^{\sigma}$ as the product of subtracting the vector $\tilde{A}^{\epsilon}$ from the vector A^{ν}

$$\breve{A}^{\sigma}=A^{\nu}-\tilde{A}^{\epsilon} \tag{26.1}$$

We can replace the vector $\tilde{A}^{\epsilon}$ in terms of vector $A^{`\mu}$ from Equation (17.1). However, replacing vector $A^{\nu}$ in terms of vector $A^{`\mu}$ is through the matrix $\bar{\Lambda}^{\nu}_{\mu}$, which is the inverse of the matrix $\left[\Lambda^{\mu}_{\nu}\right]^{-1}$ shown in Equation (4.1)

$$\breve{A}^{\sigma}=\bar{\Lambda}^{\nu}_{\mu}A^{`\mu}-\tilde{\Lambda}^{\epsilon}_{\mu}A^{`\mu} \tag{27.1}$$

$$\breve{A}^{\sigma}=\left[\bar{\Lambda}^{\nu}_{\mu}-\tilde{\Lambda}^{\epsilon}_{\mu}\right]A^{`\mu} \tag{28.1}$$

Where the result of subtracting the two matrices represents a new matrix $\breve{\Lambda}^{\sigma}_{\mu}$, which is the required transformation matrix.

$$\breve{A}^{\sigma}=\breve{\Lambda}^{\sigma}_{\mu}A^{`\mu} \tag{29.1}$$

By substituting from (4.1), (18.1), into (30.1)

$$\bar{\Lambda}^{\nu}_{\mu}-\tilde{\Lambda}^{\epsilon}_{\mu}=\begin{bmatrix}\gamma & 0 & 0 & \beta\gamma\\ 0 & 1 & 0 & 0\\ 0 & 0 & 1 & 0\\ \beta\gamma & 0 & 0 & \gamma\end{bmatrix}-\begin{bmatrix}1 & 0 & 0 & 0\\ 0 & 1 & 0 & 0\\ 0 & 0 & 1 & 0\\ 0 & 0 & 0 & 1\end{bmatrix} \tag{30.1}$$

$$\breve{\Lambda}^{\sigma}_{\mu}=\begin{bmatrix}\gamma-1 & 0 & 0 & \beta\gamma\\ 0 & 0 & 0 & 0\\ 0 & 0 & 0 & 0\\ \beta\gamma & 0 & 0 & \gamma-1\end{bmatrix} \tag{31.1}$$

By substituting from (31.1), into (29.1)

$$\begin{bmatrix}\breve{A}^1\\ \breve{A}^2\\ \breve{A}^3\\ \breve{A}^4\end{bmatrix}=\begin{bmatrix}\gamma-1 & 0 & 0 & \beta\gamma\\ 0 & 0 & 0 & 0\\ 0 & 0 & 0 & 0\\ \beta\gamma & 0 & 0 & \gamma-1\end{bmatrix}\cdot\begin{bmatrix}A^{`1}\\ A^{`2}\\ A^{`3}\\ A^{`4}\end{bmatrix} \tag{32.1}$$

Multiplying the vector $A^{`\mu}$ by the matrix $\breve{\Lambda}^{\sigma}_{\mu}$, we obtain the equations for the transformation of the components of the vector $\breve{A}^{\sigma}$

$$\breve{A}^1=(\gamma-1)A^{`1}+\beta\gamma A^{`4} \tag{33.1}$$

$$\breve{A}^2=0 \tag{34.1}$$

$$\breve{A}^3=0 \tag{35.1}$$

$$\breve{A}^4=\beta\gamma A^{`1}+(\gamma-1)A^{`4} \tag{36.1}$$

By substituting the value of each component of the vector into the coordinate form in Equation (33.1)

$$\breve{x} = (\gamma - 1)x` + \beta\gamma ct` \tag{37.1}$$

The previous equation can be reduced to the following formula

$$\breve{x} = \gamma\left(x`\left(1 - \gamma^{-1}\right) + V_s t`\right) \tag{38.1}$$

By substituting the value of each component of the vector into the coordinate form in Equations (34.1), (35.1)

$$\breve{y} = 0 \tag{39.1}$$

$$\breve{z} = 0 \tag{40.1}$$

By substituting the value of each component of the vector into the coordinate form in Equation (36.1)

$$\breve{V}\breve{t} = \beta\gamma x` + (\gamma - 1)ct` \tag{41.1}$$

$$\breve{V}\breve{t} = \gamma\frac{V_s x`}{c} + ct`\gamma - ct` \tag{42.1}$$

Assuming that the photon is moving at full speed along the $xx`$-axis, in this case we can write the velocity on the vector $\vec{\breve{\alpha}}$ in terms of the velocity on each of the vectors $\vec{\tilde{\alpha}}, \vec{\alpha}$ as an analysis of parallel velocity vectors.

$$\left(c - \tilde{V}\right)\breve{t} = \gamma\frac{V_s x`}{c} + ct`\gamma - ct` \tag{43.1}$$

$$c\breve{t} = \gamma\frac{V_s x`}{c} + ct`\gamma - ct` + \tilde{V}\breve{t} \tag{44.1}$$

But $\tilde{V}\breve{t} = \tilde{V}\tilde{t}$ because the times are in the same frame of reference and for the same event by substituting from Equation (16.1) and rearranging the equation

$$\breve{t} = \gamma\left(t` + \frac{V_s x`}{c^2}\right) \tag{45.1}$$

The set of Equations (38.1)-(40.1), (45.1) represent the inverse modified Lorentz transformations for negative subspace-time. They also represent the transformation of a four-dimensional vector from one inertial reference frame to another with a change in direction in the spatial space to make it parallel to the direction of motion of the reference frame, so the vector $\vec{\breve{\alpha}}$ is called a parallel vector, as shown in the **Figure 1**. As for the modified Lorentz transformations for negative subspace-time, they represent the transformation from the vector A^{ν} to the vector $\breve{A}^{`\gamma}$ through the transformation matrix $\breve{\Lambda}^{`\gamma}_{\nu}$, where the vector $\vec{\breve{\alpha}}`$ or $\breve{A}^{\gamma}$ is the second component is the result of analyzing the vector $\vec{\alpha}`$ or $A^{`\mu}$ into the two vectors in the reference frame S' as we mentioned above. By following the same previous steps, we obtain the modified Lorentz transformations for negative subspace-time with the following two formulas

$$\breve{x}` = \gamma\left(x\left(1-\gamma^{-1}\right)-V_s t\right) \qquad \Delta\breve{x}` = \gamma\left(\Delta x\left(1-\gamma^{-1}\right)-V_s \Delta t\right) \tag{46.1}$$

$$\breve{y}` = 0 \qquad \Delta\breve{y}` = 0 \tag{47.1}$$

$$\breve{z}` = 0 \qquad \Delta\breve{z}` = 0 \tag{48.1}$$

$$\breve{t}` = \gamma\left(t - \frac{V_s x}{c^2}\right) \qquad \Delta\breve{t}` = \gamma\left(\Delta t - \frac{V_s \Delta x}{c^2}\right) \tag{49.1}$$

To obtain the velocity of the photon on the vector $\vec{\breve{\alpha}}$ or in the spatial space of negative subspace-time, we use Equations (38.1), (45.1) of the inverse modified Lorentz transformations, but in the differential form.

$$\mathrm{d}\breve{x} = \gamma\left(\mathrm{d}x`\left(1-\gamma^{-1}\right)+V_s \mathrm{d}t`\right) \tag{50.1}$$

$$\mathrm{d}\breve{t} = \gamma\left(\mathrm{d}t` + \frac{V_s \mathrm{d}x`}{c^2}\right) \tag{51.1}$$

By dividing the distance equation by the time equation

$$\frac{\mathrm{d}\breve{x}}{\mathrm{d}\breve{t}} = \frac{\gamma\left(\mathrm{d}x`\left(1-\gamma^{-1}\right)+V_s \mathrm{d}t`\right)}{\gamma\left(\mathrm{d}t` + \frac{V_s \mathrm{d}x`}{c^2}\right)} \tag{52.1}$$

By dividing both the numerator and denominator in the equation by $\mathrm{d}t`$

$$\breve{V}_x = \frac{V_x^{`}\left(1-\gamma^{-1}\right)+V_s}{1+\frac{V_s V_x^{`}}{c^2}} \tag{53.1}$$

If the velocity of the reference frame is much less than the speed of light $V_s \ll c$, in this case the value of the inverse of the Lorentz factor is very close to the correct one $\gamma^{-1} \approx 1$ and therefore its effect is very slight and can be neglected. We also find that the amount $V_x^{`} V_s / c^2$ is very small and can also be neglected, by substituting for this in the previous equation, we get

$$\frac{V_s V_x^{`}}{c^2} \approx 0 \qquad V_x^{`}\left(1-\gamma^{-1}\right) \approx 0 \qquad \breve{V}_x = V_s \tag{54.1}$$

But if the velocity of the reference frame is equal to the speed of light, and that is from a theoretical point of view only, in this case we find that. $\gamma^{-1} = 0$, by substituting this in the previous equation, we get

$$\breve{V}_x = \frac{V_x^{`} + c}{1+\frac{V_x^{`} c}{c^2}} \qquad \breve{V}_x = \frac{V_x^{`} + c}{\frac{c}{c}+\frac{V_x^{`}}{c}} \qquad \breve{V}_x = c = V_s \tag{55.1}$$

We conclude from Equations (54.1) and (55.1) that the speed $\breve{V}_x$ is always equal to the speed of the reference frame V_s regardless of the value of $V_x^{`}$, even assuming values for the speed such as $V_x^{`} = 0$ by substituting in Equation (53.1) or $V_x^{`} = c$ by substituting in Equation (55.1); we will get the same result. This means that the observer O observes the speed of the photon on the vector $\vec{\breve{\alpha}}$, or in negative spatial space, always equal to the speed of the reference frame.

So, the inverse Lorentz transformations modified for negative subspace-time show us that all velocity vectors that occur in the spatial space of the observer O' are represented in the negative spatial space of the observer O by vectors that are uniform in the direction, which is the positive direction of the x-axis, and also uniform in the velocity magnitude, which is the velocity magnitude of the reference frame. That is, the velocity vectors in the spatial space of negative subspace-time are always parallel in direction and equal in magnitude to each other with respect to the observer O. Therefore, there are no points of intersection or connection between the vectors in this space, which means that there is no collision or physical causality in negative subspace-time. This means that the physical quantities in negative subspace-time are the quantities that are not affected by causality or affect the behavior of the physical phenomenon, that is, the negative effect. Therefore, we call it negative subspace-time. When representing the previous collision event in negative subspace-time, we find that the velocity vectors $\breve{V}_x$ for each particle have the same the direction and its magnitude along the velocity vector $\tilde{V}$, as shown in **Figure 3**.

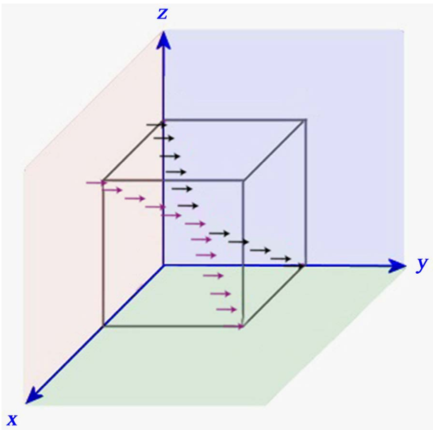


Figure 3. Shows a collision between two particles in the negative 3D subspace of the observer O.

Here we can also provide a mathematical, geometric, physical definition of negative subspace-time. Mathematically, it is a four-dimensional subspace consisting of three spatial dimensions and a fourth time dimension splintered from the total space-time. This space results from the negative modified Lorentz transformations or the negative modified inverse Lorentz transformations. Geometrically, it is the space of parallel vectors in which the velocity vectors remain without changing their direction or magnitude. Physically, it is a non-causal space in which no (causal) collisions occur between particles. It is the space of physical quantities that do not affect causality or the form of the physical law.

2.5. Splitting of the Fabric of Space-Time in the General Case

If the previous reference frames S and S' were non-inertial, *i.e.* acceleration or rotation [9]. Therefore, the transformation from vector $A^{`k} e`_k$ in the reference frame S' to vector $A^{\mu} e_{\mu}$ in the reference frame S is through the tensor transformation [17] [18] (the index here $k = \mu = 0,1,2,3$), where both vectors have the same magnitude. If the vector $A^{`k}e_{`k}$ represents a vector on a flat total space-time fabric in terms of spherical coordinates, then its magnitude is equal to multiplying the vector by itself, which represents the line element of space-time $\mathrm{d}s^{`2}$, is equal to

$$\mathrm{d}s^{`2} = g_{kl}^{`}\mathrm{d}A^{`k}\mathrm{d}A^{`l} \tag{56.1}$$

where $\mathrm{d}A^{`k}\mathrm{d}A^{`l}$ is contravariant components of the vector, $g_{kl}^{`}$ the metric tensor of flat space-time in terms of spherical coordinates, and by using the tensor transformation we get the same previous magnitude in terms of the contravariant components $\mathrm{d}A^{\mu}\mathrm{d}A^{\nu}$ of the vector $\mathrm{d}A$, which represents a vector on a total curved space-time fabric under the influence of mass, see **Figure 4(a)**.

$$\mathrm{d}s^{`2} = g_{kl}^{`}\frac{\partial A^{`k}}{\partial A^{\mu}}\mathrm{d}A^{\mu}\frac{\partial A^{`l}}{\partial A^{\nu}}\mathrm{d}A^{\nu} \tag{57.1}$$

$$\mathrm{d}s^{`2} = g_{kl}^{`}\left(\frac{\partial A^{`k}}{\partial A^{\mu}}\cdot\frac{\partial A^{`l}}{\partial A^{\nu}}\right)\mathrm{d}A^{\mu}\mathrm{d}A^{\nu} \tag{58.1}$$

$$g_{\mu\nu} = g_{kl}^{`}\left(\frac{\partial A^{`k}}{\partial A^{\mu}}\cdot\frac{\partial A^{`l}}{\partial A^{\nu}}\right) \tag{59.1}$$

Thus, we obtain $g_{\mu\nu}$ the metric tensor of the total space-time in the general case or in general relativity, which describes the curvature of the total space-time under the influence of mass [19]. To achieve symmetry of the structure of the spatial space with respect to all observers or reference frames in the general case, we perform the same previous analysis process for the vector $\mathrm{d}A$ on the curved fabric of space-time into two vectors, and thus we also obtain a split in the total curved space-time fabric, or in the general case, into positive and negative subspace-time.

2.6. The Metric Tensor of Positive Subspace-Time

Before obtaining the metric tensor of positive subspace-time in the general case, we must first obtain the metric tensor in the special case because this is a four-dimensional space split from Minkowski space-time. Therefore, the line element on this space in the special case will be exactly the same as the line element of Minkowski space-time, which is the line element of a flat or level space. So it is written in the following form [20]

$$\mathrm{d}\tilde{s}^2 = \mathrm{d}\tilde{x}^2 + \mathrm{d}\tilde{y}^2 + \mathrm{d}\tilde{z}^2 - \tilde{V}^2\mathrm{d}\tilde{t}^2 \tag{60.1}$$

In the tensor form, it is

$$\mathrm{d}\tilde{s}^2 = \eta_{\epsilon\tau}\mathrm{d}\tilde{A}^{\epsilon}\mathrm{d}\tilde{A}^{\tau} \tag{61.1}$$

$$\eta_{\epsilon\tau} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -1 \end{bmatrix} \tag{62.1}$$

$\eta_{\epsilon\tau}$ represents the metric tensor of positive subspace-time in the special case, and it represents a flat space-time where we find $\eta_{\epsilon\tau} = \delta_{ij}$ (δ_{ij} Delta Kronecker), therefore, it corresponds to the Minkowski space-time metric tensor, that is, $\eta_{\epsilon\tau} = \eta_{\mu\nu}$. We can rewrite the previous positive subspace-time metric in Cartesian coordinate $\left(\tilde{x}, \tilde{y}, \tilde{z}, \tilde{V}\tilde{t}\right)$ in terms of spherical coordinates $\left(\tilde{V}\tilde{t}, \tilde{r}, \tilde{\theta}, \tilde{\varnothing}\right)$, with the change of the index of the time dimension component from $\left(\tilde{A}^4 = \tilde{V}\tilde{t}\right)$ to $\left(\tilde{A}^0 = \tilde{V}\tilde{t}\right)$, so that this fits with the index used in general relativity equations. The positive subspace-time metric in the special case with spherical coordinates is written in the following formula

$$\mathrm{d}\tilde{s}^2 = -\tilde{V}^2\mathrm{d}\tilde{t}^2 + \mathrm{d}\tilde{r}^2 + \tilde{r}^2\mathrm{d}\tilde{\theta}^2 + \tilde{r}^2\sin^2\left(\tilde{\theta}\right)\mathrm{d}\tilde{\varnothing}^2 \tag{63.1}$$

Where the metric tensor matrix of positive subspace-time flat in terms of spherical coordinates, it is as follows

$$\tilde{g}_{\epsilon\tau} = \begin{bmatrix} -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & \tilde{r}^2 & 0 \\ 0 & 0 & 0 & \tilde{r}^2\sin^2\left(\tilde{\theta}\right) \end{bmatrix} \qquad \epsilon = \tau = 0,1,2,3 \tag{64.1}$$

As for the metric tensor of positive subspace-time in the general case, it expresses the transformation from the vector $\mathrm{d}A^{`}$ on a flat total space-time fabric to the vector $\mathrm{d}\tilde{A}$ as one of the vectors resulting from the analysis of the vector $\mathrm{d}A$ on the total curved space-time fabric, through the tensor transformation and following the same previous steps from Equation (56.1) to Equation (59.1), we get.

$$\tilde{g}_{\epsilon\tau} = g^{`}_{kl}\left(\frac{\partial A^{`k}}{\partial\tilde{A}^{\epsilon}}\frac{\partial A^{`l}}{\partial\tilde{A}^{\tau}}\right) \tag{65.1}$$

Because we also want in the general case, to maintain the symmetry of the structure of spatial space with respect to all observers or in both frames of reference, but the stability of the spatial components necessarily leads to the stability of the time component, we have previously explained this in the special case above.Therefore, we assume here also that all contravariant compounds of the vector $\mathrm{d}A^{`}$ are equal to the contravariant components of the vector $\mathrm{d}\tilde{A}$, and therefore.

$$\left(\frac{\partial A^{`k}}{\partial\tilde{A}^{\epsilon}}\frac{\partial A^{`l}}{\partial\tilde{A}^{\tau}}\right) = 1 \tag{66.1}$$

$$\tilde{g}_{\epsilon\tau} = g^{`}_{kl} \tag{67.1}$$

This means that the metric tensor of positive subspace-time in the general case is similar to the special case, and therefore Equation (64.1), represents the metric tensor of positive subspace-time in both the special and general case, meaning we can generalize Equation (63.1) and use it in the general case as well, Look at **Figure 4(b)**. As a result of the equality of the time components between the two linear elements $\mathrm{d}s^{`2} = \mathrm{d}\tilde{s}^2$, we get the following equation.

$$-\tilde{V}^2\mathrm{d}\tilde{t}^2 = -c^2\mathrm{d}\tau^2 \tag{68.1}$$

$$\tilde{V}\mathrm{d}\tilde{t} = c\,\mathrm{d}\tau \tag{69.1}$$

where $\mathrm{d}\tau$ is the change in proper time to a body moving along the total space-time fabric, $\mathrm{d}\tilde{t}$ the change in coordinate time on the fabric of positive subspace-time observed by an observer far from the source of gravity, $\tilde{V}$ the velocity of the photon on the fabric of positive subspace-time, and because the time $\mathrm{d}\tilde{t}$ and the time $\mathrm{d}t$ are in the same reference frame S and for the same event. So we will assume here also that the time transformation is similar to the time transformation in general relativity [21], and it has the following transformation

$$\mathrm{d}\tilde{t} = \frac{\mathrm{d}\tau}{\sqrt{1 - \frac{2MG}{\tilde{r}c^2}}} \tag{70.1}$$

where M is the mass of the body causing gravity, G is Gravitational constant, $\tilde{r}$ the distance of the body from a source of gravity, and by dividing Equation (69.1) by Equation (70.1)

$$\tilde{V} = c\sqrt{1 - \frac{2MG}{\tilde{r}c^2}} \tag{71.1}$$

Equation (71.1) shows us that the observer who is far from the source of gravity observes the velocity of a photon moving along the coordinate $\tilde{\boldsymbol{r}}$ in the fabric of positive subspace-time, decreasing with the decrease in the radius $\tilde{\boldsymbol{r}}$ or when the photon approaches the source of gravity until it reaches zero, when the photon reaches the Schwarzschild radius.

2.7. The Metric Tensor of Negative Subspace-Time

As for obtaining the metric tensor for negative subspace-time in the special case, because it is also a four-dimensional space split from Minkowski space-time, therefore the metric of this space in the special case is also similar to the metric of Minkowski space-time or the line element of flat space, so it is written in the same previous formula in Equation (60.1)

$$\mathrm{d}\breve{s}^2 = \mathrm{d}\breve{x}^2 + \mathrm{d}\breve{y}^2 + \mathrm{d}\breve{z}^2 - \breve{V}^2\mathrm{d}\breve{t}^2 \tag{72.1}$$

In the tensor form, it is

$$\mathrm{d}\breve{s}^2 = \eta_{\sigma\rho}\mathrm{d}\breve{A}^\sigma \mathrm{d}\breve{A}^\rho \tag{73.1}$$

$$\eta_{\sigma\rho} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -1 \end{bmatrix} \text{ or } \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 \end{bmatrix} \tag{74.1}$$

$\eta_{\sigma\rho}$ represents the metric tensor of negative subspace-time in the special case, and it expresses a flat space-time, where we also find $\eta_{\sigma\rho} = \delta_{ij}$ and it also corresponds with the metric tensor of Minkowski space-time, that is, $\eta_{\sigma\rho} = \eta_{\mu\nu}$. Because the direction of the vector in the spatial space of negative subspace-time is always parallel to the motion of the reference frame, therefore the first matrix in Equation (74.1) represents the metric tensor of negative subspace-time resulting from the motion of the reference frames relative to each other along three axes, but when the relative motion of the reference frames is on only one axis, for example the $xx`$ axis, as we assumed above, the metric tensor of negative subspace-time is expressed through the second matrix in the same equation.

To obtain the metric tensor for negative subspace-time in the general case, we take the inverse of the metric tensor of the total space-time curved under the influence of mass shown in Equation (59.1).

$$g^{\mu\nu} = g^{`kl}\left(\frac{\partial A^{\mu}}{\partial A^{`k}}\frac{\partial A^{\nu}}{\partial A^{`l}}\right) \tag{75.1}$$

By replacing the contravariant components of the differential vector $\mathrm{d}A$ with the contravariant components of the differential vectors $\mathrm{d}\breve{A},\mathrm{d}\tilde{A}$, According to the following equations $\partial A^{\mu} = \partial\tilde{A}^{\epsilon} + \partial\breve{A}^{\sigma}$, $\partial A^{\nu} = \partial\tilde{A}^{\tau} + \partial\breve{A}^{\rho}$ in the previous equation

$$g^{\mu\nu} = g^{`kl}\left(\frac{\partial\tilde{A}^{\epsilon} + \partial\breve{A}^{\sigma}}{\partial A^{`k}}\frac{\partial\tilde{A}^{\tau} + \partial\breve{A}^{\rho}}{\partial A^{`l}}\right) \tag{76.1}$$

$$g^{\mu\nu} = g^{`kl}\left[\left(\frac{\partial\tilde{A}^{\epsilon}}{\partial A^{`k}} + \frac{\partial\breve{A}^{\sigma}}{\partial A^{`k}}\right)\left(\frac{\partial\tilde{A}^{\tau}}{\partial A^{`l}} + \frac{\partial\breve{A}^{\rho}}{\partial A^{`l}}\right)\right] \tag{77.1}$$

$$g^{\mu\nu} = g^{`kl}\left[\frac{\partial\tilde{A}^{\epsilon}}{\partial A^{`k}}\frac{\partial\tilde{A}^{\tau}}{\partial A^{`l}} + \frac{\partial\tilde{A}^{\epsilon}}{\partial A^{`k}}\frac{\partial\breve{A}^{\rho}}{\partial A^{`l}} + \frac{\partial\breve{A}^{\sigma}}{\partial A^{`k}}\frac{\partial\tilde{A}^{\tau}}{\partial A^{`l}} + \frac{\partial\breve{A}^{\sigma}}{\partial A^{`k}}\frac{\partial\breve{A}^{\rho}}{\partial A^{`l}}\right] \tag{78.1}$$

Assuming here that the vectors $\mathrm{d}\breve{A},\mathrm{d}\tilde{A}$ are perpendicular, therefore the dot product of the following components equals zero.

$$\frac{\partial\tilde{A}^{\epsilon}}{\partial A^{`k}}\frac{\partial\breve{A}^{\rho}}{\partial A^{`l}} = \frac{\partial\breve{A}^{\sigma}}{\partial A^{`k}}\frac{\partial\tilde{A}^{\tau}}{\partial A^{`l}} = 0 \tag{79.1}$$

$$g^{\mu\nu} = g^{`kl}\left(\frac{\partial\tilde{A}^{\epsilon}}{\partial A^{`k}}\frac{\partial\tilde{A}^{\tau}}{\partial A^{`l}}\right) + g^{`kl}\left(\frac{\partial\breve{A}^{\sigma}}{\partial A^{`k}}\frac{\partial\breve{A}^{\rho}}{\partial A^{`l}}\right) \tag{80.1}$$

Where the first amount on the right side of the equation represents the inverse of the metric tensor for positive subspace-time $\tilde{g}^{\epsilon\tau}$ in the general case we previously explained above, and the second amount in the equation represents the inverse of the metric tensor for negative subspace-time in the general case $\breve{g}^{\sigma\rho}$ with the index followed in general relativity $\sigma = \rho = 0,1,2,3$. By rearranging the terms, we obtain the inverse of the metric tensor for negative subspace-time in terms of both the inverse of the metric tensor of total and positive space-time in the general case.

$$\breve{g}^{\sigma\rho} = g^{\mu\nu} - \tilde{g}^{\epsilon\tau} \tag{81.1}$$

Assuming here the curvature of space-time under the influence of a spherically symmetrical, non-rotating, uncharged mass, in this case we can substitute the metric tensor of the total space-time by the Schwarzschild matrix, [22] [23].

$$g_{\mu\nu}=\begin{bmatrix} -\left(1-\frac{r_s}{r}\right) & 0 & 0 & 0 \\ 0 & \frac{1}{\left(1-\frac{r_s}{r}\right)} & 0 & 0 \\ 0 & 0 & r^2 & 0 \\ 0 & 0 & 0 & r^2\sin^2(\theta) \end{bmatrix} \quad r_s=\frac{2MG}{c^2} \tag{82.1}$$

From Equation (64.1) we obtain the inverse of the metric tensor for positive subspace-time in the general case, and from Equation (82.1) we also obtain the inverse of the metric tensor for total space-time in the general case, by substituting in Equation (81.1)

$$\breve{g}^{\sigma\rho}=\begin{bmatrix} -\frac{1}{\left(1-\frac{r_s}{r}\right)} & 0 & 0 & 0 \\ 0 & \left(1-\frac{r_s}{r}\right) & 0 & 0 \\ 0 & 0 & \frac{1}{r^2} & 0 \\ 0 & 0 & 0 & \frac{1}{r^2\sin^2(\theta)} \end{bmatrix} - \begin{bmatrix} -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & \frac{1}{\tilde{r}^2} & 0 \\ 0 & 0 & 0 & \frac{1}{\tilde{r}^2\sin^2(\tilde{\theta})} \end{bmatrix} \tag{83.1}$$

$$\breve{g}^{\sigma\rho}=\begin{bmatrix} -\frac{1}{\left(1-\frac{r_s}{r}\right)}+1 & 0 & 0 & 0 \\ 0 & \left(1-\frac{r_s}{r}\right)-1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \tag{84.1}$$

The previous equation represents the inverse metric tensor matrix for negative subspace-time in the general case, and from it, we obtain the matrix of the metric tensor.

$$\breve{g}_{\sigma\rho}=\begin{bmatrix} \frac{\left(1-\frac{r_s}{r}\right)}{\left(1-\frac{r_s}{r}\right)-1} & 0 & 0 & 0 \\ 0 & \frac{1}{\left(1-\frac{r_s}{r}\right)-1} & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \tag{85.1}$$

From the metric tensor matrix, we obtain the line element of negative subspace-time in the general case, it is a curved fabric. See **Figure 4(c)**.

$$\mathrm{d}\breve{s}^2 = \breve{V}^2 \mathrm{d}\breve{t}^2 \left[\frac{\left(1-\frac{r_s}{r}\right)}{\left(1-\frac{r_s}{r}\right)-1} \right] + \mathrm{d}\breve{r}^2 \left[\frac{1}{\left(1-\frac{r_s}{r}\right)-1} \right] \tag{86.1}$$

Because $\mathrm{d}\breve{s}^2 = \mathrm{d}s^{`2}$ we can assume here that the spatial coordinates of any event on the total space-time fabric with respect to the reference frame S' are equal to the spatial coordinates of the same event on the negative subspace-time fabric with respect to the reference frame S, that is

$$\mathrm{d}r^{`2} + r^{`2}\left(\mathrm{d}\theta^{`2} + \sin^2\theta^{`}\mathrm{d}\varnothing^{`2}\right) = \mathrm{d}\breve{r}^2 \left[\left(1-\frac{r_s}{r}\right)-1 \right]^{-1} \tag{87.1}$$

In case, if the event occurs on coordinate $r^{`}$ only, that is, $\mathrm{d}\theta^{`} = \mathrm{d}\varnothing^{`} = 0$, and therefore

$$\mathrm{d}\breve{r}^2 = \mathrm{d}r^{`2} \left[\left(1-\frac{2MG}{rc^2}\right)-1 \right] \tag{88.1}$$

As a result of the previous assumption, we also find that the time coordinate of any event on the total space-time fabric with respect to the reference frame S' is necessarily equal to the time coordinate of the same event on the negative subspace-time fabric with respect to the reference frame S, that is.

$$-c^2\mathrm{d}t^{`2} = \breve{V}^2 \mathrm{d}\breve{t}^2 \left[\frac{\left(1-\frac{r_s}{r}\right)}{\left(1-\frac{r_s}{r}\right)-1} \right] \tag{89.1}$$

Assuming here that the transformation of time into negative subspace-time is according to the following equation

$$\mathrm{d}t^{`2} = \mathrm{d}\breve{t}^2 \left(1-\frac{2MG}{rc^2}\right) \tag{90.1}$$

By substituting from (90.1), into (89.1)

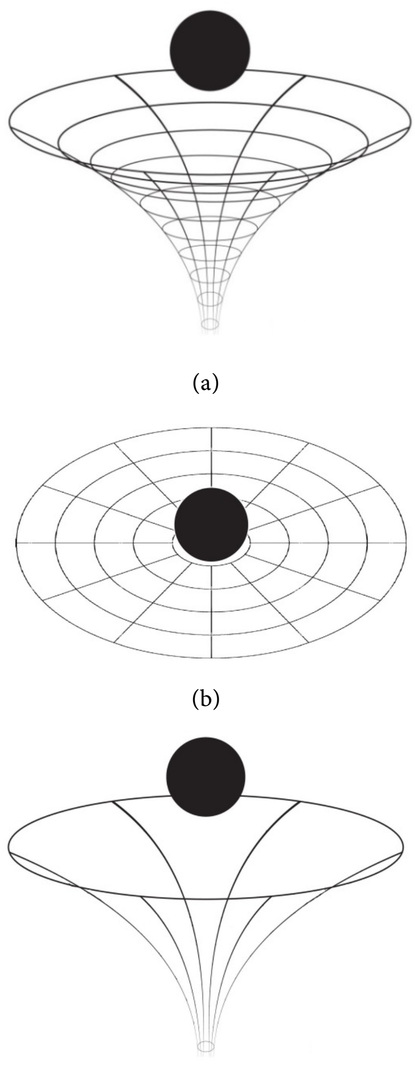
$$-c^2 = \frac{\breve{V}^2}{\left(1-\frac{r_s}{r}\right)-1} \tag{91.1}$$

$$\breve{V}^2 = -c^2 \left[\left(1-\frac{r_s}{r}\right)-1 \right] \tag{92.1}$$

By substituting for the value of the Schwarzschild radius

$$\breve{V} = \sqrt{\frac{2MG}{r}} \qquad r = r_s \qquad \breve{V} = c \tag{93.1}$$

We conclude from the last equation that the observer, who is far from the source of gravity, observes the velocity of a photon moving on the coordinate $\breve{r}$ in the fabric of negative subspace-time, which is always equal to the escape velocity from gravity of the body. It increases as the radius $\breve{r}$ decreases, or when the photon approaches the source of gravity, as the velocity of the photon reaches the speed of light when the photon reaches the Schwarzschild radius, which is an opposite result of the movement of the photon on the total curved fabric of space-time in general relativity. We also conclude that the velocity of the particle in negative subspace-time does not depend on the mass of the moving particle, but rather on the mass M that causes the curvature. This means that all particles will have the same magnitude of velocity at the same position on the $\breve{r}$ coordinate. As for the direction, it changes from one position to another on the $\breve{r}$ coordinate, because the particle moves on a curve, but the curves remain not intersecting. Therefore, negative subspace-time in the general case has the same geometric properties as negative subspace-time in the special case.



(a)

(b)

(c)

Figure 4. (a) Shows the total space-time curve under the influence of a mass; (b) The positive sub-space-time; (c) The negative sub-space-time split from the total space-time.

3. Results

Analyzing a four-dimensional displacement vector on the fabric of space-time in the special or general case into two four-dimensional displacement vectors leads to the splitting of the fabric of space-time into four-dimensional subspaces. Where the first vector is known as the identical vector because it fulfills the condition of symmetry and is represented in a four-dimensional space known as positive subspace-time and is characterized by the following properties: Symmetry of the structure of spatial space and symmetry of the laws of physics, as expressed in the space of causality. The second vector, known as the parallel vector, is represented in a four-dimensional space also known as negative subspace-time, and is characterized by the following properties: the space devoid of any causality, or the space of physical quantities that do not affect the behavior of the physical phenomenon. Thus, in the special case, we have new transformations for the coordinates of space and time called modified Lorentz transformations for positive and negative subspace-time shown in the first set ((19.1)-(22.1)), and the second set ((46.1)-(49.1)), It is characterized by the symmetries mentioned above, so length contraction disappears while time dilation remains and the speed of light is no longer a universal constant. In the general case, we have new types of metric tensor, one for positive subspace-time and the other for negative subspace-time (see Equations (64.1) and (71.1)), where we find that the velocity of the photon decreases in positive subspace-time until it reaches zero and increases in negative subspace-time until it reaches the speed of light when the photon reaches the Schwarzschild radius (see Equations (85.1) and (93.1)).

4. Discussions

Lorentz transformations or special relativity express an observational process that appears on devices, and therefore results such as the stability of the speed of light for every observer, length contraction, and time dilation can be tested experimentally. As for the observation process used in the modified Lorentz transformations, it is achieved through a process of mathematical analysis only; it is an imaginary observation process that is not achieved experimentally, but rather purely theoretically. But this does not put the new model in conflict with experiments, because the values that appear in imaginary observations are not total values, but rather partial values because they are in subspaces. By summing the values of positive and negative subspace-time, we obtain the values of the total space-time and also the same results of the Lorentz transformations, and the same in the general case. The purpose of the inverse relativity model is not to propose an alternative model to special and general relativity, but rather to reveal the complex structure of space-time, including four-dimensional subspaces as well, and the properties of those spaces that are linked to the concept of causality, symmetry, and values of the speed of light, (see the comparison Table 1).

Table 1. Comparison between lorentz transformations in special relativity and modified lorentz transformations in inverse relativity.

Transformations of space and time coordinates	Special Relativity	Inverse Relativity
Equations	Lorentz transformations of space and time coordinates $x^` = \gamma(x - V_s t)$ $y^` = y$ $z^` = z$ $t^` = \gamma\left(t - \frac{V_s x}{c^2}\right)$ Where $\gamma = \frac{1}{\sqrt{1 - \frac{V_s^2}{c^2}}}$	Positive modified Lorentz transformations $\tilde{x}^` = x$ $\tilde{y}^` = y$ $\tilde{z}^` = z$ $\tilde{t}^` = \gamma t$ Negative modified Lorentz transformations $\breve{x}^` = \gamma\left(x\left(1 - \gamma^{-1}\right) - V_s t\right)$ $\breve{y}^` = 0$ $\breve{z}^` = 0$ $\breve{t}^` = \gamma\left(t - \frac{V_s x}{c^2}\right)$

Minkowski space-time in describing causality depends on the speed factor only, as we find that the speed of light is the maximum causal speed that exists between two events in the universe [24]. But the inverse relativity model follows a different approach in describing causality that relies on a geometric factor, which is the direction of the velocity vectors, where we find that the direction and magnitude of velocity vectors are what express the presence or absence of causality in every subspace-time, and the purpose of this description of causality in subspaces is to understand the properties of every subspace-time.

Velocity transformations in the theory of special relativity were intended to obtain the law of addition of parallel velocities, while preserving the second postulate of special relativity, in order to provide a logical explanation for the problem of the constant speed of light in the experiment of Michelson and Morley [25]. As for the velocity transformations in the inverse relativity model, their purpose is to analyze parallel velocities, while also maintaining the second postulate of special relativity (see the comparison **Table 2**), to reveal the existence of subspaces in total space-time.

Table 2. Comparison between velocity transformations in special relativity and inverse relativity.

Transformations of Velocity	Special Relativity	Inverse Relativity
Purpose of transformations Equations	Addition of parallel Velocities $V_x = \frac{V_x^` + V_s}{1 + \frac{V_s V_x^`}{c^2}}$	Analysis of parallel velocities $\breve{V}_x = \frac{V_x^`\left(1 - \gamma^{-1}\right) + V_s}{1 + \frac{V_s V_x^`}{c^2}} = V_s$ $\tilde{V}_x = V_x^` \gamma^{-1}$

The inverse relativity model is more of a mathematical model than a physical one because it relies primarily on mathematical analysis. The model also does not represent an independent physical theory with its results, as in special and general relativity. However, the new model can provide some new results within the total fabric of space-time (Like the existence of other sub-spacetimes) without the need for additional hidden dimensions as in some other theories, as well as new concepts of causality. Through the new view of the structure of total space-time, the new model paves the way for solving some of the problems in which special and general relativity failed, such as problems of relativistic thermodynamics [26], quantum gravity, and others. Where we can address these problems through the geometric properties of sub-spacetimes, we will explain this in the following papers.

Conflicts of Interest

The author declares no conflicts of interest regarding the publication of this paper.

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