Asian Journal of Probability and Statistics

Volume 26, Issue 1, Page 13-24, 2024; Article no.AJPAS.112093
ISSN: 2582-0230

The Remkan Distribution and Its Applications

Uwaeme, O. R. [a*] **and Akpan, N. P.** [b]

[a] *Department of Statistics, Kingsley Ozumba Mbadiwe University, Nigeria.*
[b] *Department of Mathematics and Statistics, University of Port Harcourt, Nigeria.*

Authors' contributions

This work was carried out in collaboration between both authors. Both authors read and approved the final manuscript.

Article Information

DOI: 10.9734/AJPAS/2024/v26i1577

Open Peer Review History:
This journal follows the Advanced Open Peer Review policy. Identity of the Reviewers, Editor(s) and additional Reviewers, peer review comments, different versions of the manuscript, comments of the editors, etc are available here: https://www.sdiarticle5.com/review-history/112093

Original Research Article

Received: 12/11/2023
Accepted: 16/01/2024
Published: 22/01/2024

Abstract

Due to the ever growing demand for the development of new lifetime distributions to meet the goodness of fit demand of complex datasets, two-parameter distributions has been proposed in recent times. This study therefore aims to contribute to this demand. We propose a new two-parameter lifetime distribution known as the Remkan distribution. Important mathematical properties of the Remkan distribution such as the moments and other related measures, and moment generating function were derived and the model parameters estimated using the maximum likelihood estimate technique. Finally, the flexibility of the new Remkan distribution was illustrated using a real life dataset and the results showed that the new Remkan distribution was the best amongst other competing two parameter distributions.

Keywords: Remkan distribution; lifetime distributions; two-parameter distributions; simulation; moments; moment generating functions.

**Corresponding author: Email: onyebu57@gmail.com;*

1 Introduction

Lifetime distributions, also known as survival distributions or failure time distributions provide a statistical framework for modeling the time until an event of interest occurs [1,2]. They provide a powerful tool for analyzing and interpreting data, enabling researchers and practitioners to make informed decisions based on probabilistic models. Due to the complexities of real life dataset, the need for developing new distributions cannot be overemphasized. This is because there is no one size fit all lifetime distribution for any given complex real life dataset. The quest to get a perfect fit to a real life complex dataset has led to the development of several one-parameter lifetime distributions to model real lifetime situations across several fields including engineering, finance, biology, epidemiology, and social sciences.

Examples of the one-parameter lifetime distributions that has been proposed over the years includes the Exponential distribution [3], Lindley distribution [4], Akash distribution [5], Sujatha distribution [6], Ishita distribution [7], Akshaya distribution [8]. Others include Rama distribution [9], Pranav distribution [10], Odoma distribution [11], Nwikpe distribution [12], Iwueze distribution [13], Juchez distribution [14], Chris-Jerry distribution [15].

In recent years, new two-parameter distributions have emerged in the literature. These new two-parameter distributions have been shown to provide better fit to complex real life datasets than the one-parameter distributions. Some of the recently developed two-parameter distributions includes the Darna distribution [16], the Hamza distribution [17], the Samade distribution [18], the Alzoubi distribution [19], and the Copoun distribution [20].

It is important to note that these distributions are a mixture of the Exponential and Gamma distributions. These two distributions are known to have their weaknesses. The weakness of the Exponential distribution is that the hazard rate function is constant; hence, it cannot handle datasets with monotone non-decreasing hazard rates [13,3,21,22,23]. Furthermore, the weakness of the Gamma distribution is that the survival rate function cannot be expressed in closed form [13,5,24]. The limitations of these two distributions are what the aforementioned one-parameter and two-parameter distributions address, providing distributions whose survival rate function can be expressed in closed form and hazard rate functions capable of handling datasets with monotone non-decreasing hazard rates.

In this study, we propose a new two-parameter distribution called the Remkan distribution (RD). The subsequent sections of the paper will be arranged as follows. Section 2 discusses the new distribution, section 3 discusses the mathematical properties of the RD, section 4 discusses the maximum likelihood estimate of the RD, Section 5 discusses the simulation study, section 6 looks at the real life application of the new distribution, and section 7 concludes the paper.

2 The Remkan Distribution (RD)

This section will introduce the probability density function (pdf) and the cumulative distribution function (cdf) of the RD and illustrate the different shapes of the RD.

Definition: A random variable X is said to have a Remkan distribution (RD) with parameters η and ϕ if its probability density function, is given by

$$g(x;\eta,\phi) = \frac{\eta^2}{(\eta+2\phi+6)}[1+\phi\eta x^2+\eta^2x^3]e^{-\eta x};\ x>0,\eta>0,\phi>0 \quad (1)$$

Remark 1: The pdf in equation 1 is a three-component density of an exponential (η) gamma $(3,\eta)$, and gamma $(4,\eta)$ distribution with mixing proportions $\tau_1 = \frac{\eta}{\eta+2\phi+6}$, $\tau_2 = \frac{2\phi}{\eta+2\phi+6}$ and $\tau_2 = \frac{6}{\eta+2\phi+6}$ such that

$$g(x;\eta,\phi) = \tau_1 g_1(x;\eta) + \tau_2 g_2(x;\eta) + \tau_2 g_3(x;\eta) \quad (2)$$

The corresponding cdf of equation 1 is given by

$$G(x;\eta,\phi) = 1 - \left[1 + \frac{\eta^3x^3+(3+\phi)\eta^2x^3+(6+2\phi)\eta x}{\eta+2\phi+6}\right]e^{-\eta x} \quad (3)$$

Remark 2: It can be easily seen that the pdf in equation 1 is a proper pdf.

The graphical plots of the theoretical density and distribution function (for some selected but different real points of η and ϕ) of a Remkan distribution (RD) are shown in the Fig. 1 and Fig. 2 below.

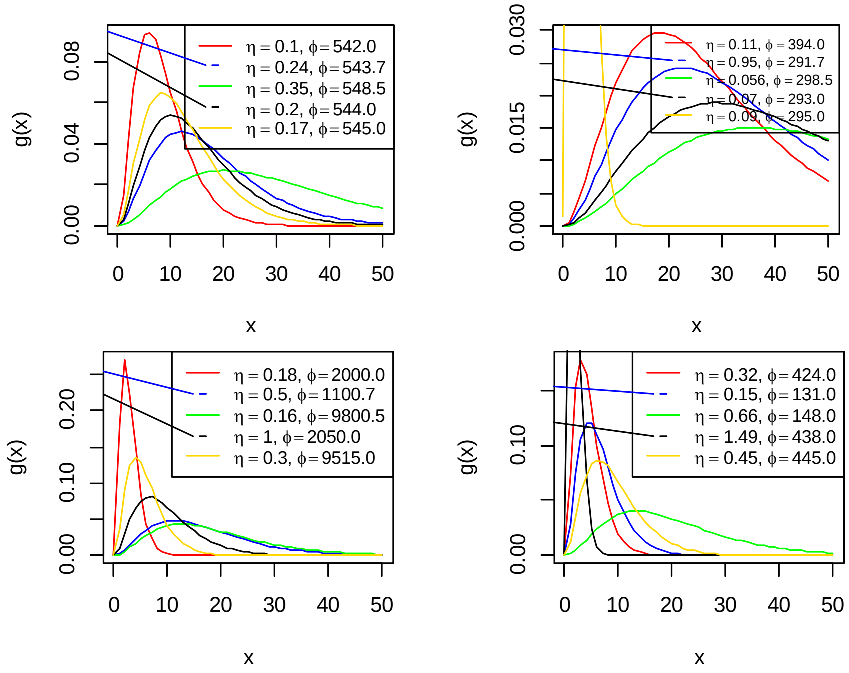

Fig. 1. The graphical plots of the probability density function (for some selected but different real points of η and ϕ) of a Remkan distribution (RD)

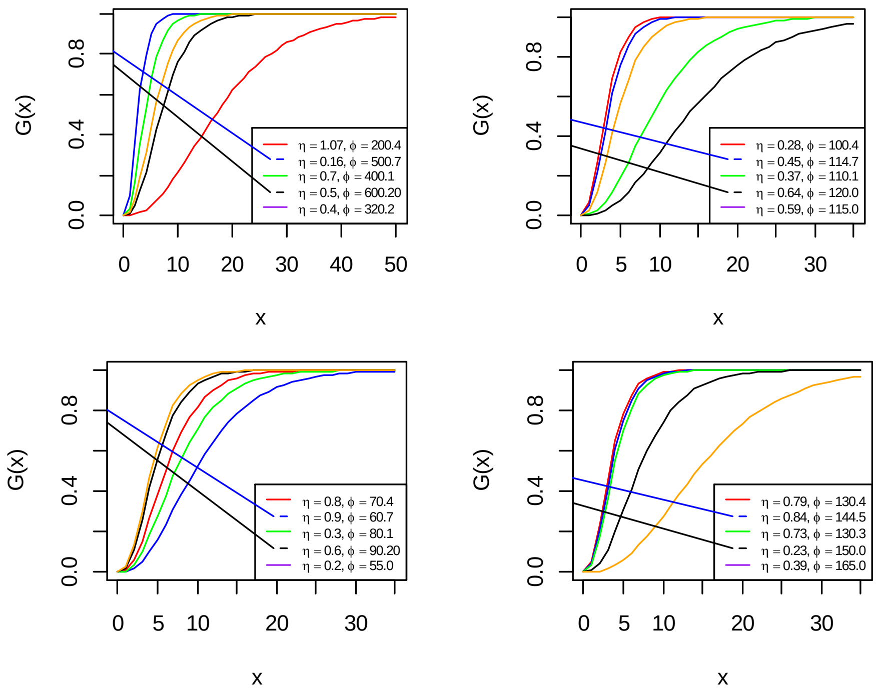

Fig. 2. The graphical plots of the cumulative distribution function (for some selected but different real points of η and ϕ) of a Remkan distribution (RD)

The curves displayed in Fig. 1 are not bell-shaped, but are positively skewed, unimodal, and right tailed. In addition, the curve shows that increasing the value of ϕ leads to a considerable increase in the peak of the curve. In addition, the curves displayed in Fig. 2 shows that the cumulative distribution function converges to one.

3 Mathematical Properties

In this section, we derive and present some of the mathematical properties of the Remkan distribution (RD) such as the moment and other related measures, and moment generating function.

3.1 Moments of the remkan distribution

We derive the r^{th} moment of the Remkan distribution (RD) in this subsection.

Theorem 1

Given a random variable X, following the Remkan distribution (RD), the k^{th} order moment about origin, $E(X^k)$ of the Remkan distribution is given by

$$\mu_k' = E(X^k) = \frac{\eta^2}{\eta+2\phi+6}\left[\frac{k!}{\eta^{k+1}} + \frac{\phi(k+2)!}{\eta^{k+2}} + \frac{(k+3)!}{\eta^{k+2}}\right] \quad (4)$$

Proof:

The k^{th} crude or uncorrected moments of a random variable and can be written as

$$\mu_k' = E(X^k) = \int_0^\infty x^k g(x;\eta,\phi)dx \quad (5)$$

$$= \int_0^\infty x^k \cdot \frac{\eta^2}{(\eta+2\phi+6)}[1+\phi\eta x^2+\eta^2 x^3]e^{-\eta x}\,dx \quad (6)$$

$$= \frac{\eta^2}{(\eta+2\phi+6)}\left[\int_0^\infty x^k e^{-\eta x}\,dx + \phi\eta\int_0^\infty x^{k+2}e^{-\eta x}\,dx + \eta^2\int_0^\infty x^{k+3}e^{-\eta x}\,dx\right] \quad (7)$$

Recall, $\frac{\Gamma(\varrho)}{\varphi^{\varrho}} = \int_0^\infty z^{\varrho-1}e^{-\varphi z}\,dz$, and $\Gamma(s) = (s-1)!$. Hence,

$$E(X^k) = \frac{\eta^2}{(\eta+2\phi+6)}\left[\frac{\Gamma(k+1)}{\eta^{k+1}} + \frac{\phi\eta\Gamma(k+3)}{\eta^{k+3}} + \frac{\eta^2\Gamma(k+4)}{\eta^{k+4}}\right] \quad (8)$$

$$\therefore E(X^k) = \frac{\eta^2}{(\eta+2\phi+6)}\left[\frac{k!}{\eta^{k+1}} + \frac{\phi(k+2)!}{\eta^{k+2}} + \frac{(k+3)!}{\eta^{k+2}}\right] \quad (9)$$

Which completes the proof.

In particular, the first four (4) moments about the origin of the Remkan distribution (RD) is obtained by substituting the values of k= 1, 2, 3, 4 into equation (9) as follows;

$$\mu_1' = E(X) = \frac{\eta+6\phi+24}{\eta(\eta+2\phi+6)} \quad (10)$$

$$\mu_2' = E(X^2) = \frac{2\eta+24\phi+120}{\eta^2(\eta+2\phi+6)} \quad (11)$$

$$\mu_3' = E(X^3) = \frac{6\eta+120\phi+720}{\eta^3(\eta+2\phi+6)} \quad (12)$$

$$\mu_4' = E(X^4) = \frac{24\eta+720\phi+5040}{\eta^4(\eta+2\phi+6)} \quad (13)$$

Furthermore, the first four (4) moments about the mean of the Remkan distribution (RD) is obtained by substituting the values of k= 1, 2, 3, 4 in (14) as follows;

$$\mu_k = E[(X-\mu)^k] \tag{14}$$

$$\sigma^2 = \mu_2 = \frac{\eta^2+16\phi\eta+12\phi^2+84\eta+96\phi+144}{\eta^2(\eta+2\phi+6)^2} \tag{15}$$

$$\mu_3 = \frac{2\left[864+\eta^3+6\eta^2(33+5\phi)+36\eta\left(\phi^2+9\phi+9\right)+24\phi^3+288\phi^2+864\phi\right]}{\eta^3(\eta+2\phi+6)^3} \tag{16}$$

$$\mu_4 = \frac{\begin{array}{c}9\eta^4+\eta^3(384\phi+2808)+\eta^2\left(1224\phi^2+12384\phi+20736\right)+\eta\left(1728\phi^3+22752\phi^2+81216\phi+93312\right)\\ +720\phi^4+11520\phi^3+58752\phi^2+124416\phi+93312\end{array}}{\eta^4(\eta+2\phi+6)^4} \tag{17}$$

The coefficient of variation, coefficient of skewness, coefficient of kurtosis, and index of dispersion of the Remkan distribution (RD), respectively, are given by

$$C.V = \frac{\sigma}{\mu_1'} = \frac{\sqrt{\eta^2+16\phi\eta+12\phi^2+84\eta+96\phi+144}}{\eta+2\phi+6} \tag{18}$$

$$\sqrt{\beta_1} = \frac{\mu_3}{(\mu_2)^{\frac{3}{2}}} = \frac{2\left[864+\eta^3+6\eta^2(33+5\phi)+36\eta\left(\phi^2+9\phi+9\right)+24\phi^3+288\phi^2+864\phi\right]}{(\eta^2+16\phi\eta+12\phi^2+84\eta+96\phi+144)^{\frac{3}{2}}} \tag{19}$$

$$\beta_2 = \frac{\mu_4}{(\mu_2)^2} = \frac{\begin{array}{c}9\eta^4+\eta^3(384\phi+2808)+\eta^2\left(1224\phi^2+12384\phi+20736\right)+\eta\left(1728\phi^3+22752\phi^2+81216\phi+93312\right)\\ +720\phi^4+11520\phi^3+58752\phi^2+124416\phi+93312\end{array}}{(\eta^2+16\phi\eta+12\phi^2+84\eta+96\phi+144)^2} \tag{20}$$

$$\gamma = \frac{\sigma^2}{\mu_1'} = \frac{\eta^2+\eta(84+16\phi)+\phi(96+12\phi)+144}{\eta(\eta+2\phi+6)(\eta+6\phi+24)} \tag{21}$$

The behaviors of C.V., $\sqrt{\beta_1}$, β_2 and γ, for varying values of the parameters η and ϕ have been shown numerically in Tables 1,2,3 and 4.

Table 1. CV of RD for varying values of parameters η and ϕ

ϕ \ η	0.2	0.5	1	2	3	4
0.2	2.2347	1.8790	1.7004	1.5854	1.5409	1.5172
0.5	1.8366	1.4927	1.3041	1.1691	1.1119	1.0797
1	1.5743	1.2561	1.0764	0.9404	0.8791	0.8430
2	1.3546	1.0622	0.8972	0.7689	0.7085	0.6717
3	1.2468	0.9662	0.8101	0.6881	0.6299	0.5939
4	1.1797	0.9053	0.7549	0.6377	0.5815	0.5465

For a given value of ϕ, CV decreases as the value of η increases

Table 2. Skewness of RD for varying values of parameters η and ϕ

ϕ \ η	0.2	0.5	1	2	3	4
0.2	0.0309	0.0111	0.0038	0.0009	0.00037	0.00018
0.5	0.0098	0.0048	0.0023	0.00086	0.0004	0.00024
1	0.0039	0.0021	0.0012	0.0006	0.0003	0.00021
2	0.0018	0.0008	0.0005	0.0003	0.0002	0.00014
3	0.0013	0.0005	0.0003	0.0002	0.0001	0.00010
4	0.0010	0.0004	0.0002	0.0001	0.00009	0.00007

For a given value of ϕ, $\sqrt{\beta_1}$ decreases as the value of η increases. Also, since $\sqrt{\beta_1} > 0$, RD is always positively skewed.

Table 3. Kurtosis of RD for varying values of parameters η and ϕ

ϕ \ η	**0.2**	**0.5**	**1**	**2**	**3**	**4**
0.2	3.7139	3.2686	3.0239	2.4711	2.0527	1.7547
0.5	6.1218	3.7139	3.3202	3.1316	2.9091	2.6806
1	11.9188	5.1868	3.7139	3.3202	3.2254	3.1316
2	26.9814	9.4135	5.1868	3.7139	3.4177	3.3102
3	44.7241	14.6245	7.1468	4.3673	3.7139	3.4789
4	63.8215	20.5343	9.4135	5.1868	4.1273	3.7139

For $0.2 \leq \phi \geq 0.5$, and $2 \leq \eta \geq 4$, $\beta_2 < 3$, and the RD is platykurtic, while the rest is leptokurtic, since $\beta_2 > 3$, RD can be both platykurtic or leptokurtic.

Table 4. Index of dispersion of RD for varying values of parameters η and ϕ

ϕ \ η	**0.2**	**0.5**	**1**	**2**	**3**	**4**
0.2	5.4032	2.3406	1.2992	0.7486	0.5461	0.4351
0.5	5.4493	2.3394	1.2857	0.7337	0.5333	0.4245
1	5.4927	2.3317	1.2652	0.7125	0.5152	0.4093
2	5.5146	2.3079	1.2310	0.6798	0.4872	0.3857
3	5.5011	2.2824	1.2039	0.6558	0.4667	0.3682
4	5.4763	2.2588	1.1823	0.6375	0.4510	0.3547

As long as $0 \leq \eta \geq 1$, and $0 \leq \phi \geq 5$, the nature of RD is over dispersed $(\sigma^2 > \mu_1')$ and for $1 \leq \eta \geq 5$, and $0 \leq \phi \geq 5$, the nature of RD is under dispersed $(\sigma^2 < \mu_1')$.

3.2 Moment generating function

Here, we propose the moment generating function for the Remkan distribution (RD) on this subsection.

Theorem 2

Given a random variable X, following the Remkan distribution (RD), the moment generating function of X, $M_X(t)$ of the Remkan distribution (RD) is given by

$$M_X(t) = E(e^{tx}) = \frac{\eta^2}{(\eta+2\phi+6)}\sum_{k=0}^{\infty}\left(\frac{t}{\eta}\right)^k\left[\frac{k!}{\eta^{k+1}} + \frac{\phi(k+2)!}{\eta^{k+2}} + \frac{(k+3)!}{\eta^{k+2}}\right] \tag{22}$$

Proof:

The moment generating function of X, $M_X(t)$ can be written as

$$M_X(t) = E(e^{tx}) = \int_0^\infty e^{tx}\, g(x;\eta,\phi)dx \tag{23}$$

$$= \int_0^\infty e^{tx}\cdot\frac{\eta^2}{(\eta+2\phi+6)}[1+\phi\eta x^2+\eta^2x^3]e^{-\eta x}\,dx \tag{24}$$

$$= \frac{\eta^2}{(\eta+2\phi+6)}\int_0^\infty e^{tx}\cdot[1+\phi\eta x^2+\eta^2x^3]e^{-\eta x}\,dx \tag{25}$$

Recall that $e^{tx} = \sum_{k=0}^{\infty}\frac{(tx)^k}{k!}$. Substituting, we obtain

$$= \frac{\eta^2}{(\eta+2\phi+6)} \sum_{k=0}^{\infty} \frac{t^k}{k!} \int_0^{\infty} x^k \cdot [1 + \phi\eta x^2 + \eta^2 x^3] e^{-\eta x}\, dx \tag{26}$$

$$= \sum_{k=0}^{\infty} \frac{t^k}{k!} E(X^k) \tag{27}$$

$$= \frac{\eta^2}{6\eta(\phi+\eta)} \sum_{k=0}^{\infty} \binom{t}{\eta}^k \cdot \frac{k!}{k!} + \frac{\phi\eta^4}{6\eta^4(\phi+\eta)} \sum_{k=0}^{\infty} \binom{t}{\eta}^k \cdot \frac{(k+3)!}{k!} \tag{28}$$

$$\therefore M_X(t) = E(e^{tx}) = \frac{\eta^2}{(\eta+2\phi+6)} \sum_{k=0}^{\infty} \frac{t^k}{k!} \left[\frac{k!}{\eta^{k+1}} + \frac{\phi(k+2)!}{\eta^{k+2}} + \frac{(k+3)!}{\eta^{k+2}} \right] \tag{29}$$

Which completes the proof.

4 Maximum Likelihood Estimation

Given a random sample of size n, $X_1, X_2, X_3, \dots, X_n$ from the RD distribution. The log-likelihood function of parameters can be obtained by

$$LL(x;\eta\phi) = \prod_{i=1}^{n} \ln g_{RD}(x) \tag{30}$$

$$LL(x;\eta\phi) = \prod_{i=1}^{n} \ln \left\{ \frac{\eta^2}{(\eta+2\phi+6)} [1 + \phi\eta x_i^2 + \eta^2 x_i^3] e^{-\eta x_i} \right\} \tag{31}$$

$$LL(x;\eta\phi) = \ln \left\{ \left(\frac{\eta^2}{(\eta+2\phi+6)} \right)^n \prod_{i=1}^{n} ([1 + \phi\eta x_i^2 + \eta^2 x_i^3] e^{-\eta x_i}) \right\} \tag{32}$$

$$LL(x;\eta\phi) = 2n \ln(\eta) - n \ln(\eta + 2\phi + 6) + \sum_{i=1}^{n} \ln(1 + \phi\eta x_i^2 + \eta^2 x_i^3) - \eta \sum_{i=1}^{n} x_i \tag{33}$$

We take the partial differentiation of (33) w.r.t η and ϕ and solve the nonlinear likelihood equations obtained so as to maximize the log-likelihood as shown below;

$$\frac{\partial LL}{\partial \eta} = \frac{2n}{\eta} - \frac{n}{(\eta+2\phi+6)} + \sum_{i=1}^{n} \frac{(2\eta x_i^3 + \phi x_i^2)}{(\eta^2 x_i^3 + \eta\phi x_i^2)} \tag{34}$$

$$\frac{\partial LL}{\partial \phi} = -\frac{2n}{(\eta+2\phi+6)} + \sum_{i=1}^{n} \frac{(\eta x_i^2)}{(\eta^2 x_i^3 + \eta\phi x_i^2 + 1)} \tag{35}$$

The above derivatives in (34) and (35) are then equated to zero and solved simultaneously to obtain the estimates of the parameters. The solutions cannot be solved analytically hence we solve numerically using the Adequacy package of Marinho et al. (2019) with “BFGS” algorithm. The R software (R Team, 2017) is used for this analysis.

5 Simulation Study

In this section, we present the results of the simulation study carried out on the new Remkan distribution to determine the efficiency of the maximum likelihood estimate. The acceptance/rejection algorithm was adopted for this simulation study [25,26].

5.1 The acceptance/rejection algorithm

In order to simulate from the density $g(x_k;\Phi)$, it is assumed that we have the envelope density $\Im$ from which we can simulate from, and that we have some $k < \infty$ such that $\underset{x}{Sup} \frac{g(x_k;\Phi)}{\Im(x_k;\Phi)} \le k$.

Step 1: Simulate X from $\Im$

Step 2: Generate $Y \sim U\big(0, k\Im(x_k;\Phi)\big)$ where $k = \frac{\eta^2}{(\eta+2\phi+6)}$

Step 3: If $Y < g(x_k; \Phi)$, then return X, otherwise go to step 1.

The simulation study will be based on generating $N = 10{,}000$ samples of size $n = 50{,}100{,}150, 200$ for $\Phi = 0.5{,}1{,}1.5\ and\ 2$ using the algorithm above. Then the following measures will be calculated.

i. Average bias of the simulated estimate

$$Average\ bias = \frac{1}{N}\sum_{i=1}^{N}\left(\widehat{\Phi_\iota} - \Phi\right). \tag{36}$$

where $\widehat{\vartheta_\iota}$ is the maximum likelihood estimate.

ii. Average mean square error (MSE)

$$MSE = \frac{1}{N}\sum_{i=1}^{N}\left(\widehat{\Phi_\iota} - \Phi\right)^2. \tag{37}$$

The average bias and average mean square error (MSE) for each of the ML estimate has been calculated and shown in Table (5), where MSE has been shown in bracket.

Table 5. The average bias and MSEs of parameter estimates with MLE for different values of parameters(η, ϕ)

n	Par (η)	$Average\ bias$	MSE	Par (ϕ)	$Average\ bias$	MSE
50	0.5	0.5061	0.2764	2.0	-1.1514	1.3371
	1	0.0062	0.0196	2.5	-1.6484	2.7284
	1.5	-0.5531	0.3212	3.0	-2.1501	4.6341
	2.0	-1.1486	1.3308	3.5	-2.6481	7.0234
100	0.5	0.5084	0.2682	2.0	-1.1507	1.3298
	1	0.00421	0.0095	2.5	-1.6499	2.7277
	1.5	-0.5490	0.3091	3.0	-2.1495	4.6258
	2.0	-1.1503	1.3289	3.5	-2.6476	7.0152
150	0.5	0.5072	0.2641	2.0	-1.1516	1.3300
	1	0.00419	0.0065	2.5	-1.6512	2.7302
	1.5	-0.5498	0.3074	3.0	-2.1498	4.6252
	2.0	-1.1504	1.3272	3.5	-2.6471	7.0108
200	0.5	0.5065	0.2615	2.0	-1.1524	1.3309
	1	0.0055	0.0049	2.5	-1.6507	2.7278
	1.5	-0.5502	0.3065	3.0	-2.1483	4.6180
	2.0	-1.1498	1.3249	3.5	-2.6473	7.0113

Source: Author's compilation, 2023

6 Application

This section discusses the flexibility and superiority of the Remkan distribution to some competing distributions using real life data sets. The dataset employed represents the lifetime data relating to time (in months from 1st January, 2013 to 31st July, 2018) of 105 patients who were diagnosed with hypertension and received at least one treatment-related to hypertension in the hospital where death is the event of interest. It was reported by Umeh & Ibenegbu, [27] and used by Enogwe et al. [28]. The data is shown below.

45, 37, 14, 64, 67, 58, 67, 55, 64, 62, 9, 65, 65,43, 13, 8, 31, 30, 66, 9, 10, 31, 31, 31, 46, 37, 46, 44,45, 30, 26, 28, 45, 40, 47, 53, 47, 41, 39, 33, 38, 26, 22, 31, 46, 47, 66, 61, 54, 28, 9, 63, 56, 9, 49, 52, 58, 49, 53, 63, 16, 67, 61, 67, 28, 17, 31, 46, 52, 50, 30, 33, 13, 63, 54, 63, 56, 32, 33, 37, 7, 56, 1, 67, 38, 33, 22, 25, 30, 34, 53, 53, 41, 45, 59, 59, 60, 62, 14, 57, 56, 57, 40, 44, 63.

This dataset is then fitted with the Remkan distribution (RD) and compared with Two-parameter Lindley distribution (TPLD) [29], Two-parameter Akash distribution (TPAD) [30], Two-parameter Rama distribution

(TPRD) [31], Two-parameter Sujatha distribution (TPAD) [32], and Samade distribution (SD) [18] with corresponding pdfs.

$$f_{TPLD}(x;\Phi) = \frac{\eta^2}{\eta\phi+1}(\phi + x)e^{-\eta x} \tag{38}$$

$$f_{TPAD}(x;\Phi) = \frac{\eta^3}{\phi\eta^2+2}(\phi + x^2)e^{-\eta x} \tag{39}$$

$$f_{TPRD}(x;\Phi) = \frac{\eta^4}{\phi\eta^3+6}(\phi + x^3)e^{-\eta x} \tag{40}$$

$$f_{TPSD}(x;\Phi) = \frac{\eta^3}{\eta^2+\phi\eta+2}(1 + \phi x + x^2)e^{-\eta x} \tag{41}$$

$$f_{SD}(x;\Phi) = \frac{\eta^4}{\eta^4+6\phi}(\eta + \phi x^3)e^{-\eta x} \tag{42}$$

This comparison is done using some measures for testing the goodness of fit of a distribution. The measures used are the parameter estimates, the log likelihood, the Akaike Information Criteria (AIC) and the Bayesian Information Criteria (BIC) -2lnL, Akaike Information Criterion (AIC) [33], Bayesian Information Criterion (BIC) [34], Consistent Akaike information criterion (CAIC), Hannan-Quinn information criterion (HQIC). In general, the smaller the values of AIC, BIC, CAIC, and HQIC the better the fit to the data.

$$AIC = 2k - 2\ln L \tag{43}$$

$$BIC = k\ln n - 2\ln L \tag{44}$$

$$CAIC = AIC + \frac{2k(k+1)}{(n-k-1)} \tag{45}$$

$$HQIC = 2k\ln(\ln(n)) - 2\ln L \tag{46}$$

where k is the number of parameters, n is the sample size of the dataset, and L is the likelihood function.

Table 6. Goodness of fit for the Hypertension Data

Distribution	MLE's	S.E	-2ln L	AIC	BIC	CAIC	HQIC
RD	**$\hat{\eta}$ =0.0936**	**0.0060**	**460.1171**	**924.2342**	**929.5422**	**924.3519**	**923.3097**
	$\hat{\phi}$ = -0.0235	**0.5267**					
TPLD	$\hat{\eta}$ =0.001	1.1021	463.7301	931.4603	931.5579	931.5579	933.6112
	$\hat{\phi}$ = 0.047	0.0035					
Samade	$\hat{\eta}$ =0.0948	0.00463	471.0754	938.1509	932.843	938.0333	939.0755
	$\hat{\phi}$ = 7428.0	1.049					
TPRD	$\hat{\eta}$ =381.2325	318.2483	472.6460	949.2920	949.4096	949.4096	951.4428
	$\hat{\phi}$ = 0.0916	0.0049					
TPSD	$\hat{\eta}$ =0.0705	0.0044	464.6302	933.2603	938.5683	933.378	932.3358
	$\hat{\phi}$ = 0.6089	2.3069					
TPAD	$\hat{\eta}$ = 0.0700	0.0039	463.7305	931.4609	936.7688	931.5786	930.5363
	$\hat{\phi}$ =10.4281	2.9755					

The parameter estimates and their goodness of fit of the different models for the dataset are presented in Table 6. From the results, the Remkan distribution (RD) performed better than the competing distributions.

7 Conclusion

We have proposed a new two-parameter distribution known as the Remkan distribution (RD). The mathematical properties of the Remkan distribution (RD) such as the moments and other related measures, and moment generating functions was derived and presented. The parameter estimates of the Remkan distribution (RD) was obtained using the method of maximum likelihood estimates. Furthermore, the simulation study of the new distribution was also conducted. Finally, the flexibility of the Remkan distribution (RD) was illustrated using a real lifetime data. The result showed that the Remkan distribution (RD) performed better than the other competing two-parameter distributions.

Competing Interests

Authors have declared that no competing interests exist.

References

[1] Chahkandi M, Ganjali M. On some lifetime distributions with decreasing failure rate. Computational Statistics & Data Analysis. 2009;53(12):4433–4440.
Available: https://doi.org/10.1016/j.csda.2009.06.016

[2] SchÄbe H. Constructing lifetime distributions with bathtub shaped failure rate from DFR distributions. Microelectronics Reliability. 1994;34(9):1501–1508.
Available: https://doi.org/10.1016/0026-2714(94)90458-8

[3] Epstein N. Exponential distribution and its role in life testing. 1958;1–21.

[4] Lindley DV. Fiducial distributions and bayes' theorem. Journal of the Royal Statistical Society Series B-Methodological. 1958;20(1):102–107.
Available: https://doi.org/10.1111/j.2517-6161.1958.tb00278.x

[5] Shanker R. Akash distribution and its applications. International Journal of Probability and Statistics. 2015a;4(3):65–75.
Available: https://doi.org/10.5923/j.ijps.20150403.01

[6] Shanker R. Sujatha distribution and its applications. Statistics in Transition New Series. 2016;17(3):391–410.
Available: https://doi.org/10.21307/stattrans-2016-029

[7] Shanker R, Shukla KK. Ishita distribution and its applications. Biometrics & Biostatistics International Journal. 2017a;5(2):39–46.
Available: https://doi.org/10.15406/bbij.2017.05.00126

[8] Shanker R. Akshaya distribution and its application. American Journal of Mathematics and Statistics. 2017a;7(2):51–59.

[9] Shanker R. Rama distribution and its application. International Journal of Statistics and Applications. 2017b;7(1):26–35.

[10] Shukla KK. Pranav distribution with properties and its applications. Biometrics & Biostatistics International Journal. 2018b;7(3):244–254.
Available: https://doi.org/10.15406/BBIJ.2018.07.00215

[11] Odom CC, Ijomah MA. Odoma distribution and its application. Asian Journal of Probability and Statistics. 2019;4(1):1–11.
Available: https://doi.org/10.9734/AJPAS/2019/V4I130103

[12] Nwikpe BJ, Isaac DE, Emeka A. Nwikpe probability distribution: Statistical properties and goodness of fit. Asian Journal of Probability and Statistics. 2021;11(1):52–61.
Available: https://doi.org/10.9734/ajpas/2021/v11i130260

[13] Elechi O, Okereke EW, Chukwudi IH, Chizoba KL, Wale OT. Iwueze's Distribution and Its Application. Journal of Applied Mathematics and Physics. Article 12. 2022;10(12).
Available: https://doi.org/10.4236/jamp.2022.1012251X

[14] Echebiri UV, Mbegbu JI. Juchez probability distribution: Properties and applications semantic scholar. Asian Journal of Probability and Statistics. 2022;20(2):56–71.
Available: https://doi.org/10.9734/AJPAS/2022/v20i2419

[15] Onyekwere CK, Obulezi OJ. Chris-jerry distribution and its applications. Asian Journal of Probability and Statistics. 2022;20(1):16–30.
Available: https://doi.org/10.9734/ajpas/2022/v20i130480

[16] Shraa DH, Al-Omari AI. Darna distribution: Properties and application. Electronic Journal of Applied Statistical Analysis. 2019;12(02):520–541.
Available: https://doi.org/10.1285/i20705948v12n2p520

[17] Aijaz A, Jallal M, Ain SQU, Tripathi R. The hamza distribution with statistical properties and applications. Asian Journal of Probability and Statistics. 2020;8(1):28–42.
Available: https://doi.org/10.9734/AJPAS/2020/v8i130196

[18] Aderoju S. Samade probability distribution: Its properties and application to real lifetime data. Asian Journal of Probability and Statistics. 2021;14(1):1–11.
Available: https://doi.org/10.9734/ajpas/2021/v14i130317

[19] Benrabia M, Alzoubi LMA. Alzoubi distribution: Properties and applications. An International Journal of Statistics Applications & Probability. 2021;11(2):1–16.
Available: https://doi.org/10.18576/jsap/ALzoubi dist nsp

[20] Uwaeme OR, Akpan NP, Orumie UC. The copoun distribution and its mathematical properties. Asian Journal of Probability and Statistics. 2023;24(1):37–44.

[21] Ronald WE, Myers RH, Myers SL, Ye KE. Probability and Statistics for Engineers and Scientists (9th Edition) (9th ed.). Prentice Hall; 2011.
Available: http://library.lol/main/09C6713DAA3BF89644499A5EB315D12E

[22] Shukla KK. Prakaamy distribution with properties and applications. JAQM. 2018a;13(3):30–38.

[23] Shukla KK. Ram awadh distribution with properties and applications. Biometrics & Biostatistics International Journal. 2018c;7(6):515–523.
Available: https://doi.org/10.15406/bbij.2018.07.00254

[24] Shanker R. Shanker distribution and its applications. International Journal of Statistics and Applications. 2015b;5:338–348.

[25] Law AM. Simulation Modeling and Analysis. McGraw-Hill Education; 2014.
Available: http://library.lol/main/028D069D632D9AD57697D89E9DB7FBAB

[26] Shanker R. Rani distribution and its application. Biometrics & Biostatistics International Journal. 2017c;6(1):256–265.
Available: https://doi.org/10.15406/bbij.2017.06.00155

[27] Umeh E, Ibenegbu A. A two-parameter pranav distribution with properties and its application. Journal of Biostatistics and Epidemiology. 2019;5(1):74–90.
Available: https://doi.org/10.18502/jbe.v5i1.1909

[28] Enogwe SU, Nwosu DF, Ngome EC, Onyekwere CK, Omeje IL. Two-parameter odoma distribution with applications. Journal of Xidian University. 2020;14(8):740–764.

[29] Shanker R, Mishra A. A two-parameter lindley distribution. Statistics in Transition New Series. 2013;14(1):45–56.

[30] Shanker R, Shukla KK. On two-parameter akash distribution. Biometrics & Biostatistics International Journal. 2017b;6(5):416–425.
Available: https://doi.org/10.15406/bbij.2017.06.00178

[31] Edith UU, Ebele TU, Henrietta AI. A two-parameter rama distribution. Earthline Journal of Mathematical Sciences. 2019;2(2):365–382.
Available: https://doi.org/10.34198/ejms.2219.365382

[32] Tesfay M, Shanker R. A two–parameter sujatha distribution. Biometrics & Biostatistics International Journal. 2018;7(3):188–197.

[33] Club TB. Akaike H. Information theory and an extension of the maximum likelihood principle. 1973;267-281 in 2nd International Symposium on Information Theory, Tsahkadsor, Armenia, USSR, September 2-8, 1971, eds. BN Petrov and F. Csáki. Budapest: Akadémiai Kiadó. Bankevich, A., S. Nurk, D. Antipov, AA Gurevich, M. Dvorkin, AS Kulikov, VM Lesin, et al. 2012. SPAdes: A new genome assembly algorithm and its applications to single. Systematics and Evolution of the Arundinoideae and Micrairoideae (Poaceae). 2016 Aug 15;18(1):139.

[34] Pollock DSG, Green RC, Nguyen T. Handbook of time series analysis, signal processing, and dynamics. Elsevier; 1999.

Peer-review history:
The peer review history for this paper can be accessed here (Please copy paste the total link in your browser address bar)
https://www.sdiarticle5.com/review-history/112093