



Modified Preston-Bennett Method for Estimation of Adult Mortality in Developing Countries

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Authors' contributions

This work was carried out in collaboration among all authors. All authors read and approved the final manuscript.

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ABSTRACT

This study proposes a modified Preston-Bennett method for the estimation of adult mortality in developing countries. Unlike the previous method, the proposed method derives estimates of adult mortality directly from the 5-year age distribution at the mid-point of the inter-censal period. In other words, it does not convert the 5-year age distributions to single years before estimation. Results obtained from empirical studies compared favourably well with those from the original Preston-Bennett method and therefore, recommend for the estimation of adult mortality in developing countries.

Keywords: Adult mortality; census; expectation of life; survival ratio; inter-censal.

1. INTRODUCTION

Mortality beyond age five, called adult mortality throughout this article, appear more difficult to estimate especially in developing countries. With good quality data from developed countries modeling of adult mortality appears easier than

in developing countries with the problem of poor data quality [1]. In most developing countries data on adult mortality come from censuses and sample surveys [2,1]. Data from these two sources have been shown to be defective, hence the resort to indirect techniques for its estimation in developing countries [3,1,4,5,6,7,8,9,10].

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Among the indirect techniques for estimation of adult mortality, the most popular ones are those based on data from two censuses age distributions. These include those of Stolnitz [11], Coale and Hoover [12], Carrier and Hobcraft [13], Preston [14], Preston and Bennett [15], and United Nations [16]. Of these methods, Stolnitz [11], Coale and Hoover[12], and Carrier and Hobcraft [13] require intercensal period of 5-years or multiples of 5-years, while Preston [14], Preston and Bennett [15], and United Nations [16] Synthetic Survival Ratio do not insist on intercensal period that is a multiple of 5-years. Of these methods based on arbitrary intercensal period, the integrated method introduced by Preston [14] requires a life table, which is not usually available for most developing countries.

There is a sluggish improvement in the expectation of life of most developing countries although there have been massive interventions by donor agencies to change the poor health conditions and improve the standard of living of people in these countries. These unimpressive estimates may be largely due to a wrong model and not diseases such as (HIV/AIDS, Malaria, etc.) as often alluded. The Preston-Bennett Method often used for estimation of Adult mortality in the developing countries due data availability first converts the 5-year age groups to single-years before using it to derive estimates of adult mortality, without minding the effects of single-years data which is well known in demographic estimation. Therefore, any method that would use the population by 5-year age groups as reported could yield a more accurate estimate.

This paper is organized as follows, Section 2 deals with Preston-Bennett Method, Section 3 is devoted to the modified Preston-Bennett Method, Section 4 presents the empirical examples and results, Section 5 talks about discussion of results while Section 6 is dedicated to conclusions and recommendations.

2. PRESTON-BENNETT METHOD

Preston-Bennett method (PBM), Preston and Bennett [15], for estimation of adult mortality is being considered for modification in this study because it does not require of:

- (a) an intercensal period that is a multiple of 5-years
- (b) a standard life table for its implementation.

Other advantages of the method are the following:

- (a) only the age and sex structures of two censuses are required,
- (b) the technique does not assume that the population is stable,
- (c) It only assumes that:
 - (i) the population is closed to migration (or migration is negligible) during the inter-censal period,
 - (ii) both population censuses have the same degree of completeness of enumeration,
 - (iii) age misreporting is the same in both censuses.

Furthermore, PB's measure for estimation adult mortality is the expectation of life at age y ,

$$e_y = \frac{T_y}{l_y} = \frac{\sum_{x=y}^{\omega} {}_5L_x + L(\omega+)}{l_y}, \quad y \geq 5 \quad (1)$$

where ω is the beginning of the open age interval,

$${}_5L_y = {}_5\bar{N}_y \exp\left[5 \sum_{x=0}^{y-5} {}_5r_x + 2.5 \cdot ({}_5r_y)\right] \quad (2)$$

number of person- years lived between ages y and $y+5$ years, $L(\omega+)$ is the person-years lived from age ω and above, and for $y = 10, 15, \dots, \omega - 5$

$$l_y = \frac{1}{10} \left[{}_5\bar{N}_y \exp[2.5 \cdot ({}_5r_y)] + {}_5\bar{N}_{y-5} \exp[-2.5 \cdot ({}_5r_{y-5})] \right] \quad (3)$$

the survivors to exact age y , with

$${}_5\bar{N}_y = \frac{[{}_5N_y^{t_2} - {}_5N_y^{t_1}]}{t \cdot {}_5r_y},$$

the mid-point population, and

$${}_5r_y = \frac{1}{t} \text{Ln} \left(\frac{{}_5N_y^{(t)}}{{}_5N_y^{(0)}} \right),$$

the age-specific growth rate. Here ${}_5N_y^{(0)}$ and ${}_5N_y^{(t)}$ are the populations at the initial and end of the interval t while Ln is simply the log to base e .

For the open age interval $(\omega+)$, Preston and Bennett [15] gave an expression for the estimate of the number of person- years lived from ω years and above as

$$L(\omega+) = \bar{N}(\omega+) \exp[R(\omega+)], \tag{4}$$

Where R is the growth rate of the population aged ω years and above, where $\bar{N}(\omega+)$ is the population aged ω years and above and

$$R(\omega+) = a(\omega) + b(\omega) \cdot r(10+) + c(\omega) \cdot Ln \left[\bar{N}(45+) / \bar{N}(10+) \right] + 5 \sum_{y=0}^{\omega-5} {}_5r_y \tag{5}$$

$a(\omega)$, $b(\omega)$ and $c(\omega)$ are constants given in Appendix A, $\bar{N}(10+)$ is the mid-period population aged 10 years and above, $\bar{N}(45+)$ is the mid-period population aged 45 years and above while $r(10+)$ is the corresponding inter-censal growth rate.

On the other hand, for the age range $5 \leq y < \omega - 5$ the life table population aged $(y, y+n)$, ${}_5L_y$, is computed from l_y using the expression

$${}_5L_y = \frac{5}{2} (l_y + l_{y+5})$$

Conversely, given ${}_5L_y$ and ${}_5L_{y-5}$, l_y may be computed as

$$l_y = \frac{1}{10} ({}_5L_y + {}_5L_{y-5}) \tag{6}$$

From equation (2),

$$\begin{aligned} {}_5L_y &= {}_5\bar{N}_y \exp \left[5 \sum_{x=0}^{y-5} {}_5r_x + 2.5 \cdot ({}_5r_y) \right] \\ &= {}_5\bar{N}_y \exp \left[5 \sum_{x=0}^{(y-5)-5} {}_5r_x + 5 \cdot ({}_5r_{y-5}) + 2.5 \cdot ({}_5r_y) \right] \\ &= {}_5\bar{N}_y \exp [2.5 \cdot ({}_5r_y)] \exp \left[5 \sum_{x=0}^{(y-5)-5} {}_5r_x + 5 \cdot ({}_5r_{y-5}) \right] \end{aligned} \tag{7}$$

Similarly, from equation (2) we have

$${}_5L_{y-5} = {}_5\bar{N}_{y-5} \exp \left[5 \sum_{x=0}^{(y-5)-5} {}_5r_x + 2.5 \cdot ({}_5r_{y-5}) \right] \tag{8}$$

Combining (6), (7) and (8), we have

$$\begin{aligned}
 l_y &= \frac{1}{10} \left\{ {}_5\bar{N}_y \exp[+2.5 \cdot ({}_5r_y)] \exp\left[5 \sum_{x=0}^{(y-5)-5} {}_5r_x + 5 \cdot ({}_5r_{y-5})\right] + {}_5\bar{N}_{y-5} \exp\left[5 \sum_{x=0}^{(y-5)-5} {}_5r_x + 2.5 \cdot ({}_5r_{y-5})\right] \right\} \\
 &= \frac{\exp\left[5 \sum_{x=0}^{(y-5)-5} {}_5r_x + 5 \cdot ({}_5r_{y-5})\right]}{10} \left\{ {}_5\bar{N}_y \exp[+2.5 \cdot ({}_5r_y)] + {}_5\bar{N}_{y-5} \exp[2.5 \cdot ({}_5r_{y-5}) - 5 \cdot ({}_5r_{y-5})] \right\} \\
 &= \frac{\exp\left[5 \sum_{x=0}^{(y-5)} {}_5r_x\right]}{10} \left\{ {}_5\bar{N}_y \exp[+2.5 \cdot ({}_5r_y)] + {}_5\bar{N}_{y-5} \exp[-2.5 \cdot ({}_5r_{y-5})] \right\}
 \end{aligned}$$

That is, for the age range $5 \leq y < \omega - 5$

$$l_y = \frac{\exp\left[5 \sum_{x=0}^{(y-5)} {}_5r_x\right]}{10} \left\{ {}_5\bar{N}_y \exp(+2.5 {}_5r_y) + {}_5\bar{N}_{y-5} \exp(-2.5 {}_5r_{y-5}) \right\} = N(y).$$

Observe that, N_y is biased with respect to the l_y defined by Preston and Bennett (1983) in (3) unless

$$\exp\left[5 \sum_{x=0}^{(y-5)} {}_5r_x\right] = 1, \quad \text{for } y = 10, 15, 20, \dots$$

Furthermore, while the estimate of T_y in (1) is a product of interpolation between the first and second censuses, the estimate N_y is a product of double interpolation. First from the two censuses to obtain ${}_5\bar{N}_y$ and second from two adjacent age groups ${}_5\bar{N}_{y-5}$ and ${}_5\bar{N}_y$. Thus, l_y given in (3) suffer from errors not only in ${}_5\bar{N}_{y-5}$ computed from ${}_5N_{y-5}^{t_1}$ and ${}_5N_{y-5}^{t_2}$ but errors from ${}_5\bar{N}_y$ computed from ${}_5N_y^{t_1}$ and ${}_5N_y^{t_2}$.

Therefore, the ultimate objective of this study is to obtain a revised model that will give improved estimate of adult mortality from the age-sex distribution of population at two points in time. Specifically, (i) to obtain the mid-point population, ${}_5\bar{N}_y$, and age-specific inter-censal growth rate, ${}_5r_y$, (ii) to obtain the life table persons-year lived, ${}_5L_y$, (iii) and hence obtained a measure of adult mortality (expectation of life

at age x , $x > 5$; (iv) to illustrate the applicability of the proposed model with empirical examples.

3. MODIFIED PRESTON-BENNETT METHOD

By analogy to the derivation made by Preston and Bennett (1983) for age in single years, given the life table survival ratio, ${}_5P_{(x-5)}^{(5)}$, from age group $(x-5, x)$ to age group $(x, x+5)$

$${}_5P_{x-5}^{(5)} = \frac{{}_5L_x}{{}_5L_{x-5}}$$

the population aged $(x, x+5)$ at time t , ${}_5N_x(t)$ may be derived from population aged $(x-5, x)$ at time $t-5$, ${}_5N_{x-5}(t-5)$ as

$${}_5N_x(t) = {}_5N_{x-5}(t-5) {}_5P_{x-5}^{(5)} \tag{9}$$

Furthermore, given the average age-specific growth rate ${}_5r_x$, the population aged $(x, x+5)$ at

time t can be derived from population aged $(x, x+5)$ at time $t-5$ as

$${}_5N_x(t) = {}_5N_x(t-5) \exp[5 \cdot ({}_5r_x)]$$

Hence,

$${}_5N_{x-5}(t) = {}_5N_{x-5}(t-5) \exp[5 \cdot ({}_5r_{x-5})]$$

or

$${}_5N_{x-5}(t-5) = {}_5N_{x-5}(t) \exp[-5 \cdot ({}_5r_{x-5})] \quad (10)$$

Substituting (10) into (9), we have

$$\begin{aligned} {}_5N_x(t) &= {}_5N_{x-5}(t) \exp[-5 \cdot ({}_5r_{x-5})] {}_5P_{x-5}^{(5)} \\ &= {}_5N_{x-5}(t) \exp[-5 \cdot ({}_5r_{x-5})] \frac{{}_5L_x}{{}_5L_{x-5}} \quad (11) \end{aligned}$$

all referring to the same time point t . Specifically, for $x=5, 10, 15, \dots, w-5$

$${}_5N_5(t) = ({}_5N_0(t) \exp[-5({}_5r_0)]) \frac{{}_5L_5}{{}_5L_0}$$

$${}_5N_{10}(t) = ({}_5N_5(t) \exp[-5({}_5r_5)]) \frac{{}_5L_{10}}{{}_5L_5}$$

$${}_5N_{\omega-5}(t) = ({}_5N_{(\omega-5)-5}(t) \exp[5 \cdot ({}_5r_{(\omega-5)-5})]) \frac{{}_5L_{\omega-5}}{{}_5L_{(\omega-5)-5}}$$

3.1 Estimation of Adult Mortality

The proposed method derives the expression for adult mortality from ${}_5L_x$, the life table population aged $(x, x+5)$, by using two census age distributions at the midpoint of the interval. From (11)

$$\frac{{}_5L_x}{{}_5L_{x-5}} = \frac{{}_5\bar{N}_x}{{}_5\bar{N}_{x-5}} \exp[5 \cdot ({}_5r_{x-5})]$$

Hence, for $x = 5, 10 \dots \omega - 5$

$${}_5L_x = {}_5L_{x-5} \frac{{}_5\bar{N}_x}{{}_5\bar{N}_{x-5}} \exp[5 \cdot ({}_5r_{x-5})] \quad (12)$$

To achieve this, we adopted an approach using chain-based method (CBM).

3.2 Chain-Base Method

In this approach the person- years lived ${}_5L_x$ in (12) is expressed in terms of the ${}_5L_{x-5}$ and ${}_5N_{x-5}$ as

$${}_5L_x = {}_5L_{x-5} \left(\frac{{}_5\bar{N}_x}{{}_5\bar{N}_{x-5}} \right) \exp[5 \cdot ({}_5r_{x-5})],$$

for $x = 5, 10 \dots \omega - 5$. Specifically, for $x = 5, 10 \dots \omega - 5$

$${}_5L_5 = {}_5L_0 \left(\frac{{}_5\bar{N}_5}{{}_5\bar{N}_0} \right) \exp[5 \cdot ({}_5r_0)]$$

$${}_5L_{10} = {}_5L_5 \left(\frac{{}_5\bar{N}_{10}}{{}_5\bar{N}_5} \right) \exp[5 \cdot ({}_5r_5)]$$

$${}_5L_{\omega-5} = {}_5L_{(\omega-5)-5} \left(\frac{{}_5\bar{N}_{\omega-5}}{{}_5\bar{N}_{(\omega-5)-5}} \right) \exp[5 \cdot ({}_5r_{(\omega-5)-5})]$$

This method is expected to perform better when fertility and mortality rates are changing over time.

3.3 Estimation of Person-Years Lived for the Open Ended Interval

To obtain estimate of the person-years lived for the open ended interval, $L(\omega+)$, we adopted the Preston-Bennett approach given in (4) and (5) with coefficients $a(\omega)$, $b(\omega)$, and $c(\omega)$ given in Appendix B.

3.4 Estimation of ${}_5L_0$

The value of person-years lived between exact ages 0 and 4, ${}_5L_0$, can be obtained from surveys with reference period close to the mid-point of the intercensal period of the two censuses. Kpedekpo (1982) gave the following expressions by ${}_1L_0$ and ${}_4L_1$ as

$${}_1L_0 = 0.3l_0 + 0.7l_1 \quad \text{and} \quad {}_4L_1 = 1.3l_1 + 2.7l_5,$$

where,

$$l_0 = 1, \text{ and } {}_5L_0 = {}_4L_1 + {}_1L_0 = 0.3 + 2l_1 + 2.7l_5.$$

4. EMPIRICAL EXAMPLES AND RESULTS

In this Section, the proposed methods were applied to data from some developing countries while comparing the results with results from PBM and some other related methods.

4.1 Nigeria 1991-2006 (Females)

The first empirical example is the application of the proposed method to the age-sex data from the 1991 and 2006 Nigeria censuses (Female population) shown in Table 1 (see appendix 1a). The quality of demographic data from Nigeria is known to be poor or incomplete. It poses a serious challenge for all direct or indirect demographic estimation. Figure 1 shows that at age 5, it appears a female in Nigeria experiencing the observed age-specific growth

rate is expected to live about 67 years while at age 80, it is approximately 13 years for the period. The result further shows that the estimates of the life expectancies obtained dropped consistently from one age group to another.

4.2 Zambia 2000 – 2010 (Females)

Table 2 (in appendix 1b), shows the results obtained by applying the proposed model to the Zambia Female population, 2000 – 2010. The results obtained exhibited the pattern observed in Figure 1. At age 5, the expectation of life is approximately 65 years while their counterpart recorded 9.5 years at age 80. The result further show that the age-specific estimates of the life expectancies obtained dropped by 5 years from 5 – 9 year age group to 35 – 39 years and subsequently by about 3 years. Zambia just like other developing countries has the problem of poor quality of demographic data.

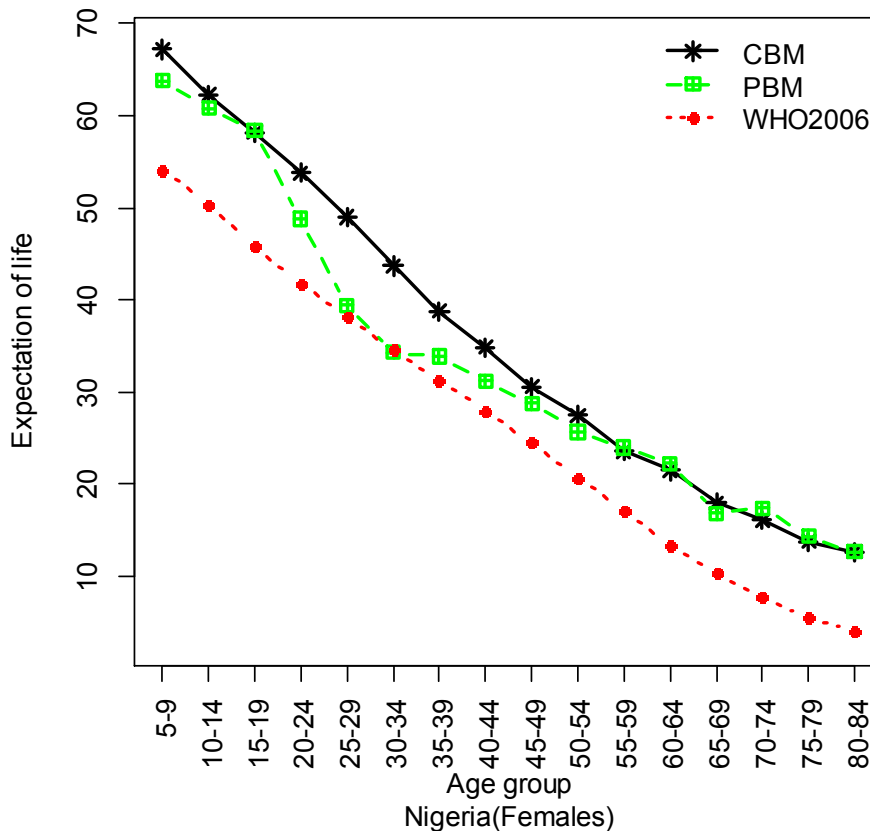


Fig. 1. Plot of age groups against Expectation of life for Nigeria female populations (1991 – 2006)

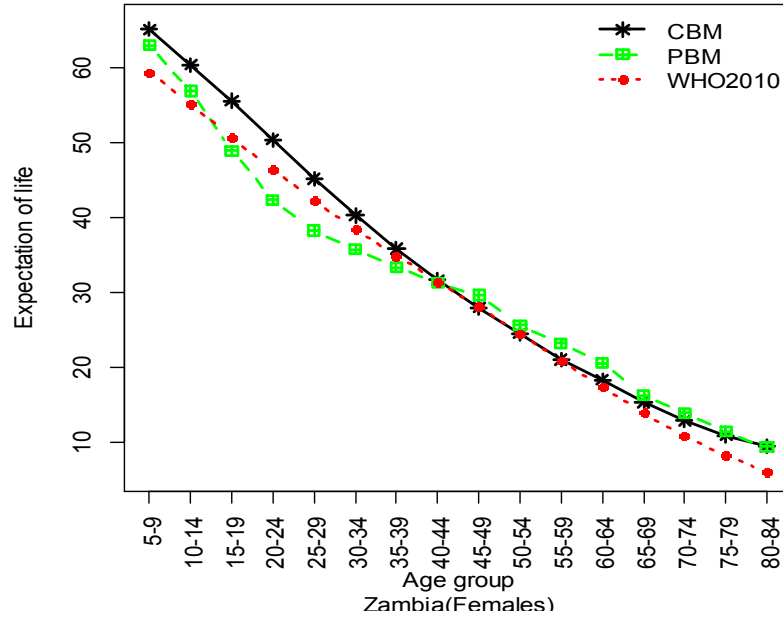


Fig. 2. Plot of age groups against Expectation of life for Zambia female populations (2000 – 2010)

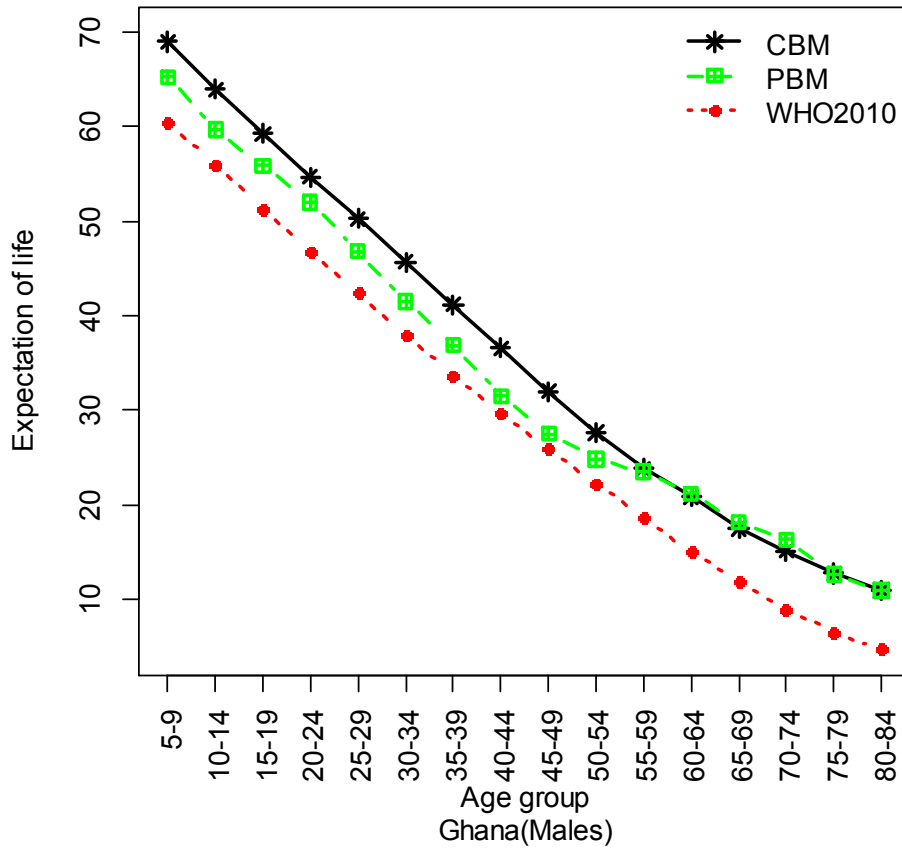


Fig. 3. Plot of age groups against Expectation of life for Ghana male populations (2000 – 2010)

Table 4. Expectation of Life (e_x) from the proposed model for selected countries and some related models

Age	Nigeria (Females)			Zambia (Females)			Ghana (Males)		
	CBM	PBM	WHO 2006	CBM	PBM	WHO 2010	CBM	PBM	WHO 2010
5-9	67.2	63.8	54.1	65.1	63.0	59.3	69.0	65.3	60.4
10-14	62.3	60.8	50.3	60.4	56.9	55.2	64.0	59.7	56.0
15-19	58.2	58.4	45.9	55.6	48.9	50.7	59.3	55.9	51.3
20-24	53.8	48.8	41.8	50.4	42.4	46.4	54.7	52.0	46.8
25-29	49.0	39.4	38.1	45.2	38.3	42.3	50.3	46.8	42.4
30-34	43.7	34.3	34.6	40.3	35.8	38.5	45.7	41.5	37.9
35-39	38.8	33.9	31.3	35.9	33.4	34.9	41.2	36.9	33.7
40-44	34.9	31.2	28.0	31.7	31.3	31.5	36.6	31.5	29.7
45-49	30.6	28.8	24.5	28.0	29.7	28.1	32.0	27.5	25.9
50-54	27.5	25.7	20.7	24.6	25.7	24.6	27.6	24.8	22.2
55-59	23.6	24.0	17.1	21.1	23.3	21.0	23.8	23.5	18.6
60-64	21.5	22.3	13.4	18.4	20.6	17.4	20.8	21.2	15.0
65-69	18.0	16.9	10.4	15.4	16.3	14.0	17.5	18.1	11.8
70-74	16.1	17.4	7.7	13.0	14.0	10.9	15.1	16.3	8.9
75-79	13.8	14.4	5.6	11.0	11.5	8.3	12.7	12.6	6.5
80-84	12.7	12.7	4.0	9.5	9.5	6.1	10.9	10.9	4.7

Note: World Health Organization (WHO) life expectancy estimates retrieved from WHO Global Health Observatory Data Repository. Life expectancy estimates of PBM were computed by the Authors

4.3 Ghana 2000- 2010 (Males)

The final empirical example is the application of the proposed method to Ghana (Male population) between 2000 and 2010. Although Ghana appears to have a better quality of demographic data than Nigeria—it is still poor compared to developed countries. Figure 3 (also see Table 3 in appendix 1c), shows that the age-specific estimates of the life expectancies obtained appears linearly distributed from ages 5 to 60, and became irregular thereafter. At age 5, it appears a male in Ghana experiencing the observed age-specific growth rate is expected to live about 69 years and approximately 11 years at age 80 respectively.

5. DISCUSSION

The proposed model here builds upon existing methods. The model was applied to age-sex data from three countries, namely Nigeria, Zambia, and Ghana. The quality of Nigeria demographic data is known to be poor or incomplete. It poses a serious challenge for all direct or indirect demographic estimations. Even at that, the life expectancy estimates obtained for Nigeria by applying the proposed model appears useful. The life expectancy estimates of the proposed model for Nigeria were fairly higher than the PBM estimates from ages 5 to 50 years; levelling up at

55 years and both dropping consistently till the end (see Table 4). The average of the first three life expectancies is 62.6 years for the proposed model, 61.0 years for PBM while WHO recorded 50.1 years. The proposed model and PBM appear to be more than 10 years higher compared to WHO estimates judging by the mean of the first three life expectancy estimates of each of the models for Nigeria. The age-specific life expectancy estimates of the proposed model from ages 5 to 50 appear to be linearly distributed.

The pattern of life expectancy estimates observed in Nigeria persisted up to age 40 for Zambia while the PBM estimates from ages 45 to 75 were relatively higher than the chain-based model for that country. For Ghana, at age 5, the expectation of life is approximately 69 years for the proposed model and 65 years for the PBM although the estimates of the proposed model are consistently higher than the PBM from ages 5 to 55 years. The age-specific estimates of the life expectancies obtained for Ghana appear linearly distributed from ages 5 to 60 and became irregular thereafter. Above all, the proposed model estimates were consistently higher than the World Health Organization (WHO) estimates. The WHO expectation of life estimates is used for assessing the estimates of the proposed model not as a direct comparison

of results because the data requirements and methods of estimation are not the same.

WHO expectation of life estimates for most developing countries are mere interpolated values derived from existing life tables such as the United Nation model life tables [17,18,10]. It is worthy to note that most of the data used in preparing such life tables are not from those countries thereby raising questions about the accuracy of the estimates of the life expectancies produced and kept for such countries by WHO [10]. The estimates of the proposed model appear reasonable compared to the original PBM. The slight differences between the proposed model and the PBM in some age groups may be due to, in part, increasing age exaggeration beyond certain ages or recent changes in mortality and fertility since the CBM takes into consideration changes in the two components over time.

It is very difficult to ascertain the model with the best estimate since each of them has its peculiar limitations. But the proposed method will be helpful in most developing countries which are known to be experiencing significant changes in fertility and mortality over time. The proposed model also has the advantage of deriving estimates of adult mortality from the population by 5-year age groups as reported not converting the age distributions reported by 5-year to single years before using it to derive estimates of adult mortality.

Just like PBM, the estimates obtained using the proposed model may suffer some of the limitations associated with the inter-censal survival methods which include: (i) sensitivity to the level of completeness of the reported census age distributions, and (ii) gross misstatement of mortality level or life expectancy at the youngest age group [15].

6. CONCLUSIONS AND RECOMMENDATIONS

The proposed model provides more insight into the quest to find a “consensus model” for estimating adult mortality, especially in developing countries. Although there is no consensus yet on the best model for estimating adult mortality in the developing world, the combination of different methods with different data requirements can be of great help to measure adult mortality levels accurately in developing countries at any given time.

We recommend the use of the proposed model for the estimation of adult mortality. We do not recommend adjustment of census data before the application of these proposed model but when the censuses are grossly misreported it may be advisable to do a moderate adjustment of one census or both of them. Although indirect techniques can be used in the estimation of adult mortality in the absence of reliable civil registration systems, efforts should be made in developing countries to improve existing vital data collection mechanisms among others.

COMPETING INTERESTS

Authors have declared that no competing interests exist.

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Appendix 1a

Table 1. Proposed Chain-base Model applied to Nigeria, Females: 1991-2006.

Age Group	Census Population		${}_5\bar{N}_x$	$\frac{{}_5\bar{N}_x}{{}_5\bar{N}_{x-5}}$	${}_5r_x$	$5 \cdot ({}_5r_{x-5})$	$\exp[5 \cdot ({}_5r_{x-5})]$	$\frac{{}_5\bar{N}_x}{{}_5\bar{N}_{x-5}} \exp[5 \cdot ({}_5r_{x-5})]$	nLx	l _x	e _x
	${}_5N_x^{t_1}$	${}_5N_x^{t_2}$									
0-4	6,999,435	11,025,749	8,860,650	-	0.03175	-	-	-	-	-	-
5-9	7,126,144	9,616,769	8,309,338	0.9378	0.02094	0.1587	1.1720	1.09912	4.8468	-	67.2
10-14	5,336,143	7,631,631	6,415,589	0.7721	0.02500	0.1047	1.1104	0.85733	4.1553	0.9002	62.3
15-19	4,806,977	7,362,887	5,994,389	0.9343	0.02979	0.1250	1.1331	1.05875	4.3995	0.8555	58.2
20-24	4,357,267	7,197,530	5,659,102	0.9441	0.03507	0.1490	1.1606	1.09571	4.8205	0.9220	53.8
25-29	4,006,932	6,676,968	5,228,822	0.9240	0.03568	0.1753	1.1917	1.10105	5.3076	1.0128	49.0
30-34	3,105,298	4,962,352	3,961,545	0.7576	0.03275	0.1784	1.1953	0.90560	4.8066	1.0114	43.7
35-39	2,007,882	3,670,622	2,756,163	0.6957	0.04215	0.1638	1.1779	0.81953	3.9391	0.8746	38.8
40-44	1,874,721	3,060,981	2,419,578	0.8779	0.03426	0.2108	1.2346	1.08384	4.2694	0.8209	34.9
45-49	1,061,332	2,029,767	1,493,585	0.6173	0.04530	0.1713	1.1868	0.73262	3.1278	0.7397	30.6
50-54	1,182,149	1,885,282	1,506,466	1.0086	0.03261	0.2265	1.2542	1.26505	3.9569	0.7085	27.5
55-59	481,394	876,477	659,324	0.4377	0.04187	0.1631	1.1771	0.51518	2.0385	0.5995	23.6
60-64	791,573	1,087,067	931,522	1.4128	0.02216	0.2093	1.2329	1.74185	3.5507	0.5589	21.5
65-69	357,400	522,612	434,787	0.4667	0.02655	0.1108	1.1172	0.52145	1.8515	0.5402	18.0
70-74	394,116	564,609	474,266	1.0908	0.02512	0.1328	1.1420	1.24566	2.3064	0.4158	16.1
75-79	156,368	252,422	200,576	0.4229	0.03346	0.1256	1.1338	0.47951	1.1059	0.3412	13.8
80-84	222,627	351,373	282,121	1.4066	0.03189	0.1673	1.1821	1.66270	1.8388	0.2945	12.7
85+	194,404	311,204	248,241	-	-	-	-	-	-	-	-

Note: The value of ${}_5L_0 = 4.40973$ from 2013 Nigeria DHS. $t = 14.312$ years

Appendix 1b

Table 2. Proposed Chain-base Model applied to Zambia, Females: 2000-2010.

Age Group	Census Population 2000	Census Population 2010	${}_5\bar{N}_x$	$\frac{{}_5\bar{N}_x}{{}_5\bar{N}_{x-5}}$	$s f_x$	$5 \cdot (s f_{x-5})$	$\exp[5 \cdot ({}_s r_{x-5})]$	$\frac{{}_5\bar{N}_x}{{}_5\bar{N}_{x-5}} \exp[5 \cdot ({}_s r_{x-5})]$	nL_x	l_x	e_x
	${}_5N_x^0$	${}_5N_x^t$									
0-4	830931	1131280	973,395	-	0.03086	-	-	-	-	-	-
5-9	731901	961955	841,695	0.8647	0.02733	0.1543	1.1668	1.0089	4.7111	-	65.1
10-14	604367	895562	740,446	0.8797	0.03933	0.1367	1.1464	1.0085	4.7513	0.9462	60.4
15-19	556676	782499	663,192	0.8957	0.03405	0.1966	1.2173	1.0903	5.1803	0.9932	55.6
20-24	492589	641375	563,713	0.8500	0.02639	0.1703	1.1856	1.0078	5.2205	1.0401	50.4
25-29	379247	559303	463,460	0.8222	0.03885	0.1320	1.1411	0.9381	4.8976	1.0118	45.2
30-34	275434	415081	340,498	0.7347	0.04101	0.1943	1.2144	0.8922	4.3696	0.9267	40.3
35-39	218631	325824	268,673	0.7891	0.03990	0.2051	1.2276	0.9687	4.2327	0.8602	35.9
40-44	164597	222823	192,243	0.7155	0.03029	0.1995	1.2208	0.8735	3.6972	0.7930	31.7
45-49	122834	187117	152,727	0.7945	0.04209	0.1514	1.1635	0.9243	3.4175	0.7115	28.0
50-54	105762	146100	124,847	0.8174	0.03231	0.2105	1.2342	1.0089	3.4480	0.6866	24.6
55-59	72933	97444	84,598	0.6776	0.02897	0.1616	1.1753	0.7964	2.7460	0.6194	21.1
60-64	68797	90262	79,044	0.9344	0.02716	0.1449	1.1559	1.0800	2.9658	0.5712	18.4
65-69	47994	66117	56,573	0.7157	0.03203	0.1358	1.1454	0.8198	2.4313	0.5397	15.4
70-74	31869	49403	39,997	0.7070	0.04384	0.1602	1.1737	0.8298	2.0176	0.4449	13.0
75-79	17348	31134	23,573	0.5894	0.05848	0.2192	1.2451	0.7338	1.4805	0.3498	11.0
80-84	10931	17029	13,755	0.5835	0.04433	0.2924	1.3397	0.7817	1.1573	0.2638	9.5
85+	10294	17711	13,669	-	0.05426	-	-	-	-	-	-

Note: The value of ${}_5L_0 = 4.6693$ from the 2013-14 Zambia DHS. $t = 10$ years

Appendix 1c

Table 3. Proposed Chain-base Model applied to Ghana, Males: 2000-2010.

Age Group	Census Population		${}_s\bar{N}_x$	$\frac{{}_s\bar{N}_x}{{}_s\bar{N}_{x-5}}$	$s r_x$	$5 \cdot (s r_{x-5})$	$\exp[5 \cdot ({}_s r_{x-5})]$	$\frac{{}_s\bar{N}_x}{{}_s\bar{N}_{x-5}} \exp[5 \cdot ({}_s r_{x-5})]$	nLx	I _x	e _x
	2000	2010									
0-4	1,379,770	1,731,787	1,549,118	-	0.02272	-	-	-	-	-	-
5-9	1,390,652	1,589,632	1,487,925	0.9605	0.01337	0.1136	1.1203	1.0761	5.0037	-	69.0
10-14	1,151,131	1,477,525	1,307,545	0.8788	0.02496	0.0669	1.0692	0.9395	4.7012	0.9705	64.0
15-19	961,162	1,311,112	1,127,097	0.8620	0.03105	0.1248	1.1329	0.9766	4.5911	0.9292	59.3
20-24	763,051	1,100,727	921,602	0.8177	0.03664	0.1552	1.1679	0.9550	4.3845	0.8976	54.7
25-29	695,494	943,213	813,074	0.8822	0.03047	0.1832	1.2011	1.0596	4.6459	0.9030	50.3
30-34	566,439	790,301	672,168	0.8267	0.03330	0.1523	1.1646	0.9627	4.4728	0.9119	45.7
35-39	490,864	676,768	578,849	0.8612	0.03212	0.1665	1.1812	1.0172	4.5497	0.9023	41.2
40-44	443,284	572,620	505,196	0.8728	0.02560	0.1606	1.1742	1.0248	4.6625	0.9212	36.6
45-49	377,315	452,975	413,993	0.8195	0.01828	0.1280	1.1366	0.9314	4.3425	0.9005	32.0
50-54	279,950	394,600	334,002	0.8068	0.03433	0.0914	1.0957	0.8840	3.8387	0.8181	27.6
55-59	182,843	258,582	218,529	0.6543	0.03466	0.1716	1.1872	0.7768	2.9818	0.6821	23.8
60-64	177,347	227,050	201,176	0.9206	0.02471	0.1733	1.1892	1.0948	3.2645	0.6246	20.8
65-69	129,090	136,244	132,635	0.6593	0.00539	0.1235	1.1315	0.7460	2.4352	0.5700	17.5
70-74	106,513	149,512	126,800	0.9560	0.03391	0.0270	1.0273	0.9821	2.3917	0.4827	15.1
75-79	74,268	89,149	81,482	0.6426	0.01826	0.1696	1.1848	0.7613	1.8209	0.4213	12.7
80-84	66,941	62,357	64,622	0.7931	-0.00709	0.0913	1.0956	0.8689	1.5822	0.3403	10.9
85+	121,268	60,691	87,513	-	-0.06922	-	-	-	-	-	-

Note: The value of ${}_sL_0 = 4.65001$ from 2014 Ghana DHS. $t = 10$ years.

Appendix B

Preston-Bennett Method applied to Nigeria, Females: 1991-2006

Age(y)	${}_sN_y^{(1)}$	${}_sN_y^{(2)}$	${}_s\bar{N}_y$	$({}_sr_y)$	S_y	${}_sL_y$	T_y	l_y	e_y
	1991	2006							
0-4	6,999,435	11,025,749	8,860,650	0.03175	0.0794	9,592,627	136,345,921	-	-
5-9	7,126,144	9,616,769	8,309,338	0.02094	0.2111	10,262,403	126,753,294	1,985,503	63.8
10-14	5,336,143	7,631,631	6,415,589	0.02500	0.3260	8,887,942	116,490,891	1,915,035	60.8
15-19	4,806,977	7,362,887	5,994,389	0.02979	0.4629	9,523,557	107,602,949	1,841,150	58.4
20-24	4,357,267	7,197,530	5,659,102	0.03507	0.6251	10,573,594	98,079,392	2,009,715	48.8
25-29	4,006,932	6,676,968	5,228,822	0.03568	0.8020	11,659,812	87,505,798	2,223,341	39.4
30-34	3,105,298	4,962,352	3,961,545	0.03275	0.9730	10,482,192	75,845,986	2,214,200	34.3
35-39	2,007,882	3,670,622	2,756,163	0.04215	1.1603	8,794,685	65,363,794	1,927,688	33.9
40-44	1,874,721	3,060,981	2,419,578	0.03426	1.3513	9,345,753	56,569,109	1,814,044	31.2
45-49	1,061,332	2,029,767	1,493,585	0.04530	1.5502	7,038,604	47,223,356	1,638,436	28.8
50-54	1,182,149	1,885,282	1,506,466	0.03261	1.7450	8,626,060	40,184,753	1,566,466	25.7
55-59	481,394	876,477	659,324	0.04187	1.9312	4,547,979	31,558,693	1,317,404	24.0
60-64	791,573	1,087,067	931,522	0.02216	2.0913	7,541,116	27,010,713	1,208,910	22.3
65-69	357,400	522,612	434,787	0.02655	2.2131	3,975,669	19,469,597	1,151,678	16.9
70-74	394,116	564,609	474,266	0.02512	2.3423	4,934,615	15,493,929	891,028	17.4
75-79	156,368	252,422	200,576	0.03346	2.4887	2,416,084	10,559,314	735,070	14.4
80-84	222,627	351,373	282,121	0.03189	2.6521	4,001,443	8,143,230	641,753	12.7
85+	194,404	311,204	248,241	0.03288	2.8145	4,141,787	4,141,787	-	-

Note: e_y was obtained using equation (1), where the numerator was computed using equations (2) and (4), and the denominator was computed using equation (3).

Coefficients for estimation of R (ω) for the open ended interval

Age ω	$a(\omega)$	$b(\omega)$	$c(\omega)$
45	0.229	20.43	0.258
50	0.205	18.28	0.235
55	0.179	16.02	0.207
60	0.150	13.66	0.176
65	0.119	11.22	0.141
70	0.086	8.77	0.102
75	0.053	6.40	0.063
80	0.025	4.30	0.006
85	0.006	2.68	0.006

Source: United Nations Manual X (1983)

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