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Research Article

Tsallisian Gravity and Cosmology

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In this paper, we adopt the Verlinde hypothesis on the origin of gravity as the consequence of the tendency of systems to increase their entropy and employ the Tsallis statistics. Thereinafter, modifications to the Newtonian second law of motion, its gravity, and radial velocity profile are studied. In addition, and in a classical framework, the corresponding cosmology and also its ability in describing the inflationary phases are investigated.

1. Introduction

The study of the relation between thermodynamics and gravity has a long history [1–7]. On the one hand, Gibbs shows that gravitational systems are not extensive [1], a conclusion in agreement with the Bekenstein entropy of black holes [2], which is a nonextensive entropy. On the other, it seems that all gravitational systems satisfy the Bekenstein entropy bound expressed as [8]

$$S_{BE} = \frac{Ac^3}{4G\hbar}, \quad (1)$$

where $A = 4\pi R^2$ and R denote the area of the system boundary and its radius, respectively, and k_B (Boltzmann constant). Using this entropy and Clausius relation, one can show that the Einstein gravitational field is in fact a thermodynamic equation of state [9]. This amazing result is valid in various gravitational and cosmological setups which lead to notable predictions about the behavior of cosmos and gravitational systems [10–30]. Motivated by the Gibbs work [1], the nonextensivity of the Bekenstein entropy, and based on the long-range nature of gravity [31], recently, the use of nonextensive statistical mechanics (based on possible generalizations of Gibbs entropy) has been proposed to model and study some phenomena such as the cosmic evolution [32–39], black holes [40–49], and Jeans mass [50, 51].

In order to find the probable thermodynamic aspects of gravity, Verlinde describes it as the implication of the tendency of systems to increase their entropy [52], an astonishing approach which attracts investigators to itself [53–65]. In the framework of generalized entropies, the Verlinde hypothesis leads to significant implications on the cosmic evolution [35, 66–68], Newtonian gravity [69], Jeans mass (as a stability criterion) [70], and also gravitational systems [71–76]. Indeed, the differences between generalized entropies and the Bekenstein entropy, originated from the nonextensive viewpoint, can (i) describe the universe inflationary phases [32–34, 39], (ii) relate Padmanabhan emergent gravity scenario to the Verlinde hypothesis [32], and (iii) propose an origin for the MOND theory [69].

Based on the Verlinde hypothesis [52], the entropy change of a system increases as

$$\Delta S = 2\pi \frac{mc}{\hbar} \Delta x, \quad (2)$$

when the test mass m has distance $\Delta x = \hbar/mc \equiv \lambda_c$ (reduced Compton wavelength) with respect to the holographic screen (boundary of system). This screen consists of N degrees of freedom calculated by

$$N = \frac{Ac^3}{G\hbar}, \quad (3)$$

in agreement with Eq.(1) and thus $S_{BE} = N/4$ [2]. Following [55, 56], we assume $\Delta x = \eta\lambda_c$ from now, and use the Unruh temperature [7]

$$T = \frac{1}{2\pi}\frac{\hbar a}{c}, \tag{4}$$

to get [55, 56]

$$F = T\frac{\Delta S}{\Delta x} = T\frac{dS}{dA}\frac{\Delta A}{\Delta x} = ma, \tag{5}$$

as the net force that source M applies to particle m, which finally brings it acceleration a. Indeed, this result is available if $\eta = 1/8\pi$ leading to $\Delta x = \lambda_c/8\pi$, to get Eq. (5). Now, combining $A = 4\pi R^2$ and Eq. (3) with

$$E = \frac{1}{2}NT = Mc^2, \tag{6}$$

and using Eq. (5), one easily reaches at Newtonian gravity

$$a = G\frac{M}{R^2}. \tag{7}$$

It is also useful to mention that it seems there is a deep connection between generalized entropies and quantum gravity scenarios, and indeed, quantum aspects of gravity may also be considered as another motivation for considering generalized entropies [77, 78]. Tsallis entropy is one of the generalized entropy measures which leads to acceptable results in the cosmological and gravitational setups [32, 36, 40, 47, 49]. In fact, there are two Tsallis entropies [40, 47, 49]. One of them has been proposed by Tsallis and Cirto [40] which is confirmed by the multifractal structure of horizon in quantum gravity [78] and modifies Eq. (1) as $S \sim A^\delta$ (δ is a free unknown parameter [77]).

The second one has recently been calculated in [49] by relying on statistical properties of degrees of freedom distributed on the holographic screen. The result is compatible with a detailed study in the framework of quantum gravity [47]. This case proposes an exponential relation between the horizon entropy and its surface, and we will focus on it in this paper. In the next section, modifications to the Newtonian second law of motion and also Newtonian gravity is derived by using the Tsallis entropy. Its implications on the radial velocity are also addressed. In the third section, after evaluating the Tsallis modification to the gravitational potential, we adopt the approach of paper [79] and find out the corresponding Friedmann first equation in a classical way in which a test mass is located on the edge of the universe, namely apparent horizon [79]. The possibility of obtaining an accelerated universe is also debated in this section. A summary of the work is presented in the last section.

2. Tsallis Gravity and Dynamics

Employing the Tsallis statistics, it has been recently shown that Eq. (1) is modified as [49]

$$S_q^T = \frac{1}{1-q}\left[\exp\left((1-q)S_{BE}\right) - 1\right], \tag{8}$$

in full agreement with quantum gravity calculations [47]. Here, q is a free parameter evaluated from other parts of physics and also observations, and Eq. (1) is recovered when $q = 1$ [31, 47, 49]. In the nonextensive scenarios, Eq. (6) takes the form [35, 80]

$$E = \frac{1}{5-3q}NT = Mc^2, \tag{9}$$

which approaches Eq. (1) at the appropriate limit of $q = 1$.

Now, following the recipe which led to Eq. (5), one can use Eq. (8) to find

$$F^T = T\frac{dS_q^T}{dA}\frac{\Delta A}{\Delta x} = ma\ \exp\left(\delta\frac{(2+3\delta)Mc^3\pi}{2\hbar a}\right), \tag{10}$$

where $\delta = 1 - q$ is the Tsallis second law of motion. Clearly, Eq. (5) is recovered whenever $\delta = 0$, and therefore, this approach claims the net force F^T that source M applies to m depends on M. In order to obtain the above result, we used $S_{BE} = N/4$ [2], and $N = ((5-3q)Mc^2)/T$. Of course, since the relation $F = ma$ works very well (classical regime), one can deduce that δ is very close to 0 meaning that the exponential factor may have nonsensible effects in the classical regime.

The modified form of Eq. (7), called Tsallis gravity, is also obtained as

$$a^T = G_q\frac{M}{R^2}\ \exp\left(\delta\frac{R_0^2}{R^2}\right), \tag{11}$$

where $R_0^2 \equiv G\hbar/c^3\pi = l_P^2/\pi$, l_P denotes the Planck length, and $G_q \equiv ((5-3q)/2)G$ in full agreement with [35]. In order to have a comparison between the Tsallis second law of motion and also the Tsallis gravity and those of Newton, let us write

$$\begin{aligned} \frac{F^T}{F} &= \exp\left(\frac{d}{a}\right), \\ \frac{a^T}{a} &= (5-3q)\ \exp\left(\frac{l}{R^2}\right), \end{aligned} \tag{12}$$

where $d \equiv \delta((2+3\delta)Mc^3\pi)/2\hbar$ and $l = \delta R_0^2$. As a crucial point, one should note that, for an event, the sign of a and a^T should be the same (the predictions of different theories about the value of accelerations should address the same motion meaning that both of a^T and a should have the same sign). It leads to this limitation $q < 5/3$ meaning that $\delta > -2/5$. Thus, l and d can be negative.

Now, let us compare Eq. (11) with the results of [55] and [56] where authors employ different entropies in the framework of the Verlinde theory and address two modifications for the Newtonian gravity. Unlike Eq. (11) of [11], the modified gravity obtained in [55] (Eq. (17)) diverges at large distances ($R \gg 1$). Of course, both of them claim that the gravitational force between the source M and test particle m can vanish for some points on their interface line, a property incompatible with the Newtonian gravity and experience. From Eq. (11), one can easily see that the obtained gravitational force does not diverge at large distances where it will be ignorable. Thus, it seems that this equation is a more reliable modification to the Newtonian gravity compared with those of [55, 56].

2.1. Velocity Profile. For a circular motion at radius r with velocity v, and thus acceleration $(v^2/r)(\equiv a^T)$ obeying Eq. (11), one reaches

$$v = \sqrt{\frac{G_q M}{r}} \exp\left(\frac{l}{2r^2}\right), \tag{13}$$

which implies that we should have $q < 5/3$ to get real values of velocity.

On the other hand, if one assumes the mass m in the gravitational field of source M feels the force GMm/r^2, then using (10), we can write

$$GMm/r^2 = F^T, \tag{14}$$

yields

$$GM/r = v^2 \exp\left(\frac{dr}{v^2}\right), \tag{15}$$

for $a \equiv (v^2/r)$, finally leading to

$$v^2 \cong \frac{Gm}{r} - dr \tag{16}$$

if we expand $\exp(dr/v^2)$ as $1 + (dr/v^2)$. For a constant d, this approximation is valid when radial acceleration (v^2/r) is small. Indeed, in this manner, the dr term leads to an increase in the velocity of particle m, compared with the Newtonian case for which $v^2 \cong (Gm/r)$, if $d < 0$.

3. A Tsallis Cosmology

In order to find the Friedmann first equation corresponding to the obtained Tsallis gravity, we follow the classical viewpoint fully described in [79]. The series expansion $\exp(l/r^2) = \sum_{n=0}^{\infty} l^n/n! r^{2n}$ leads to

$$\int \frac{\exp(l/r^2)}{r^2} dr = \sum_{n=0}^{\infty} \int \frac{l^n}{n! r^{2n+2}} dr = -\frac{1}{r} \sum_{n=0}^{\infty} \frac{l^n}{n!(2n+1) r^{2n}}, \tag{17}$$

combined with Eq. (11) to help us in calculating Tsallis gravitational potential as

$$\phi(r) = -\frac{G_q M}{r} \sum_{n=0}^{\infty} \frac{l^n}{n!(2n+1) r^{2n}}. \tag{18}$$

Considering a test particle on the edge of a flat FRW universe, and following the recipe of [79], this equation leads to

$$H^2 = \frac{8\pi G_q}{3} \rho \sum_{n=0}^{\infty} \frac{l^n H^{2n}}{n!(2n+1)}, \tag{19}$$

in which ρ is the cosmic fluid density and H denotes the Hubble parameter, and we used the fact that the apparent horizon is located at $r = 1/H$. Moreover, the standard Friedmann first equation [79] is recovered at the desired limit of $q = 1$ (or equally, $\delta = 0(\|l = 0)$).

3.1. Accelerated Universe. Bearing the fact that the Hubble parameter decreases during the cosmic evolution in mind, rewriting Eq. (19) as

$$\frac{H^2}{\sum_{n=0}^{\infty}\left(\left(l^n H^{2n}\right)/(n!(2n+1))\right)} = \frac{8\pi G_q}{3} \rho \tag{20}$$

and keeping terms up to the H^4 term in LHS (the first corrective term to the standard cosmology ($H^2 = (8\pi G_q/3)\rho$) due to Tsallis gravitational potential), one easily reaches at

$$H^2 \simeq \frac{3}{2l}\left(1 \pm \sqrt{1 - \frac{32\pi G_q l}{9}\rho}\right). \tag{21}$$

In order to have real solutions for H^2, this equation claims that there is a maximum bound on the density of cosmic fluid as $\rho_{\max} = 9/32\pi G_q l$ at which the universe feels a de-Sitter phase with $H = \sqrt{3/2l}$ when $l > 0$. As the universe expands, ρ decreases, and when $\rho = 0$, the positive branch experiences again the primary de-Sitter phase ($H = \sqrt{3/l}$ for $l > 0$), but forever, while the universe expansion rate vanishes for the negative solution. In fact, the vacuum solution ($\rho = 0$) of the above Friedmann first equation is an inflationary universe for the positive branch and a Minkowski universe for the negative branch.

4. Summary

In the framework of the Verlinde hypothesis on the origin of gravity, we employed the recently proposed Tsallis entropy [47, 49] to find its implications on the Newtonian dynamics (second law of motion) and gravity. The velocity profile in a circular motion has also been analyzed. Finally, adopting the classical approach to get the Friedmann first equation described in [79], the corresponding cosmology was achieved after finding the Tsallis gravitational potential. The obtained modified Friedmann first equation (20) includes a complex function of H.

Since the Hubble parameter decreases during the cosmic evolution, and because the standard Friedmann first equation $(H^2 = (8\pi G/3)\rho)$ has notable achievements, we only focused on the first corrective term due to the Tsallis gravitational potential (i.e., we only hold terms up to H^4 in writing Eq. (21)). We saw that, in some situations and depending on the value of δ, the resulting equation addresses the (anti) de-Sitter universes with $H = \sqrt{3\pi/2\delta l_P^2}$ and $H = \sqrt{3\pi/\delta l_P^2}$, depending on ρ. It also admits an upper bound on the energy density of cosmic fluid of order of $((l_P^{-2}G^{-1})/((2+3\delta)\delta)) \sim ((10^{81})/((2+3\delta)\delta))$. We also obtained that there are two branches for the assumed approximation. Whenever $\rho = 0$, the positive branch, depending on the value of δ, guides us to an eternal (anti) de-Sitter phase, and the negative branch addresses a Minkowskian fate for the universe.

Data Availability

There is no data used in this paper.

Conflicts of Interest

The authors declare that there are no conflicts of interest regarding the publication of this paper.

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