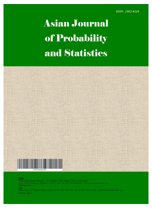

Asian Journal of Probability and Statistics

7(3): 1-13, 2020; Article no.AJPAS.57100

ISSN: 2582-0230

Nonparametric Tests for the Umbrella Alternative with Unknown Peak in a Mixed Design

Hassan Alsuhabi[1] and Rhonda Magel[2*]

[1]Department of Mathematics, Al-Qunfudah University College, Umm Al-Qura University, Mecca, KSA.
[2]Department of Statistics, North Dakota State University, Fargo, ND 58108, USA.

Authors' contributions

This work was carried out in collaboration between both authors. Author RM designed and directed the study. Author HA wrote the computer programs and conducted the study. Both authors read and approved the final manuscript.

Article Information

DOI: 10.9734/AJPAS/2020/v7i330182
Editor(s):
(1) Dr. Belkacem Chaouchi, Algeria.
Reviewers:
(1) E. H. Etuk, Rivers State University, Nigeria.
(2) Radek Matušů, Tomas Bata University, Czech Republic.
Complete Peer review History: http://www.sdiarticle4.com/review-history/57100

Original Research Article

Received: 22 March 2020
Accepted: 28 May 2020
Published: 13 June 2020

Abstract

Aims: Introducing and comparing 4 different tests for the unknown umbrella alternative in a mixed design.
Study Design: Simulation study consisting of a randomized complete block portion and a completely randomized design portion for various underlying distributions.
Place and Duration of Study: Simulation Study – conducted at North Dakota State University from September 2018 through December 2019.
Methodology: This paper proposes four non-parametric tests for testing the umbrella alternative with unknown peak when the data are mixture of a randomized complete block and a completely randomized design. The proposed tests are various combinations of a modified (unmodified) Mack-Wolfe's test and a modified (unmodified) Kim-Kim's test, respectively. In this paper, the proposed tests are an extension of Magel et al. (2010) and Hassan and Magel (2020) peak known tests to the unknown peak setting. The four proposed test statistics are compared to each other.
Results: When there were 3 populations, the unmodified versions of the test statistics did better than the modified versions. When there were 4 and 5 populations, the results varied.

**Corresponding author: E-mail: Rhonda.magel@ndsu.edu;*

Conclusion: All of the test statistics reached their asymptotic distributions quickly. The standardize first versions of the test statistics were generally better than the standardized last version of the test statistics, which meant that it was better to place equal weights on the RCBD portion and the CRD portion.

Keywords: Completely randomized; randomized complete block; mixed design; Mack-Wolfe test; Kim-Kim test; umbrella alternative; peak unknown.

1 Introduction

Non-parametric tests have been proposed for testing the umbrella alternatives when the underlying distributions are unknown. For example, testing the effectiveness of fertilization on the rate of crop yield or growth. When increasing the amount of fertilizer, the crop growth rate is likely to increase up to a point, and then the crop growth rate may start to decrease after this point. In these kinds of studies when the effects of the treatment, if they are different, are hypothesized to take the form of up and down (increasing and decreasing), an umbrella alternative is appropriate to use, and the null hypothesis of interest is:

$$H_0: \mu_1 = \mu_2 = \cdots = \mu_k$$

against the alternative

$$H_1: \mu_1 \leq \cdots \leq \mu_{p-1} \leq \mu_p \geq \mu_{p+1} \geq \cdots \geq \mu_k$$

with at least one strict inequality. Here, p is called the turning point or the peak of the umbrella by Mack and Wolfe [1], and μ_i is the location parameter. On the left side of the peak p, the means are non-decreasing, and they are non-increasing on the right side of the peak. One of the most familiar non-parametric tests for testing the umbrella alternative is the Mack-Wolfe test based on a completely randomized design (CRD). This test statistic uses the pairwise Mann and Whitney statistic [2], U_{uv}, Daniel [3]. The form of the Mack-Wolfe test statistic, A_p, is a sum of two Jonckheere-Terpstra test statistics where the Jonckheere-Terpstra test is the first nonparametric test designed to analyze ordered data [1]. The Mack-Wolfe test statistic for testing the umbrella alternatives H_1 with known peak p is given in (1):

$$A_p = \sum_{u=1}^{v-1} \sum_{v=2}^{p} U_{uv} + \sum_{u=p}^{v-1} \sum_{v=p+1}^{k} U_{vu}. \tag{1}$$

Under the null hypothesis that all population means are equal, the expected value and variance of A_p are given in (2):

$$E_0(A_p) = \frac{N_1^2 + N_2^2 - \sum_{i=1}^{k} n_i^2 - n_p^2}{4}$$

and (2)

$$var_0(A_p) = \frac{1}{72}\{2(N_1^3 + N_2^3) + 3(N_1^2 + N_2^2) - \sum_{i=1}^{k} n_i^2(2n_i + 3) - n_p^2(2n_p + 3) + 12n_p N_1 N_2 - 12n_p^2 N\},$$

where $N_1 = \sum_{i=1}^{p} n_i$, $N_2 = \sum_{i=p}^{k} n_i$ and $N = \sum_{i=1}^{k} n_i$; n_p is the sample size associated with the peak population with n_i denoting the sample size for population i.

The standardized version of the Mack-Wolfe test statistic, A_p^*, given in (3) has an asymptotic standard normal distribution when H_0 is true:

$$A_p^* = \frac{A_p - E_0(A_p)}{\sqrt{var_0(A_p)}} \tag{3}$$

The null hypothesis is rejected when $A_p^* \geq Z_\alpha$.

In the case where the peak is unknown, Mack & Wolfe [1] proposed letting

$$U_{.q} = \sum_{i \neq q} U_{iq}, \text{ for } q = 1, \dots, k \tag{4}$$

and

$$U_{.q}^* = \frac{U_{.q} - E_0(U_{.q})}{\sqrt{var_0(U_{.q})}} \tag{5}$$

where $E_0(U_{.q}) = n_q(N - n_q)/2$ and $var_0(U_{.q}) = n_q(N - n_q)(N + 1)/12$ are the null mean and variance, respectively. Then, the Mack-Wolfe test statistic for the umbrella alternative when the peak is unknown can be written as

$$A_{\hat{p}}^* = \frac{A_{\hat{p}} - E_0(A_{\hat{p}})}{\sqrt{var_0(A_{\hat{p}})}} \tag{6}$$

Where $\hat{p}$ denoted to the estimated peak for the umbrella corresponding to $U_{\hat{p}}^* = \max(U_{.1}^*, U_{.2}^*, \dots, U_{.k}^*)$, where $U_{.i}^*$ is given in Eq. (5) and $i = 1, 2, \dots, k$. Here, $A_{\hat{p}}$ is the peak-known test statistic given in Eq. (1), $E_0(A_{\hat{p}})$ and $var_0(U_{.q})$ are the corresponding null mean and variance, respectively (as given in (2)). Accordingly, $A_{\hat{p}}^*$ is the standardized peak-known statistic with the peak at i^{th} group.

Chen and Wolfe [4] considered the case of unknown umbrella peak and suggested an estimation of the peak to be at the population which maximizes the standardized version of their test statistics. They proposed to reject H_0 for large values of

$$A_{max}^* = \max(A_1^*, A_2^*, \dots, A_k^*) \tag{7}$$

where A_i^* is the standardized version of the Mack-Wolfe test which given in Eq. (3) and $i = 1, 2, \dots, k$.

Hettmansperger and Norton [5] also proposed a class of rank test versus the patterned alternative. Shi [6] went on to propose a rank test statistic comparable to the test statistic proposed by Hettmansperger and Norton [5] using various weighting schemes. Both test statistics could be used for the umbrella alternative and designed for the known peak first; then, they have been developed in case of the unknown peak.

Neuhauser et al., [7] proposed a test statistic for the nondecreasing ordered alternative, considering different weights of the Mann-Whitney statistics extended to Jonckheere and Terpstra's statistic [8,9]. They found that their proposed test statistic had better power than the Jonckheere and Terpstra test statistic in some cases. Following the results of Neuhauser et al., [7], Esra and Fikri [10] developed tests for the umbrella alternative for the completely randomized design. They applied a similar modification to the Mack and Wolfe's test [1] as Neuhauser, et al., [7] did for the Jonckheere-Terpstra test. Esra and Fikri's proposed test statistic is just the sum of two modified Jonckheere-Terpstra test statistics as introduced by Neuhauser et al., [7], namely,

$$mA_p = \sum_{u=1}^{v-1}\sum_{v=2}^{p}(v-u)U_{uv} + \sum_{u=p}^{v-1}\sum_{v=p+1}^{k}(u-v)U_{vu}. \tag{8}$$

Under the null hypothesis that all population means are equal, the expected value and variance of mA_p are given in (16):

$$E_0(mA_p) = \frac{n^2}{2}\left[\binom{p+1}{3} + \binom{k-p-2}{3}\right]$$

and (9)

$$var_0(mA_p) = \frac{n^2p^2(p^2-1)(np+1) + n^2(k-p+1)^2[(k-p+1)^2-1][n(k-p+1)+1]}{144} + \frac{n^3p(p-1)(k-p)(k-p+1)}{24}.$$

The standardized version of the Esra and Fikri test, mA_p^*, given in (17) has an asymptotic standard normal distribution when H_0 is true:

$$mA_p^* = \frac{mA_p - E_0(mA_p)}{\sqrt{var_0(mA_p)}} \tag{10}$$

The null hypothesis is rejected when $mA_p^* \geq Z_\alpha$.

In the case of an unknown peak, Esra and Fikri [10] applied the method used by Mack and Wolfe [1], in which the peak is estimated to be $U_{\hat{p}}^* = \max(U_{.1}^*, U_{.2}^*, \ldots, U_{.k}^*)$, where $U_{.i}^*$ is given in Eq. (5) and $i = 1, 2, \ldots, k$. The standardized test statistic for the unknown peak can be written as

$$mA_{\hat{p}}^* = \frac{mA_{\hat{p}} - E_0(mA_{\hat{p}})}{\sqrt{var_0(mA_{\hat{p}})}} \tag{11}$$

The null hypothesis is rejected for the large value of $mA_{\hat{p}}^*$ at the level of significance α.

In some cases when a blocking factor is introduced, when researchers may be interested in testing for the umbrella alternative, and thus a randomized complete block design (RCBD is used. For instance, when we examine the effectiveness of increasing the amount of fertilizer on the rate of crop growth, a location or plot could be a blocking factor. Kim and Kim [11] extended the Mack-Wolfe test to an RCBD. The Kim-Kim test statistic, A, is the sum of the Mack-Wolfe statistics over all blocks. It is given in (12):

$$A = \sum_{i=1}^{b} A_{ip} \tag{12}$$

where A_{ip} is the Mack-Wolfe statistic of the i^{th} block. No interaction is assumed between blocks and treatments.

Under the null hypothesis that all population means are equal, the expected value and variance of A are given in (13):

$$E_0(A) = \frac{\sum_{i=1}^{b}\left\{N_1^2 + N_2^2 - \sum_{i=1}^{k} n_i^2 - n_p^2\right\}}{4}$$

and (13)

$$var_0(A) = \sum_{i=1}^{b}\{\,2(N_{i1}^3 + N_{i2}^3) + 3(N_{i1}^2 + N_{i2}^2) - \sum_{j=1}^{k} n_{ij}^2\left(2n_{ij} + 3\right) - n_{ip}^2\left(2n_{ip} + 3\right) + 12n_{ip}N_{i1}N_{i2} - 12n_{ip}^2 N_i\}/72,$$

where $N_{i1} = \sum_{j=1}^{p} n_{ij}$, $N_{i2} = \sum_{j=p}^{k} n_{ij}$ and $N_i = \sum_{j=1}^{k} n_{ij}$.

The standardized version of the Kim-Kim statistic, A^*, given in (14) has an asymptotic standard normal distribution when H_0 is true:

$$A^* = \frac{A - E_0(A)}{\sqrt{var_0(A)}} \tag{14}$$

The null hypothesis is rejected when $A^* \geq Z_\alpha$.

Following Esra and Fikri's modification [10] to the Mack-Wolfe statistic [1] given in (8), Hassan and Magel [12] proposed a similar modification of the Kim-Kim statistic, mA, as follows:

$$mA = \sum_{i=1}^{b} mA_{ip}$$

(15)

$$mA_{ip} = \sum_{i=1}^{b}\left\{\sum_{u=1}^{v-1}\sum_{v=2}^{p}(v-u)U_{iuv} + \sum_{u=p}^{v-1}\sum_{v=p+1}^{k}(u-v)U_{ivu}\right\}.$$

where mA_{ip} denotes the modified Mack-Wolfe test statistic of the i^{th} block, $(v-u)U_{iuv}$ is the weighted Mann-Whitney test statistic applied to the observations in cell (i,u) and (i,v), k is the number of treatments, p is the known peak and the number of blocks is b. At α level of significance, we reject H_0 for the large value of mA.

When the sample sizes for each treatment per block are equal to one ($n_{11} = \cdots = n_{bk} = n = 1$) and under the null hypothesis that all population means are equal, the expected value and variance of mA are given in (16):

$$E_0(mA) = \sum_{i=1}^{b}\left\{\frac{1}{2}\left[\binom{p+1}{3} + \binom{k-p-2}{3}\right]\right\}$$

and (16)

$$var_0(mA) = \sum_{i=1}^{b}\left\{\frac{p^2(p^2-1)(p+1) + (k-p+1)^2[(k-p+1)^2-1][(k-p+1)+1]}{144} + \frac{p(p-1)(k-p)(k-p+1)}{24}\right\}.$$

The standardized version of the modified Kim-Kim statistic, mA^*, given in (17) has an asymptotic standard normal distribution when H_0 is true:

$$mA^* = \frac{mA - E_0(mA)}{\sqrt{var_0(mA)}} \tag{17}$$

The null hypothesis is rejected when $mA_p^* \geq Z_\alpha$.

Several researchers have developed statistical tests in nonparametric statistics using a mixed design consisting of both a CRD and an RCBD. Dubnicka et al., [13] considered a mixed design experiment consisting of observations that were paired, and observations from two population that were independent. They proposed a test statistic for this design that combines the Wilcoxon-signed rank test statistic for paired data and the Mann-Whitney test statistic for two independent samples. Magel et al. [14] extended the idea of Dunicka, et al., [13] to propose test statistics for the umbrella alternative, in the situation where the peak p is known, considering 3 or more mixed samples consisting of observations from a completely randomized design and observations from a randomized complete block design. Magel et al.'s first proposed statistic, A_p^{**}, [14] is given in (18):

$$A_p^{**} = A_p^* + A^* \tag{18}$$

where A_p^* is the standardized version of the usual Mack-Wolfe statistic for the completely randomized design and A^* is the standardized version of Kim-Kim statistic for the randomized complete block design. Under H_0, since the distribution of each of the test statistics of A_p^* and A^* is an asymptotically standard normal distribution, the asymptotic distribution of A_p^{**} is normal with mean zero and variance 2. The standardized version of their first proposed statistic, A^{**}, given in (19) has an asymptotic standard normal distribution:

$$A^{**} = \frac{A_p^{**} - 0}{\sqrt{2}} \tag{19}$$

The null hypothesis is rejected when $A^{**} \geq Z_\alpha$. Their second proposed statistic, A_p^{***}, is given in (20):

$$A_p^{***} = A_p + A \tag{20}$$

where A_p and A are the usual Mack-Wolfe statistic and Kim-Kim statistic [1,11], respectively. Under the H_0, the expected value and the variance of A_p^{***} are given below:

$$E_0\left(A_p^{***}\right) = E_0\left(A_p\right) + E_0(A)$$

and (21)

$$var_0\left(A_p^{***}\right) = var_0\left(A_p\right) + var_0(A)$$

where $E_0\left(A_p\right)$, $E_0(A)$, $var_0\left(A_p\right)$ and $var_0(A)$ are the expected values and variance of the usual Mack-Wolfe statistic and the Kim-Kim statistic, respectively. The standardized version of their second proposed statistic, A^{***}, given in (22) has an asymptotic standard normal distribution under H_0:

$$A^{***} = \frac{A_p^{***} - E_0\left(A_p^{***}\right)}{\sqrt{var_0\left(A_p^{***}\right)}} \tag{22}$$

The null hypothesis is rejected when $A^{**} \geq Z_\alpha$.

Hassan and Magel [12] later proposed two different test statistics for the umbrella alternative in a mixed design (CRD and RCBD). The first proposed test statistic, mA_p^{**}, is given in (23):

$$mA_p^{**} = mA_p^{*} + mA^{*} \tag{23}$$

where mA_p^{*} is the standardized version of the modified Mack-Wolfe statistic for the completely randomized design and mA^{*} is the standardized version of modified Kim-Kim statistic for the randomized complete block design. Under H_0 , and since the distribution of each of the test statistics of mA_p^{*} and mA^{*} is an asymptotically standard normal distribution, the asymptotic distribution of mA_p^{**} should be normal with mean zero and variance 2. The standardized version of the first proposed test statistic, mA^{**}, given in (24) has an asymptotic standard normal distribution:

$$mA^{**} = \frac{mA_p^{**} - 0}{\sqrt{2}} \tag{24}$$

The null hypothesis is rejected when $mA^{**} \geq Z_\alpha$. Their second proposed statistic, mA_p^{***}, is given in (25):

$$mA_p^{***} = mA_p + mA \tag{25}$$

where mA_p and mA are the unstandardized version of modified Mack-Wolfe statistic and Kim-Kim statistic, respectively. Under H_0, the expected value and the variance of mA_p^{***} are given below:

$$E_0(mA_p^{***}) = E_0(mA_p) + E_0(mA)$$

and (26)

$$var_0(mA_p^{***}) = var_0(mA_p) + var_0(mA)$$

where $E_0(mA_p)$, $E_0(mA)$, $var_0(mA_p)$ and $var_0(mA)$ are the expected values and variance of the modified Mack-Wolfe statistic and the Kim-Kim statistic, respectively. The standardized version of the second proposed statistic, mA^{***}, given in (27) has an asymptotic standard normal distribution under H_0:

$$mA^{***} = \frac{mA_p^{***} - E_0(mA_p^{***})}{\sqrt{var_0(mA_p^{***})}} \tag{27}$$

The null hypothesis is rejected when $mA^{***} \geq Z_\alpha$.

2 Materials and Methods

2.1 Four proposed tests for the mixed design for umbrella alternative with unknown peak

In the case of the unknown peak of the umbrella which is of more practical interest, the foundation of our four proposed test statistics is to use the two test statistics which were proposed by Magel et al., [14] and the two proposed test statistics that were proposed by Hassan and Magel [12] for the known peak case. We follow a proposal of Chen and Wolfe [4] and Chen [15] to estimate the peak for balanced designs, i.e. $n = n_1 = n_2 = \cdots = n_k$ which is based on calculating the maximum of the standardized test statistics for all known peaks p, $p = 1, \ldots, k$.

The first two proposed test statistics are an extension of Magel et al., [14] peak known tests to the unknown peak setting.

The first proposed test statistic is based on the maximum of the standardized test statistics calculated for all known peaks $p, p = 1, \dots, k$. Here, we use the first test statistic that was proposed by Magel et al., [14]. Then, the first proposed statistic is given by

$$A_{max}^{**} = \max(A_1^{**}, A_2^{**}, \dots, A_k^{**}), \tag{28}$$

where A_i^{**} is the first proposed statistic of the Magel et al.'s test statistics which is given in Eq. (19) and $i = 1, \dots, k$. We reject H_0 for a large value of A_{max}^{**} [14].

Similarly, the second proposed statistic follows from the first proposed test by calculating the maximum of the standardized test statistics for all known peaks $p, p = 1, \dots, k$. However, here we use the second test statistic that was proposed by Magel et al., [14]. Then, the second proposed statistic is given by

$$A_{max}^{***} = \max(A_1^{***}, A_2^{***}, \dots, A_k^{***}), \tag{29}$$

where A_i^{***} is the second proposed statistic of the Magel et al.'s test statistics which given in Eq. (22) and $i = 1, \dots, k$. H_0 is rejected for a large value of A_{max}^{***} [14].

The other two proposed test statistics are an extension of Hassan and Magel's peak known tests [12] to the unknown peak setting.

The third proposed statistic is given by

$$mA_{max}^{**} = \max(mA_1^{**}, mA_2^{**}, \dots, mA_k^{**}), \tag{30}$$

where mA_i^{**} is the first proposed statistic which is given in Eq. (24) and $i = 1, \dots, k$

Then, the fourth proposed statistic is given by

$$mA_{max}^{***} = \max(mA_1^{***}, mA_2^{***}, \dots, mA_k^{***}), \tag{31}$$

where mA_i^{***} is given in Eq. (27) and $i = 1, \dots, k$. H_0 is rejected for a large value of mA_{max}^{***}.

The asymptotical critical values were estimated when the limiting sample size proportions for the CRD portion are all equal and the number of the blocks varies for the RCBD portion (see Table 1). After obtaining an empirical cumulative distribution of A_{max}^{**}, A_{max}^{***}, mA_{max}^{**} and mA_{max}^{***} based on a sample size of $10{,}000$ from the corresponding true distribution, the estimated critical values for the A_{max}^{**}, A_{max}^{***}, mA_{max}^{**} and mA_{max}^{***} tests then correspond to the appropriate 90^{th}, 95^{th} or 99^{th} percentile of this empirical distribution. For instance, when $k = 4$, $n = 10$ for each treatment and $b = 20$, the estimated 95^{th} percentile for the asymptotic null distribution of mA_{max}^{***} is 2.19922. At α level of significance, we reject H_0 if the proposed test statistic is greater than the corresponding critical value.

Table 1 represents the critical values for $\alpha = 0.10, 0.05$ and 0.01 , $3 \le k \le 5$ where k is the number of treatments, equal sample sizes $n = 10$ for the CRD portion, the number of blocks $b = 5$ and 20 for the RCBD portion. For the RCBD portion, we consider the case when $n_{ij} = 1$ where $i = 1, \dots, b$ and $j = 1, \dots, k$. These critical values are used in the power studies in this paper. Critical Values were also generated when the number of blocks was 10. They were left out of the table, however, because it is noted that the simulated critical values for 5 blocks are about the same as the simulated critical values for 20 blocks. The test statistic appears to reach its asymptotic distribution early. The critical values that were generated when the number of blocks was 5, could also be used when the number of blocks was 20 or greater. In addition, we also used the critical values generated when the CRD sample sizes were 10, and found when the CRD sample sizes were 5, that the estimated alpha values were all approximately 0.05.

2.2 Simulation study

In this section, the type I error and the performance of the four proposed test statistics are investigated via Monte Carlo Simulation and implemented in SAS version 9.4 on 5,000 iterations. The observations are assumed to follow three different underlying distributions, which are included in this study: standard exponential, standard normal and t distribution with three degrees of freedom. These distributions were used to give two symmetric distributions, with one having a larger variance, and one skewed distribution. In this research, the data are generated from a mixed design consisting of a CRD portion and an RCBD portion.

Table 1. Selected critical values for the null distribution of the peak unknown for the mixed design (CRD & RCBD): $k = 3, 4, 5$; $n = n_1 = \cdots = n_k = 10$; b= 5, 20 for each k

k	n	b	α	Non modification		Distance modification	
				A^{**}_{max}	A^{***}_{max}	mA^{**}_{max}	mA^{***}_{max}
3	10	5	0.10	1.83212	1.83806	1.81517	1.81045
			0.05	2.11628	2.10494	2.11226	2.07612
			0.01	2.68855	2.63392	2.67566	2.58249
		20	0.10	1.79106	1.80141	1.79818	1.82436
			0.05	2.10278	2.08499	2.09161	2.08499
			0.01	2.70744	2.64967	2.67678	2.63324
4	10	5	0.10	1.93094	1.89933	1.88579	1.91270
			0.05	2.20608	2.16538	2.18840	2.19500
			0.01	2.75170	2.76266	2.74728	2.76787
		20	0.10	1.93784	1.91039	1.92245	1.92673
			0.05	2.24522	2.21680	2.20083	2.19922
			0.01	2.79538	2.77733	2.73082	2.74028
5	10	5	0.10	1.99224	1.99556	1.97741	1.96643
			0.05	2.27034	2.27303	2.26770	2.27803
			0.01	2.84581	2.84383	2.81598	2.78724
		20	0.10	2.00175	1.99456	1.96384	1.96638
			0.05	2.26909	2.28294	2.24580	2.23072
			0.01	2.83609	2.81162	2.79685	2.69453

In this paper, 3, 4 and 5 populations are considered with the assumption that the peak p is unknown. Here, the peak has to be estimated from each of the 5,000 simulated data. For every considered distribution, equal sample sizes for the CRD portion are selected so that the sample size, n, is 10. The number of blocks (complete blocks) for the RCBD is considered to be half, equal and twice the sample size for each treatment. As we mentioned in section 2, the simulated critical values found in Table 2 are used.

For the estimated powers, a variety of location parameter configurations (treatment effects shifts) are considered.

For 3 populations with unknown peak, powers were estimated in the following cases:

1. The peak is distinct and there is equal spacing between parameters (example (0.0, 0.5, 1.0) and (0.0, 0.7, 0.0)).
2. The peak is distinct and there is unequal spacing between parameters (example (0.7, 0.5, 0.0), (0.0, 0.7, 0.5)).
3. The peak is distinct and the other two parameters are equal to each other (example (0.5, 0.0, 0.0)).
4. One additional parameter equals the peak (example (0.0, 0.5, 0.5)).

For 4 populations with unknown peak, powers were estimated in the following cases:

1. The peak is distinct, and the other parameters are the same (example (1.0, 0.0, 0.0, 0.0)).
2. The peak is distinct and there is equal spacing between parameters (example (1.0, 0.75, 0.5, 0.25) and (0.0, 0.25, 0.5, 0.25)).
3. The peak is distinct and there is unequal spacing between parameters (example (0.8, 1.0, 0.75, 0.2)).
4. Two population parameters are the same, but different from the peak (example (0.5, 1.0, 0.2, 0.2), (0.2, 0.75, 1.0, 0.75) and (0.0, 0.0, 0.25, 1.0)).

5. One additional parameter equals the peak and the other parameters are different from the peak, but equal to each other (example (0.0, 0.0, 0.75, 0.75)).
6. One additional parameter equals the peak and the other parameters are different from the peak and different from each other (example (0.75, 0.75, 0.5, 0.0) and (0.0, 0.5, 0.75, 0.75)).
7. Two additional parameters are equal to the peak (example (0.5, 0.5, 0.5, 0.0)).

For 5 populations with unknown peak, powers were estimated in the following cases:

1. The peak is distinct, and the other parameters are the same (example (1.0, 0.0, 0.0, 0.0, 0.0)).
2. The peak is distinct and each two of the other parameters are equal to each other (example (0.2, 0.2, 0.5, 0.0, 0.0)).
3. The peak is distinct and three of the other parameters are equal to each other (example (1.0, 0.2, 0.2, 0.2, 0.0) and (0.0, 0.0, 0.0, 0.5, 1.0)).
4. One additional parameter equals the peak and the other parameters are different from the peak, but equal to each other (example (0.0, 0.0, 0.0, 0.5, 0.5)).
5. One additional parameter equals the peak, the two other parameters are equal to each other but not equal to the peak and one parameter is different (example (0.5, 0.5, 0.2, 0.2, 0.0) and (0.5, 0.5, 0.2, 0.0, 0.0)).
6. One additional parameter equals the peak, and the other parameters are different from the peak and different from each other (example (0.8, 0.8, 0.5, 0.2, 0.0)).
7. Two additional parameters are equal to the peak, and the other parameters are equal to each other (example (0.5, 0.5, 0.5, 0.0, 0.0) and (0.0, 0.0, 0.5, 0.5, 0.5)).

3 Results and Discussion

In this section, we present results for the four proposed test statistics in this research. These test statistics are for analyzing data in a mixed design comprising a CRD portion and an RCBD portion in the situation of having an unknown umbrella peak, which is of more practical interest. For each test conducted in this study, the stated alpha value is 0.05. All test statistics considered maintained their stated alpha value.

3.1 Three treatments with unknown peak

Selected results for three treatments with unknown peak are given in Tables 2 for the exponential. Results from the other distributions were similar as to which test was better. Results show that the Standardized First has more power than Standardized Second for both non-modified and modified test statistics, regardless of the underlying distribution, the sample size for each treatment in the CRD and the number of blocks in the RCBD.

Importantly, we notice that the test of Standardized Second with no modification, A^{***}_{max}, is better than that with distance modification, mA^{***}_{max}. Also, it is noticed that the Standardized First for both non-modified and modified test statistics (A^{**}_{max} and mA^{**}_{max}, respectively) are similar in the performance.

3.2 Four and five treatments with unknown peak

Selected results for four and five treatments with unknown peak are given in Tables 3-5 for the exponential, normal and t distribution with three degrees of freedom. For all the distributions, results show that the Standardized First is more powerful than the Standardized Second for both non-modified and modified test and the results are consistent when the proportions are changed between the number of blocks in the RCBD and sample size for each treatment in the CRD.

Importantly, we notice that the test of Standardized first with distance modification, mA^{**}_{max}, is better than that with no modification, A^{**}_{max}. Also, it is noticed that the Standardized Second for both non-modified and modified (A^{***}_{max} and mA^{***}_{max}, respectively) are similar in the performance.

Table 2. Estimated rejection percentages of tests for mixed design under the exponential distribution for 3 treatments at an unknown peak: Blocks= 10, n= 10

Location Parameter	Standardized	Non modification	Distance modification
(0.0, 0.0, 0.0)	First	0.0488	0.0506
	Second	0.0558	0.0458
(0.5, 0.0, 0.0)	First	0.4500	0.4436
	Second	0.3174	0.3014
(0.7, 0.5, 0.0)	First	0.7494	0.7476
	Second	0.5864	0.5744
(0.0, 0.5, 1.0)	First	0.9504	0.9392
	Second	0.8462	0.8188
(0.0, 0.5, 0.5)	First	0.5370	0.5468
	Second	0.4130	0.4002
(0.0, 0.7, 0.0)	First	0.8668	0.8540
	Second	0.7088	0.6690

Table 3. Estimated rejection percentages of tests for mixed design under the exponential distribution for 4 treatments at an unknown peak: Blocks= 10, n= 10

Location Parameter	Standardized	Non modification	Distance modification
(0.0, 0.0, 0.0, 0.0)	First	0.0484	0.0474
	Second	0.0522	0.0528
(1.0, 0.0, 0.0, 0.0)	First	0.8462	0.8744
	Second	0.6420	0.6684
(1.0, 0.75, 0.5, 0.25)	First	0.8442	0.8562
	Second	0.6732	0.6728
(0.8, 1.0, 0.75, 0.2)	First	0.8192	0.8398
	Second	0.6616	0.6614
(0.2, 0.75, 1.0, 0.75)	First	0.8152	0.8380
	Second	0.6568	0.6610
(0.0, 0.5, 0.75, 0.75)	First	0.8470	0.8602
	Second	0.6860	0.6840
(0.0, 0.0, 0.75, 0.75)	First	0.9186	0.9350
	Second	0.7696	0.7796

Table 4. Estimated rejection percentages of tests for mixed design under the normal distribution for 4 treatments at an unknown peak: Blocks= 20, n= 10

Location Parameter	Standardized	Non modification	Distance modification
(0.0, 0.0, 0.0, 0.0)	First	0.0478	0.0554
	Second	0.0486	0.0528
(1.0, 0.0, 0.0, 0.0)	First	0.8026	0.8240
	Second	0.5256	0.5284
(1.0, 0.75, 0.5, 0.25)	First	0.6916	0.7268
	Second	0.4280	0.4458
(0.8, 1.0, 0.75, 0.2)	First	0.7204	0.7364
	Second	0.4598	0.4746
(0.2, 0.75, 1.0, 0.75)	First	0.6998	0.7172
	Second	0.4606	0.4662
(0.0, 0.5, 0.75, 0.75)	First	0.7458	0.7796
	Second	0.4920	0.5114
(0.0, 0.0, 0.75, 0.75)	First	0.8574	0.8820
	Second	0.6072	0.6174

Table 5. Estimated rejection percentages of tests for mixed design under the t distribution for 5 treatments at an unknown peak: Blocks= 20, n= 10

Location Parameter	Standardized	Non modification	Distance modification
(0.0, 0.0, 0.0, 0.0, 0.0)	First	0.0466	0.0478
	Second	0.0484	0.0490
(1.0, 0.0, 0.0, 0.0, 0.0)	First	0.5304	0.5562
	Second	0.2806	0.2896
(0.5, 0.5, 0.2, 0.0, 0.0)	First	0.3686	0.4164
	Second	0.2152	0.2380
(0.5, 0.5, 0.5, 0.0, 0.0)	First	0.4308	0.4800
	Second	0.2538	0.2766
(0.2, 0.2, 0.5, 0.0, 0.0)	First	0.2256	0.2530
	Second	0.1442	0.1630
(0.0, 0.0, 0.0, 0.5, 0.5)	First	0.3588	0.3934
	Second	0.2068	0.2174
(0.0, 0.0, 0.5, 0.5, 0.5)	First	0.4360	0.4808
	Second	0.2580	0.2796

4 Conclusion

Results showed that regardless of the underlying distribution, the proportions between the CRD and RCBD portions in the mixed design, and the peak p, the alpha values for all test statistics are approximately 0.05. We conclude that the Standardized First versions of the test statistics were generally better than the Standardized Last versions of the test statistics.

When the study comprises three treatments with an unknown peak, we find that the test of Standardized Second with no modification (A_{max}^{***}) is better than that with distance modification (mA_{max}^{***}). Also, it is noticed that the Standardized First for both non-modified and modified test statistics (A_{max}^{**} and mA_{max}^{**}, respectively) are similar in the performance. In this case, one could use either the modified, or non-modified test statistics. When the study comprises four or five treatments with unknown peak, the modified test statistics, mA_{max}^{**}, generally have greater power. Also, it is noticed that the Standardized Second for both non-modified and modified test statistics are similar in the performance.

Acknowledgements

Thanks to Department of Mathematics, Al-Qunfudah University College, Umm Al-Qura University, Mecca, KSA for their sponsorship of Hassan Alsuhabi in the Ph.D. program at North Dakota State University.

Competing Interests

Authors have declared that no competing interests exist.

References

[1] Mack GA, Wolfe DA. K-sample rank tests for umbrella alternatives. Journal of the American Statistical Association. 1981;76:175.

[2] Mann HB, Whitney DR. On a test of whether one of two random variables is stochastically larger than the other. Ann. Math. Stat. 1947;18:50.

[3] Daniel WW. Applied nonparametric statistics. PWS-Kent Publishing Company; 1990.

[4] Chen YI, Wolfe DA. A study of distribution-free tests for umbrella alternatives. Biometrical J. 1990;32:47.

[5] Hettmansperger TP, Norton RM. Tests for patterned alternatives in k sample problems. J. Am. Stat. Assoc. 1987;82:292.

[6] Shi NZ. Rank test statistics for umbrella alternatives. Commun. Stat. 1988;17:2059.

[7] Neuhauser M, Liu PY, Hothorn L. Nonparametric tests for trend Jonckheere's test, a modification and a maximum test. Biometrical J. 1998;40:899.

[8] Jonckheere AR. A distribution-free k-sample test against ordered alternatives. Biometrika. 1954;41: 133-145.

[9] Terpstra TJ. The asymptomatic normality and consistency of Kendall's test against trend, when ties are present in one ranking. Indagationes Mathematicae. 1952;14:327-333.

[10] Esra G, Fikri G. A modified Mack–Wolfe test for the umbrella alternative problem. Communications in Statistics - Theory and Methods. 2016;45:7226-7241.

[11] Kim DH, Kim YC. Distribution-free tests for umbrella alternatives in a randomized block design. Journal of Nonparametric Statistics. 1992;1:277.

[12] Alsuhabi Hassan, Magel Rhonda. Modified nonparametric tests for the umbrella alternative with known peak in a mixed design. Journal of Progressive Research in Mathematics. 2020;16(1):2845-2860.
Available:http://www.scitecresearch.com/journals/index.php/jprm/article/view/1841

[13] Dubnicka SR, Blair RC, Hettmansperger TP. Rank-based procedures for mixed pairs and two-sample designs. Journal of Modern Applied Statistical Methods. 2002;1:32.

[14] Magel Rhonda, Terpstra Jeff, Canonizado Katrina, Park Ja In. Nonparametric tests for mixed designs. Communications in Statistics - Simulation and Computation. 2010;39:1228-1250.

[15] Chen YI. Notes on the Mack-Wolfe and Chen-Wolfe tests for umbrella alternatives. Biometrical J. 1991;33:281.

Peer-review history:
The peer review history for this paper can be accessed here (Please copy paste the total link in your browser address bar)
http://www.sdiarticle4.com/review-history/57100