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Dependence of Quantum Decoherence on Wavelengths of a Particle in One Dimensional Box

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Author's contribution

The sole author designed, analyzed and interpreted and prepared the manuscript.

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ABSTRACT

In this paper, de Broglies wave particle duality relation is applied to a particle in one dimensional box problem to find the constraint for quantum decoherence of this system. It is observed that, for a particle of mass m captured in one dimensional box of length L should follow the relation $mL \leq \frac{h}{4c}$ to have a stable quantum state where h is Planck's constant and c is the speed of light in the vacuum space. Violation of this condition would lead to quantum decoherence of this system. It is also observed that if decoherence occurs, rate of decoherence would be inversely proportional to the square of the particle's initial wavelength.

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1 INTRODUCTION

Quantum decoherence is the most important topic in quantum measurement problem [1, 2] and quantum electrodynamics [3, 4, 5]. It was first introduced by the German physicist H. Dieter Zeh in 1970 [6] and become an active field of research since the 1980s [1, 7]. The concept of decoherence is to put in place in the broader scientific community by Zurek in 1991. His article in *Physics Today* [8] elicited a series of contentious comments from the readership. There were lots of controversies, but at the end it is proved experimentally [9, 10, 11] and presently used in different fields of quantum theory, like, quantum optics [12], quantum computer [13], quantum electrodynamics [3, 4, 5], quantum gravity [14, 15] etc. Decoherence is also important to understand the quantum classical application limit [16].

Decoherence can be viewed as the loss of information from a system into the environment. In general, it is due to the coupling of a system with its surroundings. For open quantum system, decoherence occur due to dissipation. Is quantum decoherence possible for an isolated system? Very recent studies show that it is possible [17, 18]. It is reported that there is intrinsic decoherence due to random force [19]. Decoherence is also possible in an isolated system due to temperature effects [20, 21]. So far, there is no report in the literature about decoherence due to quantum tunneling. In present research work decoherence due to quantum tunneling is investigated. Particle in a one dimetional box problem is chosen for this study as it is the simplest problem in quantum mechanics. Motivation of this work is to test the possibility of quantum decoherence of a quantum particle trapped in an one dimensional box of length L and bounded by infinite potential wall at its two ends. Outcome of this study should help to understand decoherence of other isolated quantum systems. Not only that, decoherence of a isolated quantum system, like particle in a box of one dimension, may unfold new thoughts about quantum classical barrier.

2 EXPLOITATION OF DE BROGLIE RELATION ON PARTICLE IN A BOX PROBLEM

The 'particle in a box' problem is one of the most basic problems in textbook quantum mechanics. It describes a localized particle in a deep potential well. Let, a particle having rest mass, m, and momentum, p, is placed in a box of length L. Due to the confinement of the particle in this box, its energy states would be quantized. We know that energy of the i^{th} eigen state is

$$E_q = \frac{n_i^2 h^2}{8mL^2} \tag{2.1}$$

where, $n_i = 1, 2, 3, \ldots$, h is Planck's constant. If, λ_q is the wavelength corresponds to this eigen state (we can say this wavelength as quantum wavelength), we can write -

$$E_q = \frac{hc}{\lambda_q} \tag{2.2}$$

From Equation 2.1 and 2.2 we get the expression for the wavelength of the ground state ($n_i = 1$) as -

$$\lambda_q = \frac{8mcL^2}{h} \tag{2.3}$$

Now, according to our problem, the momentum of the particle is p. Hence, kinetic energy of the particle is -

$$E_p = \frac{p^2}{2m} \tag{2.4}$$

From de Broglie's wave particle duality relation we can write,

$$p = \frac{h}{\lambda_d} \tag{2.5}$$

where, λ_d is de Broglie wavelength of the particle. From Equation 2.4 and 2.5 we get

$$E_p = \frac{h^2}{2m\lambda_d^2} \tag{2.6}$$

It is obvious that, if the kinetic energy (E_p) of the particle is equal to the ground state energy ($E_{q_{(n_i=1)}}$), particle would be in the ground state of this quantum system. But, if particle's kinetic energy is less than the ground state energy, particle would not be in any quantum state. Its behavior must be like non-quantized particle. Thus, the condition for the existence of the particle in any quantum state in this system is -

$$E_p \geq E_{q_{(n_i=1)}} \tag{2.7}$$

From Equation 2.7 we get the largest value of λ_d for the existence of the particle in the quantum state as -

$$\lambda_d = \lambda_{q_{(n_i=1)}} \tag{2.8}$$

From Equation 2.2 and 2.6 we get a relation between λ_q and λ_d, as -

$$\frac{hc}{\lambda_q} = \frac{h^2}{2m\lambda_d^2} \tag{2.9}$$

or,

$$\lambda_q = \frac{2mc\lambda_d^2}{h} \tag{2.10}$$

3 CONSTRAINTS FOR DECOHERENCE

In Equation 2.10 we have a relation between de Broglie wavelength (λ_d) and the quantum wavelength (λ_q) of a particle in one dimensional box. de Broglie wavelength is directed by the initial momentum of the particle while quantum wavelength is restricted by the length of the box. Thus, it is obvious that, if a particle is trapped in such a box, its energy states would be quantized if its de Broglie wavelength is equal to the quantum wavelength of any one energy state defined by the length of the box. It is known that wavelength of the lowest energy state is the largest one. Thus, the largest de Broglie wavelength of the particle allowed for its quantum state is equal to the quantum wavelength for the lowest energy state which is double of the length of the box. If, de Broglie wavelength of the particle is larger than $2L$, then the particle would behave like a classical particle in that box.

Now, using the condition $\lambda_d = \lambda_{q_{(n=1)}}$ we get from Equation 2.10

$$\lambda_q = \frac{h}{2mc} \tag{3.1}$$

From Equation 2.3 and 3.1 we get

$$mL = \frac{h}{4c} \tag{3.2}$$

Thus, from Equation 2.7 we get the condition for the existence of the particle in any quantum state as follows -

$$mL \leq \frac{h}{4c} \tag{3.3}$$

Right hand side of Equation 3.3 is a universal constant. Thus, we have a restriction on product of the mass of the particle and the length of the box. For example, a particle of very small mass would behave as a quantum particle in a larger box compared to a heavier particle in a shorter box. On the other hand, a particle of a heavier mass having very high energy should be confined in a very short region to be in a quantum state, else, the particle would not be in any quantum state. For example, L for electron is $0.0063\mathring{A}$ which is less than the dimension of the atom. This result is consistent with reality *ie.* electron in atoms exists in quantum states. Mass of proton and neutron is nearly 1000 times than electron. Thus, L for proton or neutron is in the order of femto meter. Thus, their matter nature is more dominated than the wave nature. On the other hand, rest mass of photon is 0. Hence, L of photon is infinity which implies photon always would be in quantum state. It could not be completely converted to an object of classical nature. In the above derivation relativity condition is not considered. Thus, Equation 3.3 would not be valid for photon. But, derivation including relativity would leads to a similar equation like Equation 3.3, from which we may have similar conclusion. Equation 3.3 is obtained without taking environmental perturbation into account. Environment perturbation is the reason for decoherence. Due to quantum tunneling the particle loses its energy to the environment. Decrease of energy through tunneling cause the decoherence. We may derive an equation for decoherence considering dissipation of energy due to tunneling.

4 EQUATION FOR DECOHERENCE

In previous section, the limit of the wavelength for decoherence is mentioned. Now, let us consider that the particle is in the quantum state and its energy is gradually decreasing due to dissipation. As its energy is decreasing and wavelength is increasing it would reach to a decoherence state. We may construct its decoherence equation as follows.

If, rate of dissipation of energy is R_d we can write [19]

$$R_d = -\frac{(\Delta E)}{\Delta t} \tag{4.1}$$

As decoherence is a continuous process, we can replace ΔE by dE and Δt by dt. Thus, for decoherence, Equation 4.1 would be

$$R_d = -\frac{(dE)}{dt} \tag{4.2}$$

From Equation 2.4 we get

$$\frac{dE}{dt} = \frac{p}{m}\frac{dp}{dt} \tag{4.3}$$

From Equation 2.5 we get

$$p\frac{dp}{dt} = -\frac{h^2}{\lambda^3}\frac{d\lambda}{dt} \tag{4.4}$$

From Equation 4.2, 4.3 and 4.4 we get

$$R_d = \frac{h^2}{m\lambda^3}\frac{d\lambda}{dt} \tag{4.5}$$

If decoherence time is t, we get from Equation 4.5

$$R_d \times t = \frac{h^2}{m}\int_{\lambda_i}^{2L}\frac{d\lambda}{\lambda^3} \tag{4.6}$$

Equation 4.6 is the decoherence equation for a particle in a box of one dimension. In this derivation uncertainty principle is not considered. We can reformulate the decoherence equation using uncertainty principle.

From uncertainty principle we can write,

$$\Delta p \times \Delta\lambda \geq \frac{h}{2\pi} \tag{4.7}$$

Thus, from Equation 4.1 and 4.7 we get-

$$R_d\Delta t \geq -\frac{h^2}{4\pi^2 m(\Delta\lambda)^2} \tag{4.8}$$

5 CONCLUSION

Applying de Broglie's wave particle duality relation to a particle in one dimensional box a constraint is obtained for quantum decoherence. This constraint should help to discriminate quantum classical border of an object which has dual behavior in different region. This also helps to investigate quantum to classical or classical to quantum transformation. In this article only one dimensional box is used, but, one can use a similar methodology for three dimensional box also. Taking time dependent wave function of a particle in three dimensional box, one can test its decoherence imposing the condition mentioned in Equation 3.3. That would be very much productive to understand decoherence from a basic view point. At present, decoherence for a particle in one dimensional box is reported and it is observed that decoherence rate is inversely proportional to the square of its initial wavelength.

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COMPETING INTERESTS

Author has declared that no competing interests exist.

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