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Mathematical Modeling of Intra-Communal Violence and Risk-Level Analysis. Case Study: Obiaruku Community in Delta State, Nigeria

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Authors' contributions

This work was carried out in collaboration among all authors. All authors read and approved the final manuscript.

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Abstract

This paper aims to capture the dynamics of intra-communal violence in a deterministic model of ordinary differential equations, accordingly, the Authors found some interesting results. Lack of quality education, insecurity, bad roads, drugs and alcoholism, unequal representation in government and religious decay have been identified as key factors supporting intra-communal violence over the years. In this research work we built all these factors into a deterministic model describing intra-communal violence and performed some basic mathematical analysis such as positivity of solutions, existence of invariant region, violence-free equilibrium, violence-persistent equilibrium, basic reproduction number, sensitivity analysis, stability analysis and bifurcation analysis. It was revealed that the violence-free equilibrium is globally asymptotically stable. The model exhibits a forward bifurcation. The sensitivity analysis revealed that injustice and

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insecurity are highly sensitive parameters of the basic reproduction number. We also designed a questionnaire to ascertain the violence risk level of Obiaruku community in Delta State, Nigeria and the analysis revealed that the community is at the medium high risk level and thus violence may occur in most cases in the community. The results of the stability analysis and the sensitivity analysis showed that under certain conditions, a community can be brought to the maximum low risk level and the maximum high peace level.

Keywords: Modeling; violence; risk-level; stability; and sensitivity.

1 Introduction

Galles and Straus [1] stated that violence is concerned with carrying out an activity with the intent of hurting another individual physically. Violence can also be defined as any form of verbal, physical, sexual, or visual abuse experienced by someone that has the effect of hurting the person's feelings and will indirectly affect the person's behavior. Galtung [2] opines that violence are avoidable, and they are abuses to fundamental human desires, and lowers the level of satisfaction. He added that violence is cultural, structural, and direct; and that direct violence has to do with an event, structural violence is a process with ups and downs, and cultural violence has to do with an invariant permanence. Galtung further revealed that psychological and verbal abuse are direct violence. Stark, Anne, and William [3] defined structural violence as the confiscation of someone' rights through the use of ideas, and direct violence as the confiscation of someone' rights or interest through the use physical violence.

Coker-Appiah and Cusack [4] categorized the varieties of causes of violence into three: verbal alteration that can escalate into violent behavior, financial issues that can lead to disagreement and violence, and offensive conduct. Children that were victims or witness physical violence are more likely to become perpetrators of violence as adults than children that were not victimized. Shewafera and Birhanu [5] revealed that any condition even individuals' behavior which can spread among humans, can bring about a similar epidemiological disease condition. Patten and Arboleda-Flórez [6] said that violence is a condition where behaviors is contagious, and this has been seen in large groups and in places with high density.

In understanding dynamical systems of real-world, it has been revealed that mathematical modeling plays a fundamental role which remains effective till date Khan, Ali, Bonyah, Okosun, Islam & Khan, [7] Khan, Ullah & Farhan, [8] Karthikeyan, Karthikeyan, Baskonus, Venkatachalam & Chu, [9] Jin, Qian, Chu & Rahman, [10]. Mathematical models have been formulated and analyzed in different disciplines including the social science (Lazarus, [11] De la Poza, Jódar, & Barreda, [12] Dominioni, Marasco&Romano, [13] Lemecha & Feyissa, [14] Delgadillo-Aleman, Ku-Carrillo, Perez-Amezcua & Chen-Charpentier, [15] Danford, Kimathi & Mirau, [16] Mamo, [17] Mamo, [18] Ossaiugbo & Okposo, [19] Okposo, Jonathan, Okposo & Ossaiugbo, [20] Fantaye &Birhanu, [21]. Mathematical models have been applied tosocial situations. Mathematical models on violence include (Lazarus, [11] De la Poza, Jódar, & Barreda, [12] Wiley, Levy & Branas, [22] Delgadillo-Aleman, Ku-Carrillo, Perez-Amezcua & Chen-Charpentier, [15] Tsetimi, Ossaiugbo, & Atonuje, [23]. In this research work, we constructed a 3-compartment deterministic model for intra-communal violence, and perform some mathematical analysis on the model. Violence risk level analysis and peace level analysis were also presented. We also obtained the violence risk level perception of Obiaruku community, Delta State, Nigeria, via a questionnaire distributed to and equally retrieved from the residents of the community.

2 Methodology

We present the design of the research, model formulation, assumptions of the model, parameter descriptions, population and population sample, sampling technique, data collection instrument, basic mathematical analyses on the model and method of questionnaire analysis. The purpose of the study and the answers needed to critically validate the model has guided the researcher to choose a survey research design. We designed a questionnaire titled "Causes of Intra-Communal Violence" and distributed same to a sample of residents in Obiaruku community in Delta State, Nigeria. The accessible population includes reachable 100 residents of the community which is made up of 19 business men/women, 19 commercial motorcyclists, 20 students, 16 farmers, 15 civil servants, and 11 traditional rulers. This is considered sufficient enough to represent and generalize the entire community owing to the population size of the community. Moreover, all groups existing in the community were taken into consideration. The questionnaires were retrieved and analyzed. The questionnaire has four sections. Section 1 is on infrastructural developments within the Obiaruku community. Section 2 is on level of injustice melted on the less privileged/exposed within the community. Section 3 is on the security strength of the community, while section 4 is the level of threat to life and properties of the residents of the community. A deterministic model of ordinary differential equations is constructed to study intra-communal violence and basic mathematical analysis performed on the model. The questionnaire which was subjected to face validation and content validation was structured on a 2-point scale, which includes “Yes” and “No”. The respondents were instructed to tick the appropriate answer to the questions contained in the instrument. Calculations and model analyses were done using the version 12 Mathematica Programming Software, while charts were generated with Microsoft Excel Software.

2.1 Model formulation

The following assumptions were considered in the model formulation. Firstly, the population is uniformly mixed so that every peaceful resident is equally susceptible to infection. Secondly, natural death and violent-induced death happens in all classes; and lastly, not all brutal individuals can be completely peaceful. The model considered stratified the human community into three mutually exclusive classes. The Peaceful class (P), the Aggressive class (A) and the Brutal class (B). Peaceful individuals are residents who are neither aggressive nor brutal but are susceptible to attack, injustice or violence at any time. Aggressive individuals are residents of the community who are not satisfied by the way and manner in which they are treated or marginalized, and they can easily react. The Aggressive individuals manifest their greed/dissatisfaction at all time. They can engage in quarrels that can pull crowd but they will not destroy life or property. Their dissatisfaction may due to political marginalization, land and assets deprivation and some application of physical forces from other individuals. While Brutal individuals are those residents of the community who are ready to destroy lives and properties at all cost. They feel they are not answerable to anyone and they can express their anger and dissatisfaction by any means pleasing to them. The Brutal individuals often disregard other people’s rights. They are determined and energetic in pursuit of their ends. The per capital recruitment rate into the peaceful class is Λ. Peaceful individuals join the Aggressive class at rate$=\kappa\psi\varphi\omega\xi\left(\frac{A+\gamma B}{N}\right)$, force of infection. The parameters descriptions clearly reveal that the force of infection χ of the model has been constructed to imply all three forms of violence - the cultural, the structural and the direct form. Aggressive residents become brutal at rate α. Due to well-meaning and positive interventions from concerned individuals and/or organizations, the aggressive individuals become peaceful at the rateδ, while the brutal individuals become peaceful and aggressive at rates β and ζ respectively.It is assumed that violence-induced death and natural death occur in all classes at the rate η and μ respectively. The mathematical model is given as system (1) while the schematic diagram is given as figure 1.

$$\left.\begin{aligned} \frac{dP}{dt} &= \Lambda + \delta A + \beta B - (\chi + \eta + \mu)P \\ \frac{dA}{dt} &= \chi P + \zeta B - (\alpha + \delta + \eta + \mu)A \\ \frac{dB}{dt} &= \alpha A - (\beta + \zeta + \eta + \mu)B \end{aligned}\right\} \tag{1}$$

Initial conditions:$P(t) \geq 0, A(t) \geq 0, B(t) \geq 0.$

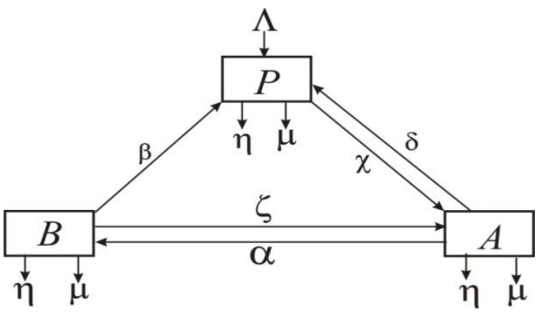

Fig. 1. Schematic diagram

Table 1. Parameters description and values

Parameter	Description	Value	Source
Λ	Per capita recruitment rate into the peaceful class	0.6	Mohammed & Musa (2019)
κ	Effective contact rate with aggressive and brutal residents	0.6	Mohammed & Musa (2019)
γ	Infection coefficient of the brutal class	0.6	Assumed
ψ	Rate of injustice	0.5	Assumed
φ	Level of insecurity on a scale of $0-1$	0.8	Assumed
ω	Level of threat to life and property on a scale of $0-1$	0.6	Assumed
ξ	Level of negligence of infrastructural development in the community by the government	0.7	Assumed
ζ	Rate at which brutal individuals refines to aggressive	0.4	Assumed
α	Rate at which aggressive residents become brutal	0.7	Assumed
δ	Rate at which aggressive peaceful	0.4	Assumed
β	Rate at which brutal individuals become peaceful	0.3	Assumed
η	Violent induced death rate	0.003	Mohammed & Musa (2019)
μ	Natural death rate	0.0124	Kotola & Mekonnen (2022)

3 Mathematical Analysis of Model

3.1 Positivity of solutions

We shall establish the positivity of solutions via the following theorem.

Theorem 1 (Positivity of Solution)

Suppose $\Gamma = \{(\mathrm{P}, \mathrm{A}, \mathrm{B}) \in \mathbb{R}^3 : \mathrm{P}(0) > 0, A(0) > 0, B(0) > 0\}$, then the solution set $\{\mathrm{P}, \mathrm{A}, \mathrm{B}\}$is positive for all$\mathrm{t} \geq 0$.

Proof:

Observe the equation,

$$\frac{dP}{dt} = \Lambda + \delta A + \beta B - (\chi + \eta + \mu)P.$$

See that

$$\frac{dP(t)}{dt} \geq -(\chi + \eta + \mu)P.$$

Since$\mathrm{P}(0) \geq 0$, we obtain$\mathrm{P}(\mathrm{t}) \geq \mathrm{P}(0)e^{-(\chi+\eta+\mu)t} \geq 0$. Similarly, $\mathrm{A}(\mathrm{t}) \geq 0, \mathrm{B}(\mathrm{t}) \geq 0 \ \forall \ \mathrm{t} \geq 0$. $\boxdot$

3.2Invariant region and boundedness of solution

The total number of individuals who are susceptible, aggressive and brutal cannot grow indefinitely. Independent of the initial number of these individuals, there is an upper bound for the population growth. Thus, at any point in time, the total number of susceptible, aggressive and brutal individuals is contained in a region. This is the invariant region. We now establish that the whole population size is bounded.

Theorem 2: The set

$$\Gamma = \left\{(P, A, B) \in \mathbb{R}^3_+ : 0 \leq P + A + B = N \leq \frac{\Lambda}{\eta + \mu}\right\} \tag{2}$$

is positively-invariant.

Proof:

$$\begin{aligned} &N(t) = P(t) + A(t) + B(t).\\ &\frac{dN(t)}{dt} = \Lambda - (\eta + \mu)P, \\ &N(t) \leq \frac{\Lambda}{\eta + \mu} + ce^{-(\eta+\mu)t}. \end{aligned} \tag{3}$$

As $t \to \infty$, we obtain

$$N(t) \leq \frac{\Lambda}{\eta + \mu}. \tag{4}$$

It follows that the model's feasible solution set remains in the region: $\Gamma = \left\{(P, A, B) \in \mathbb{R}^4_+ : 0 \leq P + A + B = N \leq \frac{\Lambda}{\eta+\mu}\right\}$. Observe that if the population is higher than the threshold level $\frac{\Lambda}{\eta+\mu}$, the population reduces to the carrying capacity. If $N \leq \frac{\Lambda}{\eta+\mu}$, then the solution remains in the invariant region for all $t > 0$. This completes the proof. ⊡

3.3 Violence-Free Equilibrium (VFE)

The VFE is obtained by equating the right-hand side of the model (1) to zero, substituting $A = B = 0$, and solving the resulting system. This gives the VFE as:

$$\mathbb{E}_{\mathbf{0}} = \left(\frac{\Lambda}{\eta + \mu}, 0,0\right) \text{(5)}$$

3.4 Violence-Persistent Equilibrium (VPE)

We obtained this equilibrium point by simply setting the right-hand side of the model (1) to zero. Thereafter, we solved the resulting non-linear system and obtained.

$$\left.\begin{aligned} P &= \frac{\Lambda(\alpha(\beta + \eta + \mu) + (\delta + \eta + \mu)(\beta + \zeta + \eta + \mu))}{\kappa(\eta + \mu)(\beta + \alpha\gamma + \zeta + \eta + \mu)\xi\varphi\psi\omega}, \\ A &= -\frac{\Lambda(\beta + \zeta + \eta + \mu)((\beta + \zeta + \eta + \mu)(\delta + \eta + \mu - \kappa\xi\varphi\psi\omega) + \alpha(\beta + \eta + \mu - \gamma\kappa\xi\varphi\psi\omega))}{\kappa(\eta + \mu)(\alpha + \beta + \zeta + \eta + \mu)(\beta + \alpha\gamma + \zeta + \eta + \mu)\xi\varphi\psi\omega}, \\ B &= \frac{\alpha\Lambda(-\alpha(\beta + \eta + \mu) - (\delta + \eta + \mu)(\beta + \zeta + \eta + \mu) + \kappa(\beta + \alpha\gamma + \zeta + \eta + \mu)\xi\varphi\psi\omega)}{\kappa(\eta + \mu)(\alpha + \beta + \zeta + \eta + \mu)(\beta + \alpha\gamma + \zeta + \eta + \mu)\xi\varphi\psi\omega}. \end{aligned}\right\} \tag{6}$$

3.5 Basic reproduction number$(\mathbf{R_0})$

This is the average number of secondary violence cases caused by an aggressive or brutal individual within an entirely peaceful population during his/her infective period. We employ the method due to Driessche and

Watmough [24]) to obtain the expression for R_0. Here, we consider the violence class of individuals$X'(t) = \mathcal{F}(t) - \mathcal{V}(t)$ where

$$\mathcal{F} = \begin{pmatrix} \chi P \\ 0 \end{pmatrix}, \mathcal{V} = \begin{pmatrix} -\zeta B + (\alpha + \delta + \eta + \mu)A \\ -\alpha A + (\beta + \zeta + \eta + \mu)B \end{pmatrix} \tag{7}$$

denote on new infection terms and old infection terms respectively. Next, we obtain the Jacobian matrix for $\mathcal{F}$ and $\mathcal{V}$, at the disease-free equilibrium, to obtain the matrices F and V below.

$$F = \begin{pmatrix} \kappa\xi\varphi\psi\omega & \gamma\kappa\xi\varphi\psi\omega \\ 0 & 0 \end{pmatrix}, \quad V = \begin{pmatrix} \alpha + \delta + \eta + \mu & -\zeta \\ -\alpha & \beta + \zeta + \eta + \mu \end{pmatrix}.$$

Furthermore, we have

$$V^{-1} = \begin{pmatrix} \frac{\beta + \zeta + \eta + \mu}{\alpha(\beta + \eta + \mu) + (\delta + \eta + \mu)(\beta + \zeta + \eta + \mu)} & \frac{\zeta}{\alpha(\beta + \eta + \mu) + (\delta + \eta + \mu)(\beta + \zeta + \eta + \mu)} \\ \frac{\alpha}{\alpha(\beta + \eta + \mu) + (\delta + \eta + \mu)(\beta + \zeta + \eta + \mu)} & \frac{\alpha + \delta + \eta + \mu}{\alpha(\beta + \eta + \mu) + (\delta + \eta + \mu)(\beta + \zeta + \eta + \mu)} \end{pmatrix}.$$

and hence

$$FV^{-1} = \begin{pmatrix} \frac{\kappa(\beta + \alpha\gamma + \zeta + \eta + \mu)\xi\varphi\psi\omega}{\alpha(\beta + \eta + \mu) + (\delta + \eta + \mu)(\beta + \zeta + \eta + \mu)} & \frac{\kappa(\zeta + \gamma(\alpha + \delta + \eta + \mu))\xi\varphi\psi\omega}{\alpha(\beta + \eta + \mu) + (\delta + \eta + \mu)(\beta + \zeta + \eta + \mu)} \end{pmatrix}.$$

The eigenvalues of the matrix $FV^{-1}\lambda$ are obtained as

$$\lambda_1 = 0, \quad \lambda_2 = \frac{\kappa(\beta + \alpha\gamma + \zeta + \eta + \mu)\xi\varphi\psi\omega}{\alpha(\beta + \eta + \mu) + (\delta + \eta + \mu)(\beta + \zeta + \eta + \mu)}.$$

It follows that R_0, which is the spectral radius, is

$$R_0 = \frac{\kappa(\beta + \alpha\gamma + \zeta + \eta + \mu)\xi\varphi\psi\omega}{\alpha(\beta + \eta + \mu) + (\delta + \eta + \mu)(\beta + \zeta + \eta + \mu)} \tag{8}$$

4 Stability Analysis of the Violence-Free Equilibrium

The stability analysis of the model tells how stable the violence-free equilibrium can be over time owing to the initial number of people who are susceptible, aggressive and brutal. This again is needed for a proper management and eradication of violence in the community. It is the utmost desire of any goodhearted and well-meaning individual or organization saddled with the responsibility of crisis management within the community to ensure the achievement of the global stability of the VFE of the said intra-communal violence. We start by finding the Jacobian matrix of the above system which is given as.

$$J = \begin{pmatrix} \frac{\partial f_1}{\partial P} & \frac{\partial f_1}{\partial A} & \frac{\partial f_1}{\partial B} \\ \frac{\partial f_2}{\partial P} & \frac{\partial f_2}{\partial A} & \frac{\partial f_2}{\partial B} \\ \frac{\partial f_3}{\partial P} & \frac{\partial f_3}{\partial A} & \frac{\partial f_3}{\partial B} \end{pmatrix},$$

Where

$$f_1 = \Lambda + \delta A + \beta B - (\chi + \eta + \mu)P,$$
$$f_2 = \chi P + \zeta B - (\alpha + \delta + \eta + \mu)A,$$
$$f_3 = \alpha A - (\beta + \zeta + \eta + \mu)B.$$

Theorem 3 (Local stability of $\mathbb{E}_0$)

The VFE ($\mathbb{E}_0$) is locally asymptotically stable if $R_0 < 1$, otherwise it is unstable.

Proof:

The Jacobian matrix is

$$J_{\mathbb{E}_0} = \begin{pmatrix} -\eta-\mu & \delta-\kappa\xi\varphi\psi\omega & \beta-\gamma\kappa\xi\varphi\psi\omega \\ 0 & -\alpha-\delta-\eta-\mu+\kappa\xi\varphi\psi\omega & \zeta+\gamma\kappa\xi\varphi\psi\omega \\ 0 & \alpha & -\beta-\zeta-\eta-\mu \end{pmatrix}. \quad (9)$$

Observe that

$$\mathrm{Trace}\left(J_{\mathbb{E}_0}\right) = \kappa\xi\varphi\psi\omega-\alpha-\beta-\delta-\zeta-3\eta-3\mu < 0,$$

and

$$\mathrm{Det}\left(J_{\mathbb{E}_0}\right) = -(-\eta-\mu)(\alpha(\beta+\eta+\mu)+(\delta+\eta+\mu)(\beta+\zeta+\eta+\mu))(-1+R_0) > 0.$$

Recall that

$$R_0 = \frac{\kappa(\beta+\alpha\gamma+\zeta+\eta+\mu)\xi\varphi\psi\omega}{\alpha(\beta+\eta+\mu)+(\delta+\eta+\mu)(\beta+\zeta+\eta+\mu)}.$$

Therefore,

$$\omega = \frac{(\alpha(\beta+\eta+\mu)+(\delta+\eta+\mu)(\beta+\zeta+\eta+\mu))R_0}{\kappa(\beta+\alpha\gamma+\zeta+\eta+\mu)\xi\varphi\psi}.$$

Substituting this into the expression for $\mathrm{Det}\left(J_{\mathbb{E}_0}\right)$, we obtain

$$\mathrm{Det}\left(J_{\mathbb{E}_0}\right) = (-\eta-\mu)(\alpha(\beta+\eta+\mu)+(\delta+\eta+\mu)(\beta+\zeta+\eta+\mu))(1+R_0) > 0.$$

Solving for R_0, we obtain $R_0 < 1$.Therefore, the VFE is locally asymptotically stable. This completes the proof.

Remarks:

1. Theorem 3 implies that as long as the initial sizes of the peaceful individuals, aggressive individuals and brutal individuals are within the basin of attraction of the violence-free equilibrium, violence can be eradicated from the community. Furthermore, global stability of the VFE guarantees that that eradication of violence does not depend on the initial sizes of the compartments. Thus, it is important to establish that with $R_0 \leq 1$, that is, the VFE is globally asymptotically stable.
2. In order to study the global asymptotic stability of the VFE, an appropriate Lyapunov function can be constructed (Ana & James, [25] and Michael & Liancheng, [26] but we shall employ the method introduced by Carlos and Song [27]. Here, we rewrite the model (1) in the form

$$\begin{cases} \dfrac{dX}{dt} = L(X,Z) \\ \dfrac{dZ}{dt} = M(X,Z), \quad M(X,0) = 0 \end{cases} \quad (10)$$

where$X = (P)$ denotes the uninfected individuals and $Z = (A,B)$ denotes the violent individuals.

3. By equation (10), we would denote an equilibrium $\mathbb{E} = (X, Z)$. The $VFE(\mathbb{E}_0)$ is thus represented as $\mathbb{E}_0 = (X^*, 0)$ where $X^* = (P)$.
4. If the following two conditions are satisfied, then the VFE is globally asymptomatically stable:

C1: For $\left.\frac{dX}{dt}\right|_{Z=0} = L(X, 0)$,

$X^* = (P)$ is globally asymptomatically stable.

C2: $\frac{dZ}{dt} = D_Z M(X^*, 0)Z - \widehat{M}(X, Z)$,

where $\widehat{M}(X, Z) \geq 0$ for all $(X, Z) \in \Gamma$.

5. Γ is the region where the model is feasible, and $D_Z M(X^*, 0)$ is known as the Metlzer matrix with nonnegative off-diagonal elements.

Theorem 4 (Global stability of the VFE):

$\mathbb{E}_0 = (X^*, 0)$ is globally asymptotically stable if $R_0 < 1$, and conditions (C1) and (C2) are satisfied.

Proof:

First let us introduce the recruitment term Λ in the Peaceful Class. Observe that

$$\frac{dX}{dt} = L(X, Z) = [\Lambda + \delta A + \beta B - (\chi + \eta + \mu)P], \tag{11}$$

$$\frac{dZ}{dt} = M(X, Z) = \begin{bmatrix} \chi P + \zeta B - (\alpha + \delta + \eta + \mu)A \\ \alpha A - (\beta + \zeta + \eta + \mu)B \end{bmatrix}, \tag{12}$$

$$\left.\frac{dX}{dt}\right|_{Z=0} = L(X, 0) = [\Lambda - (\eta + \mu)P]. \tag{13}$$

Equating the right hand side of equation (13) to zero and solving, we see that $X^* = \left(\frac{\Lambda}{\eta+\mu}\right)$ is the only equilibrium point. Solving the system of ordinary differential equation given by (13) for $P(t)$, we obtain

$$P(t) \leq \frac{\Lambda}{\eta + \mu} + \left(P_0 - \frac{\Lambda}{\eta + \mu}\right) e^{-(\eta+\mu)t}. \tag{14}$$

As $t \to \infty$, we have that $P(t) \to \frac{\Lambda}{\eta+\mu}$. Thus global convergence of $X = (P)$ is implied. Hence $X^* = \left(\frac{\Lambda}{\eta+\mu}\right)$ is globally asymptotically stable for $\left.\frac{dX}{dt}\right|_{Z=0}$. We now obtain $D_Z M(X^*, 0)Z$.

$$D_Z M(X^*, 0) = \begin{pmatrix} -\zeta - \eta + \beta\kappa\xi\sigma\varphi\omega & \delta - \delta\tau + \beta\kappa\xi\varrho\sigma\varphi\omega & \delta + \beta\gamma\kappa\xi\sigma\varphi\omega \\ \zeta - \beta\kappa\xi(-1 + \sigma)\varphi\omega & -\alpha - \delta - \eta - \beta\kappa\xi\varrho(-1 + \sigma)\varphi\omega & -\beta\gamma\kappa\xi(-1 + \sigma)\varphi\omega \\ 0 & \alpha & -\delta - \eta \end{pmatrix}$$

By the condition (C2), we obtain

$$\widehat{M}(X, Z) = \begin{pmatrix} (A + B\gamma)\kappa\xi\varphi\psi\omega - P\chi \\ 0 \end{pmatrix}$$

We observe that the condition $\widehat{M}(X, Z) \geq 0$for all $(X, Z) \in \Gamma$ holds. Thus, the condition $(C2)$ is satisfied. Therefore, given $R_0 < 1$, since only $(C1)$and $(C2)$are satisfied, then the VFE is globally asymptotically stable. This completes the proof.

Remark: The global stability of the violence-free equilibrium assures us that when the right approach is followed in managing violence/crisis (or maintaining peace)within the community, long-lasting peace can be achieved, no matter the number of brutal individuals or aggressive individuals existing at that point in time.

5 Sensitivity Analysis

The result of the sensitivity analysis is a pointer to violence eradication in the community. It tells which parameters highly influence violence outbreak in the community. The sensitivity indices of the parameters of R_0reveals the influence of small changes in parameter values on the extent of intra-communal violence/crises. By this analysis, we shall be able to tell which parameters are highly sensitive to small perturbations. This analysis shall guide decision-making pertaining crisis management. We employ the approach used by Kizito and Tumwiine [28]. The normalized forward sensitivity index of R_0 that depends on the differentiability index of a parameter υ, is

$$\zeta_{\upsilon}^{R_0} = \frac{\partial R_0}{\partial \upsilon} \times \frac{\upsilon}{R_0} \tag{15}$$

Thus, we obtain the following sensitivity indices:

$$\begin{aligned}
\zeta_{\kappa}^{R_0} &= 1 > 0,\\
\zeta_{\xi}^{R_0} &= 1 > 0,\\
\zeta_{\varphi}^{R_0} &= 1 > 0,\\
\zeta_{\psi}^{R_0} &= 1 > 0,\\
\zeta_{\omega}^{R_0} &= 1 > 0,\\
\zeta_{\gamma}^{R_0} &= \frac{\alpha\gamma}{\beta + \alpha\gamma + \zeta + \eta + \mu} > 0,\\
\zeta_{\beta}^{R_0} &= -\frac{\alpha\beta(\zeta + \gamma(\alpha + \delta + \eta + \mu))}{(\beta + \alpha\gamma + \zeta + \eta + \mu)(\alpha(\beta + \eta + \mu) + (\delta + \eta + \mu)(\beta + \zeta + \eta + \mu))} < 0,\\
\zeta_{\alpha}^{R_0} &= -\frac{\alpha(\beta + \zeta + \eta + \mu)(\beta + \eta + \mu - \gamma(\delta + \eta + \mu))}{(\beta + \alpha\gamma + \zeta + \eta + \mu)(\alpha(\beta + \eta + \mu) + (\delta + \eta + \mu)(\beta + \zeta + \eta + \mu))} < 0,\\
\zeta_{\delta}^{R_0} &= -\frac{\delta(\beta + \zeta + \eta + \mu)}{\alpha(\beta + \eta + \mu) + (\delta + \eta + \mu)(\beta + \zeta + \eta + \mu)} < 0,\\
\zeta_{\zeta}^{R_0} &= \frac{\alpha\zeta(\beta + \eta + \mu - \gamma(\delta + \eta + \mu))}{(\beta + \alpha\gamma + \zeta + \eta + \mu)(\alpha(\beta + \eta + \mu) + (\delta + \eta + \mu)(\beta + \zeta + \eta + \mu))} < 0,\\
\zeta_{\mu}^{R_0} &= -\frac{\mu(\beta^2 + \alpha^2\gamma + \alpha\beta\gamma + \alpha\zeta + 2\beta(\zeta + \eta + \mu) + (\zeta + \eta + \mu)^2 + \alpha\gamma(\delta + \zeta + 2(\eta + \mu)))}{(\beta + \alpha\gamma + \zeta + \eta + \mu)(\alpha(\beta + \eta + \mu) + (\delta + \eta + \mu)(\beta + \zeta + \eta + \mu))}\\
&\quad < 0,\\
\zeta_{\eta}^{R_0} &= -\frac{\eta(\beta^2 + \alpha^2\gamma + \alpha\beta\gamma + \alpha\zeta + 2\beta(\zeta + \eta + \mu) + (\zeta + \eta + \mu)^2 + \alpha\gamma(\delta + \zeta + 2(\eta + \mu)))}{(\beta + \alpha\gamma + \zeta + \eta + \mu)(\alpha(\beta + \eta + \mu) + (\delta + \eta + \mu)(\beta + \zeta + \eta + \mu))}\\
&\quad < 0,
\end{aligned}$$

The sensitivity analysis reveals that seven parameters of R_0have positive sensitivity indices; these parameters are the effective contact rate (κ),the level $(\xi)$of negligence of infrastructural development in the community by the government, the level $(\varphi)$of insecurity on a scale of $0 - 1$, the rate of injustice(ψ),the level $(\omega)$of threat to life and property on a scale of $0 - 1$, and the infection coefficient $(\gamma)$of the brutal class. It is pertinent to point out here that small increments in the values of these parameters will greatly increase R_0. Thus, in order to minimize or eradicate violence within the community, we recommend that:

(i) at all possible cost, peaceful individuals must avoid any form of business or dealings with brutal and aggressive residents of the community. In order words, effective contact with the infectious individuals should be avoided or minimized. This recommendation is because the effective contact rate (κ) has been shown to have a positive sensitivity index.

(ii) proper attention/consideration should be given to the provision of good infrastructural development within the community. Also, existing infrastructures must be properly maintained. The government and traditional rulers must understand that lack of these infrastructures can cause agitation within the members of the community.

(iii) the security of lives and properties of residents of the community must be on top of the scale of preference of the community residents, the traditional rulers and the government. Security issues must not be taken with negligence, so as to ensure that there is zero tolerance to insecurity and threat to life and property within the community.

(iv) all individuals involved a conflict should be treated fairly according to the laws of the land. There should be no form of injustice either by the traditional rulers or those involved in the process of crisis settlement.

(v) the law enforcement agencies must work together with the goal of ensuring that aggressive individuals and brutal individuals are properly punished/prosecuted. This will reduce the strength of spread of violence from these classes.

(vi) in a conflict or violence situation within the community, an immediate and peace-targeted response must be given and channeled to the appropriate quarter. Agitations or little conflicts should not be allowed to grow out of proportions before a proper and adequate response is dished out. In other words, irascible individuals should be bounded from the right, in order that they may not become aggressive, and aggressive individuals should be appeased on time so as to avoid increasing the brutal class.

The parameters of R_0 with negative sensitivity indices are the rate (β) at which brutal individuals become peaceful, the rate (ζ)at which brutal individuals refines to aggressive, the rate (δ)at which aggressive peaceful, the rate (α)at which aggressive residents become brutal, the natural death rate $(\mu)$and the violence-induced death rate (η).An increment in the magnitude of these negatively sensitive parameters will cause a reduction in the value of R_0, hence we recommend that the cause of agitation or conflict within the community, especially among the aggressive class and the brutal class, should be properly addressed with the motive of bringing peace and order to the community. This measure will ensure that the aggressive individuals and brutal individuals become peaceful.

5.1 Bifurcation analysis

We now examine the bifurcation of the model (1). This will help us to ascertain whether the model exhibits a forward bifurcation or a backward bifurcation. The result of this analysis will help us to know if the condition "$R_0 < 1$"is enough to guarantee the "non-appearance" of the violence-persistent equilibrium. We shall establish this via the Centre Manifold Theorem as presented by Castillo-Chavez and Song (2004).[27]. The centre manifold theorem gives the local dynamics of the model around the violence-free equilibrium point, as we consider various values of a parameter of the model. Here, our interest is the dynamics around the violence-free equilibrium point with varying values of R_0.

Theorem 5 (Centre Manifold Theorem)

Consider the following general system of ordinary differential equations with a parameter ϕ.

$$\frac{dy}{dt} = f(y,\phi), \;\; f\colon \mathbb{R}^n \times \mathbb{R} \;\; and \quad f \in C^2(\mathbb{R}^n \times \mathbb{R}), \tag{16}$$

where 0 is an equilibrium point of the system (that is, $f(0,\phi) \equiv 0$ for all ϕ) and assume

A1: $A = D_y f(0,0) = \left(\frac{\partial f_i}{\partial y_i}(0,0)\right)$ is the linearization matrix of the system (29) around the equilibrium point 0 with ϕ evaluated at 0. Zero is a simple eigenvalue of A and other eigenvalues of A have negative real parts;

A2: Matrix A has a right eigenvector w and a left vector v (each corresponding to the zero eigenvalue).

Let f_k be the k^{th} component of f and

$$a = \sum_{i,j,k=1}^{n} v_k w_i w_j \frac{\partial^2 f_k}{\partial x_i \partial x_j}(0,0), \tag{17}$$

$$b = \sum_{i,k=1}^{n} v_k w_i \frac{\partial^2 f_k}{\partial x_i \partial \phi}(0,0), \tag{18}$$

The local dynamics of the system (16) around 0 is totally determined by the signs of a and b:

1. $a > 0,\ b > 0$. When $\phi < 0$ with $|\phi| \leq 1$,0 is locally asymptotically stable, and there exists a positive unstable equilibrium; when $0 < \phi \ll 1$,0 is unstable and there exists a negative and locally asymptomatically stable equilibrium.
2. $a < 0,\ b < 0$. When $\phi < 0$ with $|\phi| \ll 1$, 0 is unstable; when $0 < \phi \ll 1$, 0 is locally asymptotically stable, and there exists a negative unstable equilibrium.
3. $a > 0,\ b < 0$. When $\phi < 0$ with $|\phi| \ll 1$, 0 is unstable, and there exists a locally asymptotically stable negative equilibrium; when $0 < \phi \ll 1$, 0 is stable, and a positive unstable equilibrium appears.
4. $a < 0,\ b > 0$. When ϕ changes from negative to positive, 0 changes its stability from stable to unstable. Correspondingly*, a negative unstable equilibrium becomes positive and locally asymptotically stable.*

Particularly, if $a > 0$ and $b > 0$, then a backward bifurcation occurs at $\phi = 0$.

Proof:

We set

$$P = x_1, \qquad A = x_2, \qquad B = x_3.$$

Thus, the model (1) becomes

$$\left.\begin{aligned} \dot{x_1} &= \Lambda + \delta x_2 + \beta x_3 - (\chi + \eta + \mu)x_1 \\ \dot{x_2} &= \chi x_1 + \zeta x_3 - (\alpha + \delta + \eta + \mu)x_2 \\ \dot{x_3} &= \alpha x_2 - (\beta + \zeta + \eta + \mu)x_3 \end{aligned}\right\} \tag{19}$$

The Jacobian matrix evaluated at the VFE is given by

$$J_{\mathbb{E}_0} = \begin{pmatrix} -\eta - \mu & \delta - \kappa\xi\varphi\psi\omega & \beta - \gamma\kappa\xi\varphi\psi\omega \\ 0 & -\alpha - \delta - \eta - \mu + \kappa\xi\varphi\psi\omega & \zeta + \gamma\kappa\xi\varphi\psi\omega \\ 0 & \alpha & -\beta - \zeta - \eta - \mu \end{pmatrix} \tag{20}$$

Let $\kappa = \kappa^*$ be the bifurcation parameter. From the expression for R_0, we get

$$\kappa = \frac{(\alpha(\beta + \eta + \mu) + (\delta + \eta + \mu)(\beta + \zeta + \eta + \mu))}{(\beta + \alpha\gamma + \zeta + \eta + \mu)\xi\varphi\psi\omega} R_0. \tag{21}$$

When $R_0 = 1$, we get

$$\kappa = \frac{(\alpha(\beta + \eta + \mu) + (\delta + \eta + \mu)(\beta + \zeta + \eta + \mu))}{(\beta + \alpha\gamma + \zeta + \eta + \mu)\xi\varphi\psi\omega}. \tag{22}$$

From the characteristic equation of $J_{\mathbb{E}_0}$ given by $\left|J_{\mathbb{E}_0} - \lambda I\right| = 0$, I is the 3×3 identity matrix, we obtain the eigenvalues:

$$\begin{cases}\lambda_1 = 0, \\ \lambda_2 = -\eta - \mu, \\ \quad \lambda_3 = -\dfrac{\beta^2 + \alpha^2\gamma + \alpha\beta\gamma + \alpha\zeta + 2\beta(\zeta+\eta+\mu) + (\zeta+\eta+\mu)^2 + \alpha\gamma(\delta+\zeta+2(\eta+\mu))}{\beta + \alpha\gamma + \zeta + \eta + \mu}.\end{cases} \tag{23}$$

0 is a simple eigenvalue of $J_{\mathbb{E}_0}$. Now the right eigenvector $(w_1, w_2, w_3)^T$of $J_{\mathbb{E}_0}\big|_{\kappa=\kappa^*}$ is

$$\begin{cases} w_1 = -\dfrac{(\alpha+\beta+\zeta+\eta+\mu)}{\alpha} w_3 \\ w_2 = \dfrac{(\beta+\zeta+\eta+\mu)}{\alpha} w_3, \\ w_3 = g_3 > 0. \end{cases} \tag{24}$$

Similarly, the left eigenvector $(v_1, v_2, v_3)$of $J_{\mathbb{E}_0}\big|_{\kappa=\kappa^*}$ is

$$\begin{cases} v_1 = 0 \\ v_2 = \dfrac{(\beta+\alpha\gamma+\zeta+\eta+\mu)}{\zeta+\gamma(\alpha+\delta+\eta+\mu)} v_3, \\ v_3 = v_3 > 0. \end{cases} \tag{25}$$

Now, from equations (24) and (25), considering only the non-zero components of the left eigenvectors, we obtain:

$$a = -\frac{2(\eta+\mu)(\alpha\gamma+\beta+\zeta+\eta+\mu)(\alpha(\beta+\eta+\mu)+(\delta+\eta+\mu)(\beta+\zeta+\eta+\mu))(\alpha^2\gamma+(\beta+\zeta+\eta+\mu)^2)}{\alpha^2\Lambda(\beta+\alpha\gamma+\zeta+\eta+\mu)(\zeta+\gamma(\alpha+\delta+\eta+\mu))} {w_3}^2 v_3 < 0,$$

$$b = \frac{(\alpha\gamma+\beta+\zeta+\eta+\mu)(\beta+\alpha\gamma+\zeta+\eta+\mu)\xi\varphi\psi\omega}{\alpha(\zeta+\gamma(\alpha+\delta+\eta+\mu))} w_3 v_3 > 0.$$

Since$a < 0$ and $b > 0$, when κ changes from negative to positive (correspondingly, when R_0alters from $R_0 < 1$ to $R_0 > 1$), the VFE$\mathbb{E}_0$ changes its stability from stable to unstable. Furthermore, the negative unstable VPE becomes positive and locally asymptotically stable.See figure2.

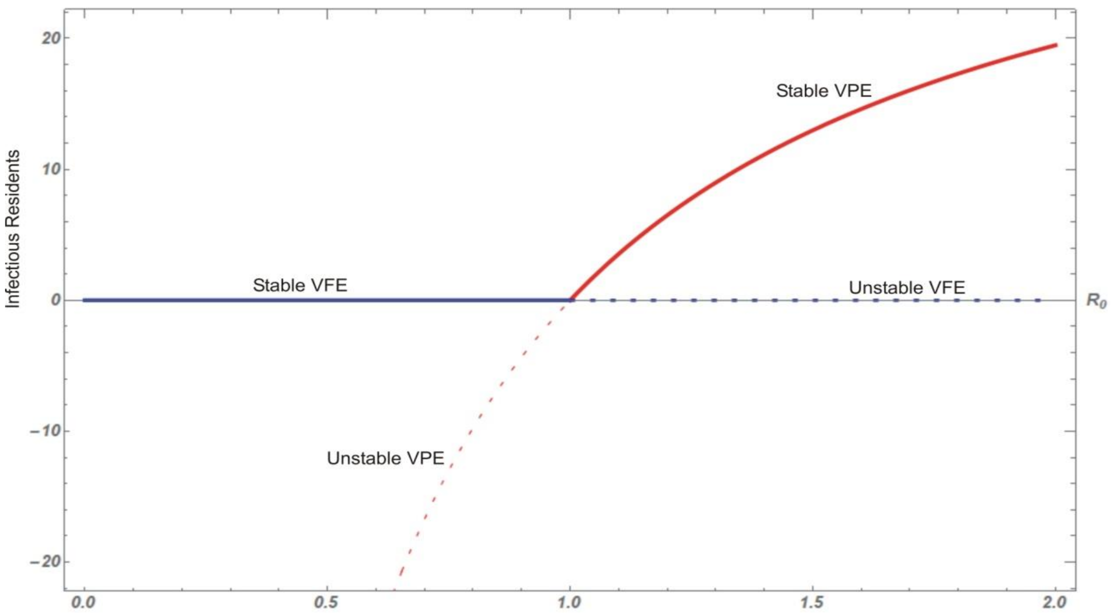

Fig. 2. Bifurcation plot

From the bifurcation plot, we see a forward bifurcation, and it becomes obvious that $R_0 < 1$ is enough to minimize the spread of violence and bring about peace stability in the community. The most sensitive parameters of R_0 have been detected and clearly stated in the sensitivity analysis of R_0.

5.2 Risk-level analysis

We present the violence-risk level analysis. Firstly, the questionnaire is presented. In line with the mathematical model, the questionnaire captures the parameters of the model such as infrastructural development (ζ), injustice (ψ), insecurity (φ), and threat to life and property (ω). The questionnaire was reviewed by some experts in the field for its validation.

Let

$$\left.\begin{aligned} A_1 &= Number\ of\ YES\ in\ section\ 1 \\ A_2 &= Number\ of\ NO\ in\ section\ 1 \\ B_1 &= Number\ of\ YES\ in\ section\ 2 \\ B_2 &= Number\ of\ NO\ in\ section\ 2 \\ C_1 &= Number\ of\ YES\ in\ section\ 3 \\ C_2 &= Number\ of\ NO\ in\ section\ 3 \\ D_1 &= Number\ of\ YES\ in\ section\ 4 \\ D_2 &= Number\ of\ NO\ in\ section\ 4 \end{aligned}\right\} \tag{26}$$

If $A_1 + C_1 > A_2 + C_2$, then the community is at low risk of intra-communal violence. Also, if $B_2 + D_2 < B_1 + D_1$, then the community is at low risk of intra-communal violence. Thus, the mean scores are:

$$x = \frac{A_2 + C_2 + B_1 + D_1}{4}, \tag{27}$$

$$y = \frac{A_1 + C_1 + B_2 + D_2}{4}. \tag{28}$$

We note that x which ranges from $0 - 5$determines the violence risk-level, while y which also ranges from $0 - 5$determines the peace-level of the community. Furthermore, if

$$\begin{aligned} x_1 &= \text{Risk level obtained from first respondent,} \\ x_2 &= \text{Risk level obtained from second respondent,} \\ &\vdots \\ x_n &= \text{Risk level obtained from nth respondent,} \end{aligned}$$

then the average risk level of the entire sample is given by

$$\bar{x} = \frac{x_1 + x_2 + \cdots + x_n}{n}. \tag{29}$$

Similarly, if

$$\begin{aligned} y_1 &= \text{Peace level obtained from first respondent,} \\ y_2 &= \text{Peace level obtained from second respondent,} \\ &\vdots \\ y_n &= \text{Peace level obtained from nth respondent,} \end{aligned}$$

then the average peace level of the entire sample is given by

$$\bar{y} = \frac{y_1 + y_2 + \cdots + y_n}{n}. \tag{30}$$

We make the following assumptions on the risk levels and clarify same using figure 3.

Risk level 0 (Maximum low risk)	-	Violence will not occur at all
Risk level 1 (Medium low risk)	-	Violence will not occur in most cases
Risk level 2 (Minimum low risk)	-	Violence may not occur in most cases
Risk level 3 (Minimum high risk)	-	Violence may occur in most cases
Risk level 4 (Medium high risk)	-	Violence will occur in most cases
Risk level 5 (Maximum high risk)	-	Violence is certain

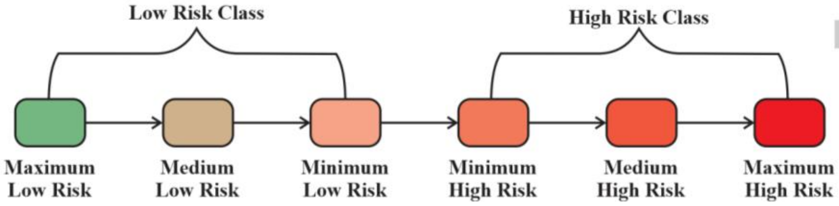

Fig. 3. Risk Levels

5.3 Questionnaire analysis

We analyze the questionnaires retrieved from the 100 respondents residing in the Obiaruku Community of Delta State, Nigeria. The analysis is based on the preliminaries presented in the risk level analysis.

Table 2. Community Risk Levels obtained from Respondents

Respondent	Section 1		Section 2		Section 3		Section 4		Risk l	Peace
	A_1	A_2	B_1	B_2	C_1	C_2	D_1	D_2		
Business men/women										
1.	0	5	2	3	3	2	4	1	3.25	1.75
2.	5	0	2	3	3	2	3	1	1.75	3.00
3.	2	3	2	3	3	2	4	1	2.75	2.25
4.	2	3	1	4	2	3	2	3	2.25	2.75
5.	3	2	2	3	3	2	4	1	2.50	2.50
6.	0	5	0	5	2	3	2	3	2.50	2.50
7.	4	1	3	2	3	2	2	3	2.00	3.00
8.	4	1	3	2	3	2	2	3	2.00	3.00
9.	0	5	0	5	2	4	3	2	3.00	2.25
10.	0	5	1	4	2	3	3	2	3.00	2.00
11.	1	4	1	4	3	2	2	3	2.25	2.75
12.	4	2	3	2	1	4	2	3	2.75	2.50
13.	2	3	2	3	2	3	4	1	3.00	2.00
14.	2	3	0	5	3	2	2	2	1.75	3.00
15.	2	3	2	3	1	4	3	2	3.00	2.00
16.	0	5	1	4	2	3	3	2	3.00	2.00
17.	0	5	0	5	2	3	2	3	2.50	2.50
18.	2	3	2	3	1	4	1	4	2.50	2.50
19.	0	5	1	4	4	1	3	2	2.50	2.50
Commercial Motorcyclists										
20.	0	5	1	4	2	3	3	2	3.00	2.00
21.	2	3	2	3	3	2	4	1	2.75	2.25
22.	5	0	5	0	2	3	2	3	2.50	2.50
23.	5	0	1	4	2	3	3	2	1.75	3.25
24.	2	3	2	3	3	2	3	2	2.50	2.50
25.	0	5	4	1	3	2	4	1	3.75	1.25
26.	1	4	0	5	3	2	2	3	2.00	3.00
27.	2	3	1	4	2	3	1	4	2.00	3.00

Respondent	Section 1		Section 2		Section 3		Section 4		Risk l	Peace
	A_1	A_2	B_1	B_2	C_1	C_2	D_1	D_2		
28.	3	2	0	5	1	4	1	4	1.75	3.25
29.	1	4	2	3	2	3	4	1	3.25	1.75
30.	1	4	0	5	2	3	2	3	2.25	2.75
31.	3	2	1	4	2	3	3	2	2.25	2.75
32.	1	4	1	4	2	3	2	3	2.50	2.50
33.	2	3	1	4	3	2	2	3	2.00	3.00
34.	2	3	1	4	2	3	2	3	2.25	2.75
35.	2	3	2	3	2	3	2	3	2.50	2.50
36.	2	3	1	4	2	3	2	3	2.25	2.75
37.	1	4	1	4	3	2	3	2	2.50	2.50
38.	2	3	2	3	2	3	3	2	2.75	2.25
Students										
39.	0	5	0	5	2	3	4	1	3.00	2.00
40.	3	2	1	4	3	2	3	2	2.00	3.00
41.	4	1	3	2	3	2	2	3	2.00	3.00
42.	3	2	2	3	3	2	2	3	2.00	3.00
43.	4	1	3	2	3	2	2	3	2.00	3.00
44.	1	4	0	5	2	3	2	3	2.25	2.75
45.	0	5	1	4	2	3	3	2	3.00	2.00
46.	5	0	4	1	3	2	2	3	2.00	3.00
47.	0	5	1	4	2	3	3	2	3.00	2.00
48.	0	5	2	3	0	5	3	2	3.75	1.25
49.	0	5	1	4	2	3	3	2	3.00	2.00
50.	0	5	1	4	2	3	3	2	3.00	2.00
51.	2	3	0	5	2	3	2	3	2.00	3.00
52.	1	4	0	5	2	3	2	3	2.25	2.75
53.	2	3	0	5	2	3	2	3	2.00	3.00
54.	4	1	3	2	4	1	3	2	2.00	3.00
55.	0	5	1	4	2	3	3	2	3.00	2.00
56.	2	3	2	3	2	3	2	3	2.50	2.50
57.	4	1	2	3	2	3	3	2	2.25	2.75
58.	3	2	1	4	3	2	2	3	1.75	3.25
Farmers										
59.	3	2	2	3	3	2	2	3	2.00	3.00
60.	3	2	2	3	3	2	2	3	2.00	3.00
61.	3	2	1	4	4	1	3	2	1.75	3.25
62.	2	3	2	3	4	1	3	2	2.25	2.75
63.	2	3	3	2	2	3	4	1	3.25	1.75
64.	3	2	0	5	2	3	2	3	1.75	3.25
65.	3	2	0	5	2	3	2	3	1.75	3.25
66.	1	4	1	4	2	3	2	3	2.50	2.50
67.	0	5	1	4	2	3	2	3	2.75	2.25
68.	0	5	2	3	2	3	3	2	3.25	1.75
69.	0	5	1	4	2	3	4	1	3.25	1.75
70.	2	3	1	4	4	1	3	2	2.00	3.00
71.	3	2	2	3	4	1	2	3	1.75	3.25
72.	1	4	0	5	4	1	4	1	2.25	2.75
73.	1	4	3	2	2	3	2	3	3.00	2.00
74.	2	3	1	4	2	3	2	3	2.25	2.75
Civil Servants										
75.	4	1	2	3	3	2	3	2	2.00	3.00
76.	4	1	3	2	3	2	2	3	2.00	3.00
77.	2	3	2	3	4	1	3	2	2.25	2.75
78.	3	2	2	3	3	2	4	1	2.50	2.50
79.	3	2	1	4	3	2	3	2	2.00	3.00

Respondent	Section 1		Section 2		Section 3		Section 4		Risk l	Peace
	A_1	A_2	B_1	B_2	C_1	C_2	D_1	D_2		
80.	3	2	0	5	4	1	3	2	1.50	3.50
81.	3	2	0	5	3	2	4	1	2.00	3.00
82.	2	3	1	4	3	2	2	3	2.00	3.00
83.	4	1	3	2	3	2	4	1	2.50	2.50
84.	5	0	1	4	2	3	3	2	1.75	3.25
85.	5	0	3	2	3	2	3	2	2.00	3.00
86.	1	4	0	5	3	2	2	3	2.00	3.00
87.	2	3	0	5	1	4	2	3	2.25	2.75
88.	2	3	1	4	2	3	2	3	2.25	2.75
89.	4	1	2	3	2	3	3	2	2.25	2.75
Traditional Rulers										
90.	5	0	4	1	4	1	2	3	1.75	3.25
91.	3	2	1	4	3	2	2	3	1.75	3.25
92.	4	1	2	4	1	4	4	1	2.75	2.50
93.	0	5	1	3	2	3	3	1	3.00	1.50
94.	1	4	1	4	4	1	2	3	2.00	3.00
95.	1	4	1	4	4	1	2	3	2.00	3.00
96.	1	4	1	4	5	0	2	3	1.75	3.25
97.	3	2	1	4	3	2	2	3	1.75	3.25
98.	0	5	1	3	2	3	3	1	3.00	1.50
99.	1	4	1	4	5	0	2	3	1.75	3.25
100.	2	3	1	4	2	3	2	3	2.25	2.75

It follows from equations (3) and (4) that the average risk level ($\bar{x}$)and the average peace level ($\bar{y}$)for the entire sample are:

$$\bar{x} = 2.36 \tag{33}$$

$$\bar{y} = 2.63 \tag{34}$$

Thus, the respondents perceived that Obiaruku is at the minimum low risk level and violence may not occur in most cases in the community. Pertaining to peace level, equation (24) reveals that the respondents perceived that Obiaruku is at minimum high peace level. The maximum low risk level and the maximum high peace level are achievable and it is required that residents, indigenes of the community, well-meaning individuals and the government, should wholeheartedly swing into action to ensure that the maximum high peace level and the maximum low risk level are achieved in the Obiaruku.

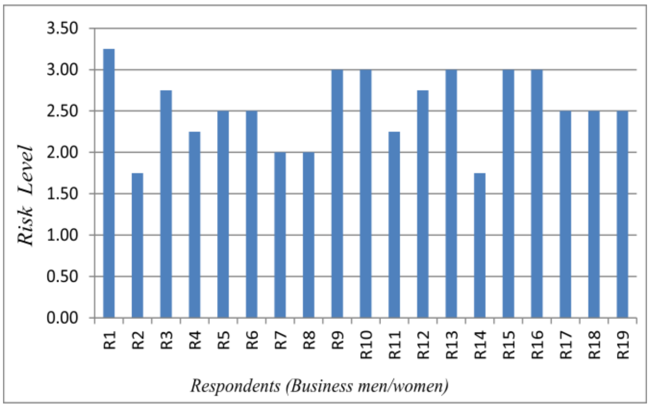

Fig. 4. Risk level as perceived by business men/women

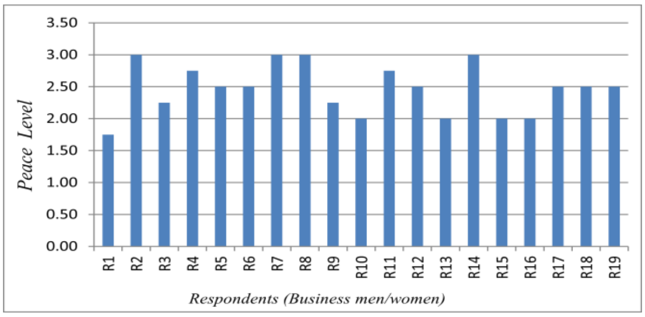

Fig. 5. Peace level as perceived by business men/women

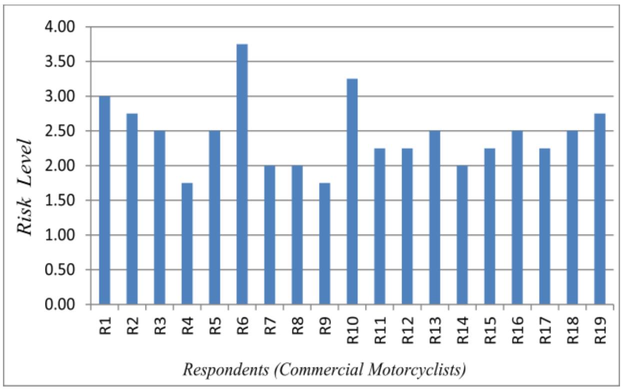

Fig. 6. Risk level as perceived by commercial motorcyclists

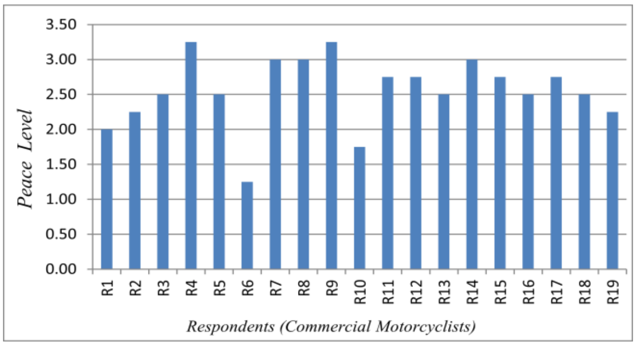

Fig. 7. Peace level as perceived by commercial motorcyclists

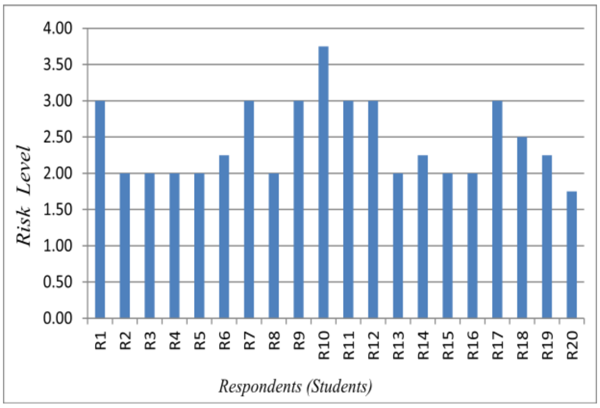

Fig. 8. Risk level as perceived by Students

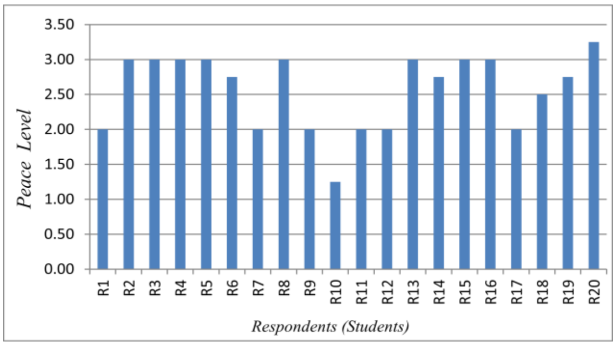

Fig. 9. Peace level as perceived by students

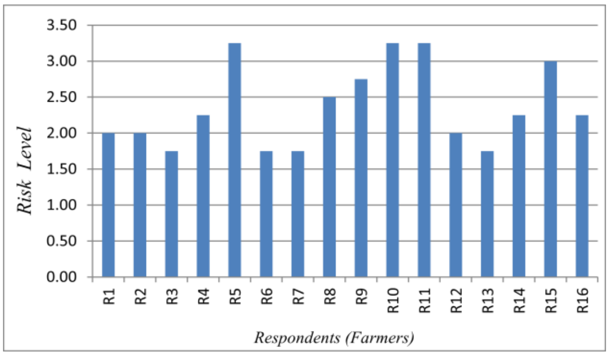

Fig. 10. Risk level as perceived by farmers

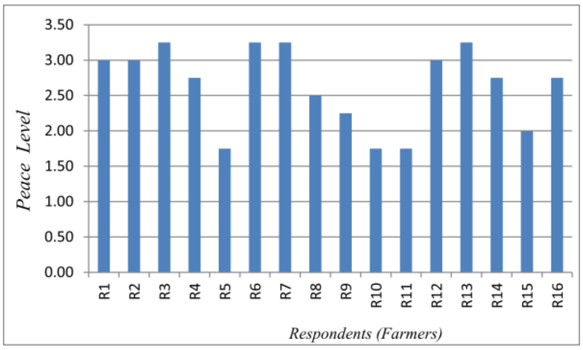

Fig. 11. Peace level as perceived by farmers

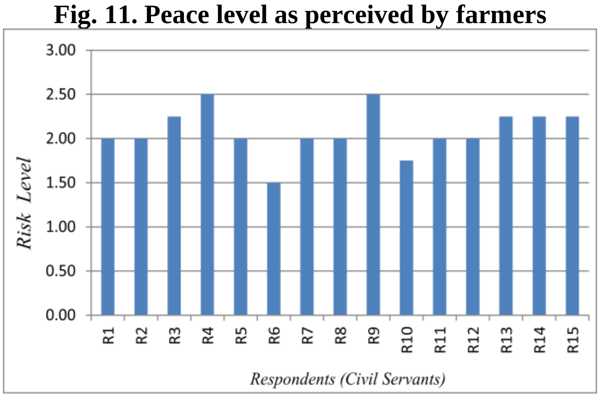

Fig. 12. Risk level as perceived by civil servants

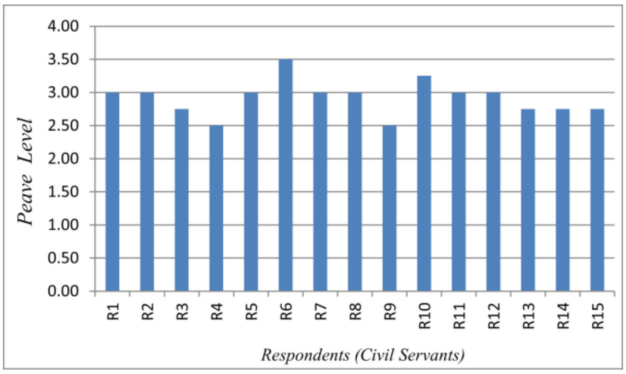

Fig. 13. Peace level as perceived by civil servants

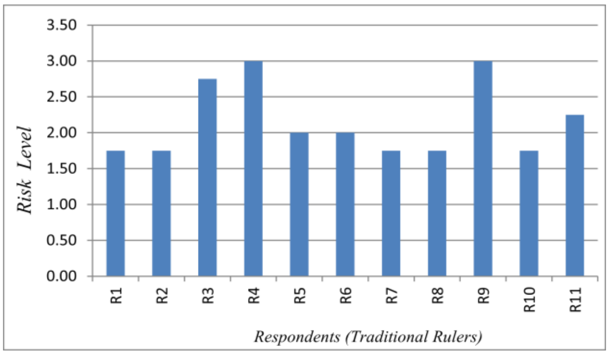

Fig. 14. Risk level as perceived by traditional rulers

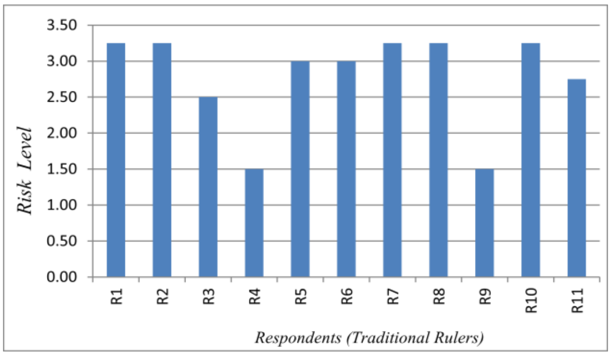

Fig. 15. Peace level as perceived by traditional rulers

The global stability of the VFE obtained guarantees that the maximum low risk level and the maximum high peace level can be achieved no irrespective of the size of the aggressive or brutal class. The recommendations given by the researcher under the sensitivity analysis will guide anyone saddled with the responsibility of restoring the maximum low risk level and the maximum high peace level to the community.

The perceptions of the different categories of the respondents are presented in the following charts.

6 Discussion and Conclusion

We have constructed a 3-compartment deterministic model to study intra-communal violence, where we have partitioned the residents of the community into the Peaceful Class, the Aggressive Class, and the Brutal Class. The mathematical analyses carried out on the model include the non-negativity of solutions, the invariant region and boundedness of solution, the violence-free equilibrium, the basic reproduction number, the violence-persistent equilibrium, the stability analysis, the sensitivity analysis, and the bifurcation analysis. The expression for the average number of secondary violence cases caused by a single aggressive or brutal individual within an entirely peaceful population during his/her infective period, was obtained via the method of next generation matrix. The violence-free equilibrium is locally and globally asymptotically stable, hence violence can be completely eradicated from the community, regardless of the initial population sizes of the peaceful individuals, the aggressive individuals and the brutal individuals. The bifurcation analysis revealed a forward bifurcation, thus $R_0 < 1$ is enough to minimize the spread of violence and bring about the stability of the violence-free equilibrium in the community. The computational software used is the Version 12 Mathematica Programming Software.

Under the sensitivity analysis we presented some vital suggestions that can help bring a community to the maximum low risk level and the maximum high peace level. The most sensitive parameters of the basic reproduction number have been detected and clearly stated. Injustice and insecurity are highly sensitive parameters of the basic reproduction number; hence a small increment in the values of these parameters can greatly trigger violence and offset peace within the community.

In order to obtain the violence risk level of Obiaruku community in Delta State, Nigeria, we also designed a questionnaire titled “Causes of Intra-Communal Violence”, and distributed 100 copies of the questionnaire to residents of the community. We analyzed and showed through charts obtained with the Microsoft Excel Software, the perceptions of the different categories of our respondents. Figure 3 reveals that about 68% of the business men and women perceived that the community is at minimum high risk level. In other words, they perceived that violence may occur in most cases in the community. While pertaining to peace level, we see from figure 4 that 63% of the farmers perceived minimum high peace level. 5% of the commercial motorcyclists (figure 5) perceived that the community is at the medium high risk level and so violence will occur in most cases, while about 47% perceived the minimum high risk level that violence may occur in most cases.About 74% (figure 6) of the commercial motorcyclists perceived that the community is at the minimum high peace level. From figure 7, we that 5% of the students perceived that the community is at the medium high risk level and so violence will occur in most cases, while 60% perceived that the community is at the minimum high risk level and so violence may occur in most cases. Pertaining peace level, 65% (figure 8) of the students perceived that the community is at minimum high peace level. Figure 9shows that about 63% of farmers perceived that the community is that minimum low risk level and so violence may not occur in most cases in the community, while about 38% of the farmers perceived that the community is at the minimum high risk level and that violence may occur in most cases in the community. From figure 9 we see that about 69% of the farmers perceived that the community is at the minimum high peace level. About 87% percent (figure 11) of the civil servants perceived that the community is at the minimum low risk level and that violence may not occur in most cases in the community. Figure 11 reveals that about 93% perceived that the community is at the minimum high peace level.73% (figure 13) of the traditional rulers perceived that the community is at the minimum low risk level and that violence may not occur in most cases in the community. About 82% (figure 14) of the traditional rulers perceived that the community is at the minimum high peace level.

The result of the questionnaire analysis revealed that the average perception of the residents of Obiaruku community in Delta State, Nigeria, is that the community is at the minimum low risk level and violence may not occur in most cases in the community. Thus, the community is not yet at the maximum low risk level where violence will not occur at all. The Obiaruku community in Delta State, Nigeria should employ the results of this research work in the community violence management and eradication, so as to ensure that the maximum high peace level and the maximum low risk level are achieved in the community.

We have shown that violence within a community can be studied theoretically in the mathematical sense, and the results of these analyses are important guidelines to individuals/organizations that are saddled with the responsibility of violence/crisis management in a community.

Competing Interests

Authors have declared that no competing interests exist.

REFERENCES

[1] Galles RJ, Straus MA. Determinants of violence in the family: Towards a theoretical integration. Chapter 21 in Contemporary Theories About the Family. edited by Wesley R. Burr, Reuben Hill, F. Ivan Nye, and Ira L. Reiss. Volume 1. Free Press. New York; 1979.

[2] Galtung J. Cultural violence. Journal of Peace Research. 1990;27(3):291-305.

[3] Stark E, Anne F, William F. Medicine and Patriarchal Violence: The Social Construction of a 'Private' Event. International Journal of Health Services. 1979;98:461-91.

[4] Coker-Appiah D, Cusack K. Breaking the silence and challenging the myths of violence against women and children in Ghana: report on a national study of violence. Accra, Gender Studies & Human Rights Documentation Centre; 1999.

[5] ShewaferaWT, Birhanu BT. Mathematical Modeling Investigation of Violence and Racism Coexistence as a Contagious Disease Dynamics in a Community, Computational and Mathematical Methods in Medicine; 2022.
Article ID 7192795, 13 pages, 2022.
Available: https://doi.org/10.1155/2022/7192795

[6] Patten SB, Arboleda-Flórez JA. Epidemic theory and group violence," Social Psychiatry and Psychiatric Epidemiology. 2004;39(11):853–856.

[7] Khan MA, Ali K, Bonyah E, Okosun KO, Islam S, and Khan A. Mathematical modeling and stability analysis of pine wilt disease with optimal control," Scientific Reports. 2017;7(1):1–19.

[8] Khan MA, Ullah S, Farhan M. The dynamics of Zika virus with Caputo fractional derivative," AIMS Mathematics. 2019;4(1):134–146.

[9] Karthikeyan K., Karthikeyan P, Baskonus HM, Venkatachalam K, Chu YM. Almost sectorial operators on Ψ-Hilfer derivative fractional impulsive integro-differential equations, Mathematical Methods in the Applied Sciences. 2021;44.

[10] Jin F, Qian ZS, Chu YM, Rahman MU. On nonlinear evolution model for drinking behavior under Caputo-Fabrizio derivative," Journal of Applied Analysis & Computation. 2022;12(2):790–806.

[11] Lazarus SM. Mathematical Modeling of Ethnic Violence Using Differential Equations and Designing of Violence Risk-Level Determinant Case Study of Nasarawa South Senatorial, Nasarawa State; 2014.

[12] De la Poza E, Jódar L, Barreda S. Mathematical modeling of hidden intimate partner violence in Spain: a quantitative and qualitative approach, Abstract and Applied Analysis. 2016; 8.

[13] Dominioni G, Marasco A, Romano A. A mathematical approach to study and forecast racial groups interactions: deterministic modeling and scenario method," Quality & Quantity. 2018;52(4):1929–1956.

[14] Lemecha L, Feyissa S. Mathematical modeling and analysis of corruption dynamics," Ethiopian Journal of Science and Sustainable Development. 2018;5(2):13–25.

[15] Delgadillo-Aleman S, Ku-Carrillo R, Perez-Amezcua B, Chen-Charpentier B. A mathematical model for intimate partner violence," Mathematical and Computational Applications. 2019;24(1):29.

[16] Danford O, Kimathi M, Mirau S. Mathematical modelling and analysis of corruption dynamics with control measures in Tanzania, Journal of Mathematics and Informatics. 2020;19:57–79.

[17] Mamo DK. Modeling the spread dynamics of racism in cyberspace," Journal of Mathematical Modeling. 2020;8(2):105–122.

[18] Mamo DK. Modeling the transmission dynamics of racism propagation with community resilience, Computational Social Networks. 2021;8(1):1–18.

[19] Ossaiugbo MI, Okposo NI. Mathematical modeling and analysis of pneumoniainfection dynamics. Science World Journal. 2021;19(2):73-80.

[20] Okposo NI, Jonathan AM, Okposo EN, Ossaiugbo M. Existence of solutions and stability analysis for a fractional helminth transmission model within the framework of Mittag-Leffler kernel. Nigerian Journal of Science and Environment. 2021;19(1):67-80.

[21] Fantaye AK, Birhanu ZK. Mathematical model and analysis of corruption dynamics with optimal control," Journal of Applied Mathematics. 2022;16.

[22] Wiley SA, Levy MZ,Branas CC. The impact of violence interruption on the diffusion of violence: a mathematical modeling approach, Advances in the Mathematical Sciences, Springer, Cham. 2016;225–249,

[23] Tsetimi J, Ossaiugbo MI, Atonuje A. Bifurcation Analysis of a Mathematical Model for the Covid-19 Infection among Pregnant and Non-Pregnant Women. European Journal of Pure and Applied Mathematics.2022;15(2):537-556.
Available: https://doi.org/10.29020/nybg.ejpam.v15i2.4312.

[24] Van den Driessche P, Watmough J. Reproduction Numbers and Sub-Threshold Endemic Equilibria for Compartmental Models of Disease Transmission," Mathematical Biosciences. 180:29-48.
Available: https://doi.org /10.1016/S0025-5564(02)00108-6, 2002.

[25] Ana L, James Y. A deterministic model for gonorrhea in a non-homogeneous population," Mathematical Biosciences. 1976;28:221–236.

[26] Michael YL, John RG, Liancheng W, Janos K. Global dynamics of a SEIR model with varying total population size," Mathematical Biosciences. 1999;160:191–213.

[27] Castillo-Chavez C, Song B. Dynamical models of tuberculosis and theirapplications, Mathematical Biosciences and Engineering. 1:361-404
Available: https://doi.org/10.3934/mbe.2004.1.361,2004.

[28] Kizito M, Tumwiine J. A Mathematical Model of Treatment and Vaccination Interventions of Pneumococcal Pneumonia Infection Dynamics, Journal of Applied Mathematics; 2018.
Available: https://doi.org/10.1155/ 2018/2539465, pp 1-15, 2018.

Appendix 1

CAUSES OF INTRA-COMMUNAL VIOLENCE

Instruction: Please answer the following questions by ticking ONLY the appropriate box provided.
Category: Traditional Ruler , Civil Servant , Student , Commercial Motorcyclist , Business Man/Woman , Farmer , Others (Please specify):______________

Section 1 (Infrastructural Development)

1. Do you have a standard public primary school in your community? Yes No
2. Do you have a standard public secondary school in your community? Yes No
3. Do you have a recognized higher institution in your community? Yes No
4. Are the roads within your community usable by motor owners? Yes No
5. Do you have good tarred roads that link to your community? Yes No

Section 2 (Injustice)

6. Are political appointments domiciled in some families in your community? Yes No
7. Has anyone in your family held a political position in your community? Yes No

8. Do all families equally benefit from government resources in your community? Yes No
9. Do all the traditional rulers in your community have good sense of judgment? Yes No
10. Have you or anybody from your family ever been deprived of a right by members of families? Yes No

Section 3 (Security Strength)

11. Do you have a standard police station in your community? Yes No
12. Does the police respond promptly and positively when duty calls? Yes No
13. Are there other security groups apart from the police in your community? Yes No
14. Are the police able to prevail over all crime scenes in your community? Yes No
15. Do people take hard drugs or smoke weeds in public in your community? Yes No

Section 4 (Threat To Life And Property)

16. Are there cases of robbery, kidnapping, rape or killing in your community? Yes No
17. Has anybody in your family been confronted by civilian(s) with sophisticated weapon(s) in your community? Yes No
18. Have you ever had a misunderstanding with any person over a right/property within the community? Yes No
19. Can a person walk freely at night in your community without any harm? Yes No
20. Do you think that a person can be afflicted with sickness or harmed with spiritual powers in your community? Yes No

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