The Role of Mathematics and Geometry in Formation of Persian Architecture

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Abstract

Geometry is one of the main features in formation of Persian architecture. Research in Persian architecture geometry is impossible without familiar with geometry. This research tries to demonstrate the relationship between the progress of Persian architecture, geometry and mathematics evolution. In this regard, seeks to address the following questions: What is the relationship between the progress of Persian architecture and geometry evolution? To this end first, a detailed survey is conducted on the Persian Architecture and Mathematicians in the pre-Islamic and Islamic History. The Persian empires and dynasties are analyzed and chronologically organized to determine the evolution of mathematics and geometry impacts on the formation and development of Persian architecture. The research hypothesis is that evolution of mathematics and geometry, have a direct impact on the formation and development of Iranian architecture. This study depicts by development of mathematic geometric, Persian architecture has progressed and With the Fading communication between mathematicians and architects, manufacturing techniques of buildings has deteriorated.

Keywords: Persian architecture, Islamic geometry, mathematic

1. Introduction

Research in Persian architecture without knowledge of the geometry is impossible. Muslims considered geometry as an important science, balanced with mathematics, astrology and music. In Iranian architecture, geometrical and abstract patterns were emphasized as a forming factor (of whole structure), a decoration method (of components), alternatives and sense transformer. The development of geometry in Islamic World started during 8th and 9th centuries from *Sanskrit* and Greek texts to Arabian texts and till 10th century, Muslims achieved great successes in the field. An investigation of reasons of geometry importance in Islamic art and architecture and its reflection forms in these structures' whole body and components, considering cultures and values governing society and understanding meanings, is important. In the field of g eometry, great changes were occurred as a result of *Umar al-Khayyām*[Note 1], *Abū al-Wafā' Būzjānī*[Note 2], *al-Khwārizmī*[Note 3] and *Ibn al-Haytham's* efforts [Note 4]. It seems that there has been a relationship between geometricians and architects in Islamic World and linguistic description of this relationship can be seen in Arabic sense of Geometry, which is not only based on geometry but also meaning engineering. Muslim mathematicians such as *Būzjānī* and *Al-Kashi* [Note 5] provided manuals with the aim of describing fundamental principles of geometry and its applications in architecture. Such manuals are technical, and their aim is to make principles of mathematics easier and comprehensible for non-professionals of the area. It seems that developments in mathematics and consequently, in geometry and advent of new tools during historical dynasties of Iran have influenced developments of Iranian architecture, directly.

2. Method

Present investigation is theoretical-fundamental, carried out in historical-descriptive method. Needed information was gathered through studying valid historical sources and field observations. What we know about relation between architecture, and mathematic is largely based upon interpretive studies that investigate the progress of the Muslims in the Islamic and pre-Islamic historical texts. The study was conducted in the form of a historic-interpretative, with data being gathered via Conformity of history, art and science architecture. In the Islamic era and earlier, using available resources and library information. The aim of present investigation is to compare art, mathematics, geometry and the application of mathematics in architecture to obtain a definition and interpretation of component influencing Persian architecture from geometric point of view, through classifying mathematicians

and scientists influencing Persian architecture, considering history and eliciting mathematical and geometric principles governing designs and principles of drawing.

3. Literature Review

Studies of science's history and architecture indicate two distinguished viewpoints with the issue of relationship between mathematics and geometry with architecture. Scholars like *Saliba* (Saliba, 1995) and Holod (Renata, 1988) suspect the role of mathematicians and the existence of relationship between mathematic texts and architecture. They considered mathematic knowledge of architecture companion in oral form, seeing insignificant relationship between these two groups. Though, other scholar including Bulatov (1978), Chorbachi (1989), Özdural (2002), and Necipoglu (1995) emphasize the role of sciences, mathematical texts and mathematicians in architecture specially in Islamic era (Chorbachi, 1989). For example *Özdural* relates the main roles of developments in architecture and related industries to mathematicians (Özdural, 1995). *Özdural* presents a series of quotations from these two sources, which show that mathematicians taught geometry to artisans by means of cut-and-paste methods and of geometrical Figures that had the potential of being used for ornamental purposes (Özdural, 2000). In a detailed study, *Necipoglu* also, clearly mentions the share of mathematics in architecture through emphasizing the role of mathematicians in writing mathematical texts which are related to architecture (Denny, Necipoglu, & al-Asad, 1998). Recently Iranian scholars, Such as *Taheri* and *Hejazi* also, have proceeded on studying and investigation in this regard, providing contents in the form of books and valid articles (Taheri, 2008). *Hejazi* shows that A comprehensive utilisation of proportions in Persian architecture, such as in the design of plans, elevations, geometric and architectural patterns, and mechanical and structural features, can be proved through geometrical analysis of Persian historical buildings (Hejazi, 2005). *Taheri* showed that how the mathematical knowledge of a Persian scientist was utilized in architecture and craftsmanship, he shed some light on the hidden layers of *Kâshânî*'s architectural life, who until now has been considered only a mathematician and astronomer (Taheri, 2011).

4. History of Employing Geometry before Islam

In order to get familiar with Iranian people's knowledge of Geometry specially applied geometry in art and investigating geometrical developments used in Iranian Architecture before Islam, we will present several historical eras, here, to show samples indicating achieving developments in applied geometry in architecture, scientists' knowledge of geometry and its feedback in architecture art of that era.

4.1 Before Achaemenid Empire [Note 6]

Evidence of architecture and urban development in this era indicate that using geometry and architecture was not ineffective in urban design and structures of this era. According to *Herodotus* [Note 7], *Ecbatana* [Note 8] city (contemporary *Hamadān* or *Hamedān*), capital city of *Medians*, was completely in circular form and the city was surrounded by seven concentric walls. Moreover, these walls had been various balanced pathways and structures were constructed in its extension (Zarei, 2000, p. 77). In This period, Iranian architecture makes use of abundant symbolic geometry, using pure forms such as circles and squares, and plans are based on often symmetrical layouts featuring rectangular courtyards and hall.

4.2 Achaemenid Empire

Achaemenian were among the greatest architects and geometers of Iranian History. Here is *Shahkarami*'s [Note 9] note about *Persepolis* [Note 10]: experts' investigations on structures of especially its columns and its conical forms indicate that: Achaemenian were using p-value 2500 years before mathematics scientists. They were completely familiar with pure mathematics and engineering. They identified the secret of p-value to build conical masses. Geometry is latent in Achaemenian's architectural decorations with simple or frequent repetition of natural or abstract motifs, which were based on axial symmetry, motif's places and their distance of each other with specified geometrical structures (Kharazmi & Sarhangi, 2013). In decoration part related to architecture, Lotus design or vortex curl was administrated using simple geometrical divisions relying on spiral development based on such divisions (Hejazi, 2005, p. 83) $\sqrt{2}$ ratios were used in Achaemenian's places specially in *Apadana* [Note 11] palace, in a way that hall involves 36 central columns as It's basic square and its diameters illuminate three sides of palace in width of columned balconies (See Figure 1) (Bemanian, Okhovat, & Baghaei, 2011, p. 175). In *Pasargadae* complex [Note 12] and royal garden, also, the geometrical network has been used in garden designing which is according to some scholars the origin of advent of Iranian *Chaharbagh* [Note 13] (Naima, 2006, p. 27). So it can be concluded that geometry was influential from micro scales e.g. decorations to macro scales, urban designing and royal complexes.
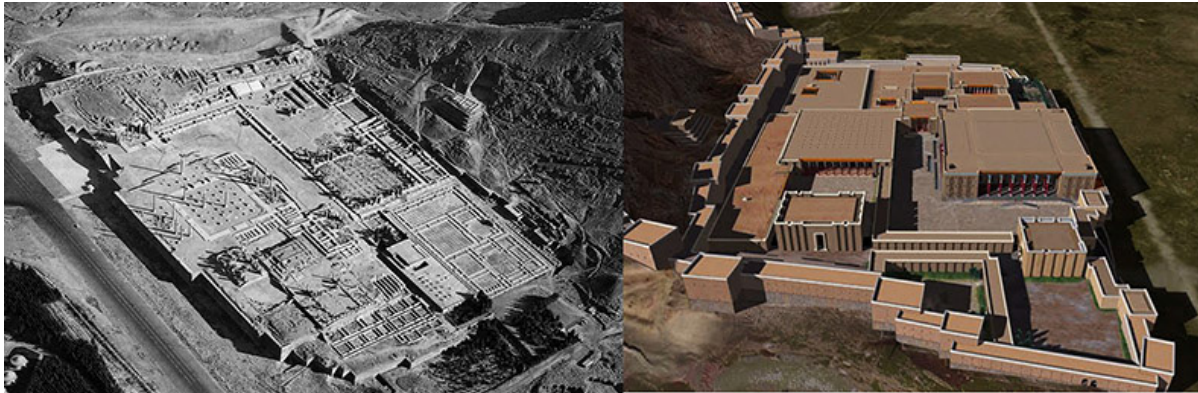
Figure 1. Geometric order and proportion in design *Persepolis* (*Takht-e Jamshid or Pārseh*), *Shiraz, Fars*, ceremonial capital of the *Achaemenid Empire* of *Iran* (ca. 550–330 BC)

4.3 Parthian Empire [Note 14]

Part tribes were fully skilled in using various designs, advanced techniques of arch and dome, through applying local materials (Zarei, 2000, p. 106). This requires applying strong geometry. The architecture has major Greek influences, while some Iranian elements from the beginning are observable. Architects of Parthian Empire started new innovations in the field of art especially in architecture, trying to revive Iranian design symbolic features and Iranian art through combining Iranian and Greek elements (Pope, 1971, p. 47). In this Empire art, motifs [Note 15] are more geometric and structures are more complicated due to influences of Greek art (Kharazmi, Afhami, & Tavoosi, 2012). Creating border and placing motifs inside specific cadres are the main influences Greek art on Iranian art of Parthian Empire (Godard, 1965). For example the architectural decorations in *Nisa* are usually purely Greek. There were Ionic and Corinthian capitals proportion with acanthus leaves. *Nisa*'s battlements and fortifications, however, have their origin in a more Iranian fashion. Moreover through developments in urban fields of Sassanid and Parthian Empire, the tradition of making circular cities in monuments of *Darabgerd, City of Goor* and *Khoreh Ardeshir* [Note 16] cities continued. This indicated applied usage of geometry in large scales (See Figure 2 & 3).

Figure 2. The use of Euclidean geometry in the Parthian Empire urbanism, ancient round city of *Darabgerd*, *Iran* (Kermani, 2012)

4.4 Sasanian Empire [Note 17]

On the rise of Sassanid dynasty, a national government was established, which provided the bed for great artistic changes through taking Achaemenian's art as their model and using experiences obtained from influences of Rome and Greek civilizations on Iran (Ghirshman, 1954). In Sassanid art change in method is from employing geometry from latent state to elaborate geometrical structures, increase in rhythms from frequent repetition to elaborate rhythms with various motifs and utilizing central symmetry, which provided the bed for great changes in architectural changes of next Empire specially changes in geometrical motifs of Islamic Architecture (Kharazmi, Afhami, & Tavoosi, 2012). In this Empire, geometric feature far from former art is eliminated, gradually, and we come across with free motifs. With such organizing, all motifs rely on using geometric infrastructure (Ibid). Dome has been established on square base in dome structures of Sassanid Empire with specific techniques of Iranian architecture, which starts with tightening four conical three-corner on four corners of square to convert it to octagon ($\sqrt{2}$ ratios) and sixteen-gone till at final phase this main tetrahedron transfer converts to be multilateral with corners highly close to circle (Bemanian, Okhovat, & Baghaei, 2011, p. 172). Moreover, central porch of *Kasra* palace follows $\sqrt{3}$ ratios. In a way that mentioned saloon with length and width of 60.34 and 30.25 has the $\sqrt{3}$ ratios (Ibid, p. 176). The unique characteristic of Sassanid architecture, was its distinctive use of space. The Sassanid architect conceived his building in terms of geometric masses and surfaces; hence the use of massive walls of brick decorated with molded or carved stucco. All of these samples from the essence of the hypothesis which are emphasizing the issue of geometry in Iranian Architecture.

Figure 3. The use of *Euclidean* geometry in the Sasanian Empire urbanism, Ancient City of *Goor, Firuzabad, Fars, Iran* (Farnam, 2012)

5. History of Employing Geometry after Islam

Islamic era architect one of whose condition to step into this profession is being familiar with principles of mathematics and geometry, is completely familiar with principles of employing scales in structure and using a scale, he could use ratios of scientific geometry artistically, depending on environmental conditions (Sadeghi & Ahmadi, 2011, p. 107). Doing so he could start unique creativities, through using his interest, genius on one hand, and employing geometrical component one the other hands who are generally considered as support of his work appropriateness.

5.1 Since an arrival of Islam till the Ghaznavid dynasty [Note 18]

In early Islam's era, with gradual development in mathematics, the art of architecture developed too. The reason for reliability of this issue can be investigated with environmental evidence of *Topkapı Scroll* [Note 19] and relating his geometrical language to historical developments of mathematics. Unlike Latin West, Islamic World achieved the book of principles of Greek mathematicians at early 2nd century. At that time, this was had been translated for *Caliph Harun al-Rashid*. (Denny, Necipoglu, & al-Asad, 1998). New writings have been published in different fields of mathematics, astronomy, geometry, optics, algebra, and trigonometry. These publications lead geometry to the world which was unknown earlier, resulting in employing it in new affairs, including architecture and decoration arts (such as dome and arch form, geometrical and Muqarnas motifs (Ibid). Doing so with an increase in Muslim's knowledge and development and progress in geometry, especially in applied and theoretical sections, an appropriate ground was appeared and formed for using theoretical knowledge, mathematics and geometry in applied fields of architecture.

In 4th century, an unprecedented emphasis of *Baghdad* school [Note 20] on balancing theoretical sciences with practical work has had an undeniable effect on visual culture of Islam. One of the most important and the oldest book which was written about close relationship between theoretical and applied sciences, is the book *Ihsa' al-'ulum* written by *Al-Farabi* [Note 21] in 4th century. This book is a pseudo- encyclopedia of theology and sciences, explaining all sciences in a fluent and simple word, mentioning branches of each science (al-Farabi, 2005). Moreover the books *Nejārat* and geometrical acts were written by *Abū al-Wafā' Būzjānī* [Note 22]. *Būzjānī* had a prominent place in applied geometry and architecture. Especially he is of higher position due to his issue of dividing a planet with the aid of grand circles, which requires higher understanding of spatial imagination and its application in architecture (Tavassoli, 2004, p. 25). On the other hand, geometry has stepped into the decorative fields of architectural

monuments in a way that the origin of views limited to geometry in Iranian architecture i.e. 4th century brickwork school is related to North and North East of Iran (Bemanian, Okhovat, & Baghaei, 2011, p. 194). *Renata Hold*, master of art history, Islamic Architecture and town history, points to the book "*Iranian geometry: Application of Geometry in practice* [Note 23]" when investigating applied geometry and Islamic Architecture. He considers it as one kind of works, which can come in handy for most craftsmen (Renata, 1988, pp. 3-4). In this dynasty, valuable monuments, including the *Samanid Mausoleum* [Note 24] and *Gonbad-e Qabus* (tower) [Note 25] were built which according to *Pirnia* have geometry relying on creative ratios and compounds (Pirnia, 2004) (Table. 1). The craftsmanship of the ornamentation and the refinement of the patterns, combined with their rhythm for geometrical contours, produce an excellent technique for fretwork on alabaster. Side by side with this, some ornaments are preserved which date back to the early middle ages: a roller with scaly ornament and strips of intertwined vine stems (See Figure 4).

Figure 4. The rightness of the geometric patterns and the construction of their intersections indicate a knowledge of applied geometry, The *Samanid Mausoleum, Bukhara, Uzbekistan*. (Sergeev, 2012)

In first half of 5th century AH, the most brilliant period of Islamic Architecture began in Iran. In this dynasty, we can see the culmination of an architectural style (*sabk*) called "*Razi* [Note 26]". *Pope* relates the advent of geometrical school to after this century in Grand Iran. Moreover, he, undoubtedly, considers geometrical decoration in Iran as a result of direct employment of mathematical systems (Pope, 1971). In 5th century mathematics and consequently, geometry and its application in architecture developed and found a place that some rulers were proud of being skillful in it. Among these kings and rulers, we can point to *Sultan Mahmud of Ghazni*, who was not only skillful at geometry, but also in architectural drawings. Regarding royal palaces build by *Sultan Mahmud of Ghazni* in *Ġaznīn*, *Abul-Fazl Bayhaqī*[Note 27] sees him as a knowledgeable man of architecture, writing so: all people built and drew lines with their own magnificent, hands based on their knowledge. There are signs in such tools, especially in geometry. (Beyhaqi, 2011). Moreover, *Abdillah Ahmed Ibn Yusuf Ibn Hud*, the ruler of *Saragossa* (*Zaragoza*) in *Andalusia* compiled a textbook of geometry, using a huge set of mathematic books gathered in his library (Denny, Necipoglu, & al-Asad, 1998) (See table2).

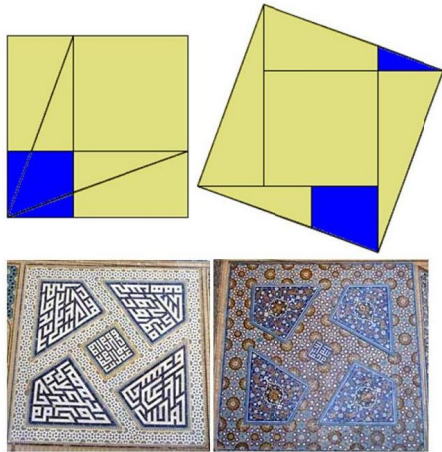Table 1. Scientists and scholars of 4th and 5th centuries

Scientist	essay	Contents			explanation
Abū Naṣr Fārābī	*Ihsa' al-'ulum*	*Geometry*	Practical	It deals with lines that *carpenter deals with wood.*	Classification of knowledge Having practical applications of geometry in any profession
			Theoretical	---	
Ikhwan al-safa	*Rasa'il Ikhwan al-safa*	*Geometry*	Practical	Emphasis on practical geometry impact on the professional skills required.	54 papers about the sciences and their classification (Nasr, 1964)
			Theoretical	Entry qualification's theoretical Science for entrance in the theoretical science of the supernatural Science	
Abū Abdallāh al-Khwārizmī,	*Mafatih al-Ulum (Key to the Sciences)*	Classification of books	Religious and Arabic Sciences	*Religious jurisprudence,* Poetry and Offers, Philosophy, Logic, Medicine, Sciences, Music	Studied Islamic sciences based on classified technical data (al-Khwārizmī, 2011).
			Greek and Egyptian science and		
Abū al-Wafā' Būzjānī	*Amal-al hendese*	Profile design to prove the theorem of Pythagoras			Radial array in the western porch of the mosque Jameh Mosque of Isfahan Tiling
		Compilation of literature outlining the method of the two-dimensional geometric *girih*.			Start by using two-dimensional *girih* (node) and a basis for future progress.
		Introduced three main tool for drawing geometry: ruler, compass and Bracket (Assarzadegan, 2008).			Addressing the needs of craftsmen
		The main direction of Būzjānī geometry books is, chapter of sphere divisions.			Impact on domes on subsequent periods
Ibn al-Nadim	*The Fihrist*	It says of teachings of engineers, mathematicians and other scientists.			This book in 377 AH. BC. Was performed. Book includes extensive information about the books and lines of the Sasanian period. This book is also called *al-Ulum* inventory list.

Table 2. Introducing 5th century scientists and effects of their works on architecture

Scientist	essay	explanation
Muhammad ibn Jarir al-Tabari	*Miftah Al Moamelat*	Useful information about the names and units of measurement of building and surveying (al-Tabari, 1970)
Abū Bakr ibn Muḥammad al-Karajī	*Extraction of hidden waters (Estexra-e abha-ye penhami)*	The first person who proposed the mapping.
		the first Engineer and surveyor in Iran (Al-Karaji, 1345 H (1966-67))
---	*Amal o Ashkal* essay	The only treatise on how to draw geometric patterns nodes
		Vogue by different types of nodes in local government

5.2 From Seljuk [Note 28] *to Ilkhanate* [Note 29] *Dynasty*

On the rise of Seljuk dynasty paying attention to sciences, especially applied sciences boomed drastically. In Seljuk dynasty, the presence of Viziers [Note 30] such as *Amidolmolk Kondori* [Note 31] and *Khawja Nizam Al-Mulk Tusi* [Note 32], and knowledge training thoughts and establishing *Al-Nizamiyya* Schools caused a drastic boom in developing Islamic culture. The evolution and development of *Elm Al-Hiyal* (Science of Contrivances) especially geometry and architecture occurred in this dynasty. In references of 6th century, the place of practical geometry of previous centuries is considered when training craftsmen and architectures. *Beyhaqi* narrates a quote of *Esfazari* [Note 33], which indicates that he considers geometry as basis and foundation of architecture, architect and requires bricklayer to follow it (Neyestani, 2005, p. 45). Similarly, at that time, the main parts of a node were substantiated and developed in different forms in various local rulings, which employed geometry. "Under these conditions, it was one of the tradition revivers of Seljuk dynasty who revived sudden node style…" (Denny, Necipoglu, & al-Asad, 1998). The rulers support of scientists, and artists caused great, architectural and scientific appear in this dynasty through a corporation of these two groups (Table 3) (See Figure 5).

Table 3. Introducing scientist of 6th century and effect of their works on architecture

Scientist	essay	explanation
Abu Hatam Esfarazi	---	In science geometry, paid to the Science of topography and various aspects of it
---	*Yavaghif al-Ulum*	Expression of geodesy (Norouzzadeh Chegini, 1999)
---	*Amal v Ashkal* essay	The only monograph about Geometric Knot

During early 7th AH century, with the establishment of Ilkhanate dynasty y, Mongols adapted the culture and traditions of defeated countries, becoming one of the supporters of art and Islamic artists. One of the main features of this dynasty is more employment of geometry in architectural design. The variety of designs in this method is more than others. Using geometry and variety can be seen in building's tint, and its *Nahāz and Nakhīr* [Note 34] house (Pirnia, 2004, p. 214). In this dynasty, geometry was used as a tool for balancing and generating pattern for construction of monuments with maximum speed and minimum error, in a way that modular enables architect to build several monuments in different parts of a country simultaneously without fearing of error, even in smallest components. In this dynasty, *Khawja Nizam Al-Mulk Tusi* was among updated scientists who made *Maragheh* observatory. (Taheri, 2011, p. 48) *Maragheh* observatory, was an institutionalized astronomical observatory which was established in 1259 CE under the patronage of the *Ilkhanid Hulagu Khan* and the directorship of *Nasīr al-Dīn Tūsī*, an Iranian scientist and astronomer.
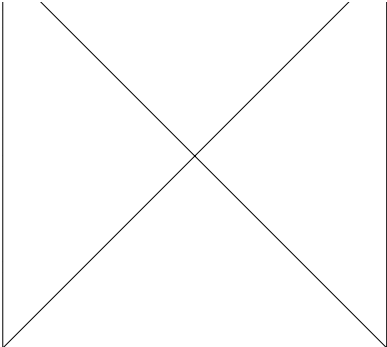
Figure 5. Beautiful brick dome in harmony with geometric proportions, *Taj-ol-molk* Dome, *Jamé Mosque of Isfahan, Isfahan, Iran* (1088-1089) (Alagheband, 2012)

5.3 From Timurid [Note 35] to Safavid [Note 36] Dynasty

Ibn Khaldūn [Note 37] in 8th AH century, in his famous introduction of describing masonry artifacts, points to the importance of recognizing geometry among architects, believing that it is the knowledge of geometrical problems and thinking about it along with relying on accurate calculations that make craftsmen successful in establishing huge monuments (Ibn Khaldûn, 1989). In this era, plans were sent from Shiraz to Yazd for *amir Ghiyāth al-Dīn'*s school construction (Afshar & Jafari, 2005, p. 119). Moreover, *Muhammad ibn Mahmud Amuli* [Note 38], the author of the book "*Nafa'is al-funun fi 'ara'is al-'uyun* [Note 39]" believes that mathematics is composed of techniques involving Architect. 9th century and Timurid dynasty is the most brilliant historical dynasties of Iranian art and architecture, which obtained more progresses using fundamental thoughts and culture of past architecture. One of the main features of this dynasty is more employment of geometry in architectural designs, which are substantiated in *Nahāz and Nakhīr* of plans, brickwork *girih-chīnī* in the form of modular in view decorations and geometrical motifs forming muqarnas and diaphoretic tiling (See Figure 6). *Ghiyāth al-Dīn Jamshīd al-Kāshī* [Note 40] is one of the most prominent scientists and mathematics of this dynasty who has proceeded on different aspects of mathematics and geometry application in architecture (table 4).

Table 4. Introducing scientists of 9th century and their effects on architecture

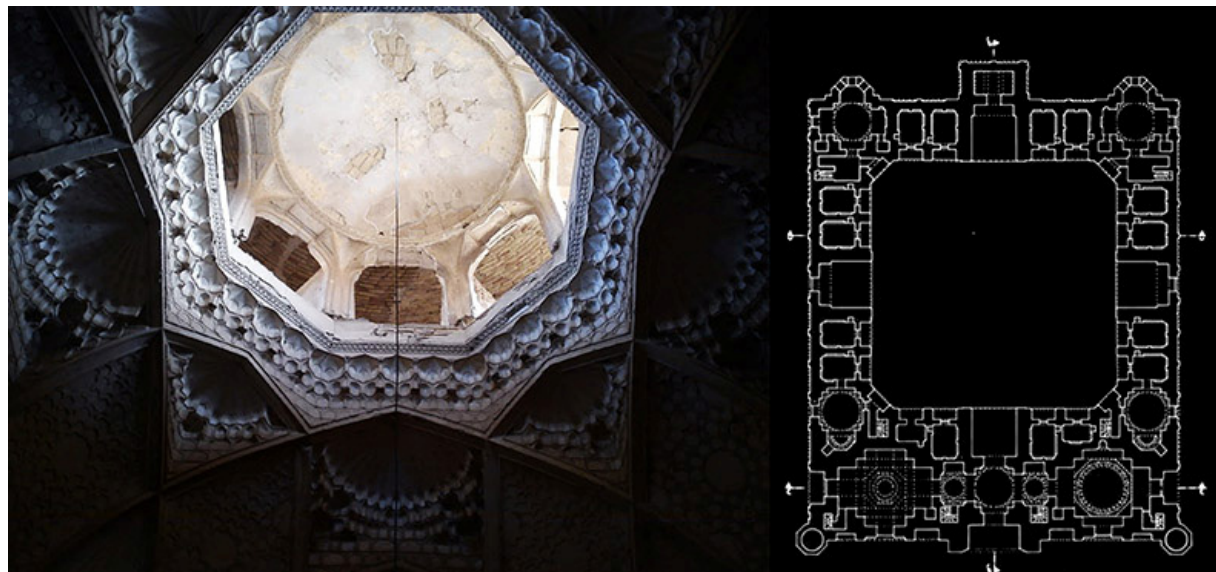| Scientist	essay	Geometry	Architecture
Ghiyāth al-Dīn Jamshīd al-Kāshī	Miftâh al-Hisâb (Key of Arithmetic) (AL-KÂSHÎ, 1977)	The fourth article: Measure the level and volume of geometric shapes	Geometry used in buildings. Addressing the scientific aspects of architecture and building.
		Article 9 of Chapter 4: Architectural components in terms of al-Kāshī; Vault. Dome. Muqarnas	Calculate the area of regular polygons Based on the square modulus. Describe Making Muqarnas. Addressing architects to work on the practical way in a side of theoretical ways. Compile the application of the theoretical foundations of geometry in architecture.
	From Samarkand to Shiraz (Ghiyāth al-Dīn letters to his son)	Letters: Identified valid documents about his attitude to architecture (Bagheri, 1997).	
Ghiyāth al-Dīn Ali Sepahany	Dor al-Masahe	Books for teaching science in architecture	

Figure 6. *Kharghard Madrasa* is accomplished by the energy of the brick floral patterns and brick geometrical schemes; by the emphatic rhythm of the arcades, open galleries and deep recesses; and especially by the contrast of the light and shadow. *Ghavam-al-Din* and *Ghiyas-al-Din Shirazi , Ghiyathiyeh Kharghard School, Khaf, Iran* (Mehrabiani, 2011)

5.4 From Safavid to Qajar [Note 41] *Dynasty*

On the rise of Safavid dynasty in the early 10th century, we can see a culmination of Iranian architecture art, generated by that time exulting and art lover kings, i.e. "*Shah Tahmasp, I* [Note 42] and *Shāh 'Abbās I* [Note 43]**". Scientists like *Bahā' al-dīn al-'Āmilī*[Note 44] was living at that dynasty presenting important writings in architecture, mathematics and geometry, taking valuable action in Isfahan in the field of architecture. Moreover, during these years handbooks of *Meftah ol Saadah* and *Meftah ol-Siyadah* were published by an unknown author in the field of mathematics and architecture (table5). Following *Eric Schroeder*, *Oleg Grabar* in his book "*The Great Mosque of Isfahan*", believes that *Khayyām* [Note 45] played a role in drawings resulted in creation of the north dome of Grand Mosque of Isfahan. He sees him as its inventor (Graber, 1990). Since using Pentagon and five headed stars on the outer surface of the dome requires scientific domination and proficiency on round triangles to be able to convert direct to deviant lines, so this confirms the probability of idea design by Khayyām. Moreover, according to article entitled: "*Jamé Mosque of Isfahan*" it was identified in Aerial Photography of historical monument's summit (Athena1974) that north dome ratio of this mosque is exactly equal to *Khayyām*'s triangle ratios (Ghazali, 1975). *Chorbachi* [Note 46] in his article of Bible Tower considers the asymmetries in Islamic designs of photos of *Khayyām*'s handbook of indiscrete shapes, more complicated than *Abū al-Wafā*'s polygons and shapes version, knowing *Khayyām* as the main criteria of decorative geometry and developments in that dynasty applied geometry (Chorbachi, 1989). Reports of the handbook of indiscrete shapes indicate that triangle explored by *Khayyām* was used, in fact, as a decorative design by craftsmen (Özdural, 2000). This indicates the fact that principles and rules of geometry are not only used in architecture, but also geometrical shapes are used to prove scientific problems. They can be, moreover, used as decorative geometrical shapes due to having specific logic (See Figure 7).
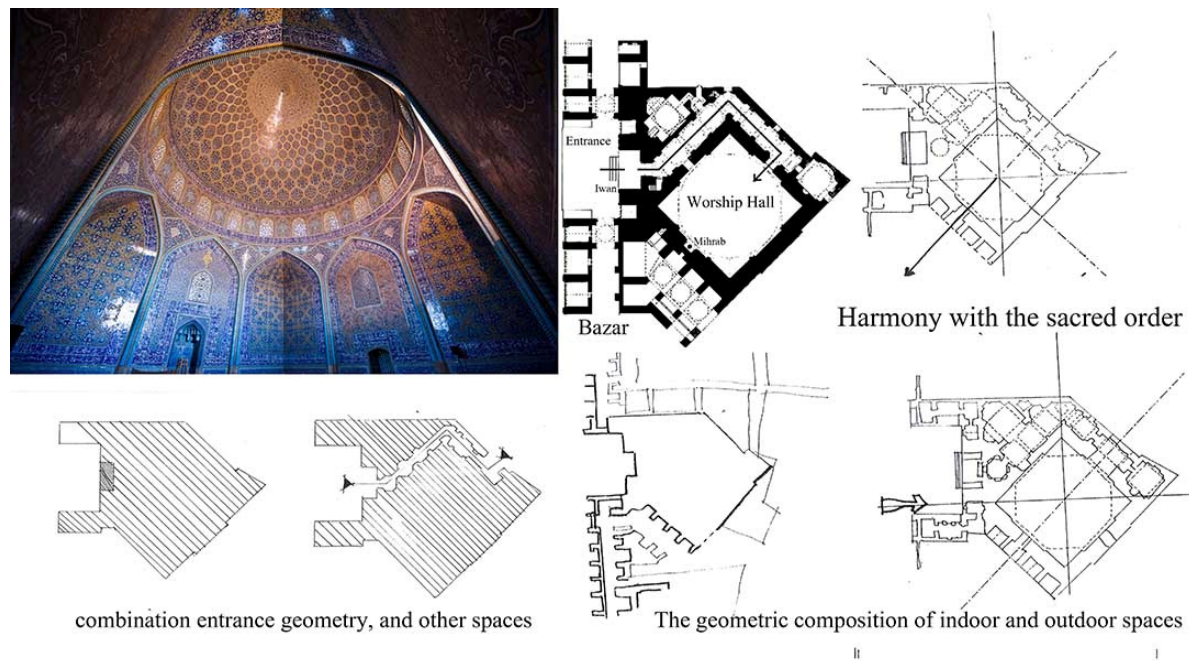

Figure 7. Prolific and impressing by the geometry of the pattern and plan, Geometric Analysis of *Sheikh Lotfollah* Mosque, *Ustad Mohammad Reza Isfahani* architect, *Isfahan*, *Iran*

Table 5. Introducing 11th century scientists and effect of their works on architecture

Scientist	essay	explanation			Architectural result
Shaykh‑i Bahā᾽ī	Architectural General Knowledge	Chapter one			The relationship between total and component architectures.
		Chapter two			Describe the psychological aspects of space.
	Mathematics of Architecture	Khulasat al-hisab	Chapter one: Subterranean parts		designing triangle tools with Plummet of architect
			Chapter two: Discover how to obtain high buildings altitude		Recognizing height of symbols and signs in cities.
			Chapter three: Discover the the width of rivers and depth of the wells.		Design and build bridges, Dam…
		Meftah Al-Falah	Books on prayer and Ritual Acts night and day		Methods for determining Prayer times.
		Fi- Tahghigh Jahat Al-Ghebleh (in Arabic)			---
		Resaleye Ostorlab			---
		footprint Sheikh Bahai in Safavid architecture			River water sharing in Esfehan
---	Meftah ol Saadah and Meftah ol-Siyadah	Mathematics	Aspects of Geometry		knowledge building scenarios and situations
					Science Building and strengthening and beautification. Geometry employed in the construction of buildings, houses and castles.

Table 6. Introducing 12th century scientists and effect of their works on architecture

Scientist	essays	Architectural result	samples	
Umar al-Khayyām	Characterization of irrational numbers in math	Using the properties of irrational numbers in the design of the dome	North Dome	Jameh Mosque of Isfahan
	Treatise on Algebra (1074)	Employment Khayyām on architecture and related arts of Domes.	Five apical and Pentagon on the South Dome (Ghazali, 1975)	
	Khayyām Triangle (Pascal's Triangle)	Equal proportions of the dome with Pascal's Triangle (Chorbachi, 1989)	North Dome	
		Applications of Pascal's Triangle as a decorative array	---	
---	---	Designed by using of sacred geometry (Pirnia, 2004)		Khan School of Shiraz

5.5 From Qajar Dynasty to Contemporary Era

Studying educational mathematics of *Dar ul-Fonun* can be carried out in two forms:

5.5.1 Safavid dynasty and investigating handbook of *Khulasat al-hisab* (based on a manuscript, compiled in 1114 AH) (al-ʿĀmilī) compiled by *Bahāʾ al-dīn al-ʿĀmilī*, surnamed *Shaykh‑i Bahāʾī*.

The reason for importance of this handbook is that it is the only handbook used as a textbook for the introductory mathematics course to train students of Safavid dynasty in the school house. In Qajar dynasty this book was translated, corrected and reviewed by several famous mathematicians, and then it was available for students with the name of *Kanz Al-Hisab* [Note 47]. Mathematic books authored in *Dar ul-Fonun* [Note 48] presented issues, which had not been seen in Islamic era and according to western and theoretical methods, these books caused Iran to develop in this field. In this era, mathematics goes far from issues of practical geometry, tending toward pure and proving issues (Pasandideh, 2007). In this era, some conditions that generated the ideas leading to the Dar al-Funun, the pivotal Iranian educational institution in the nineteenth century, as well as the people and the cultural and political environment that nurtured it.

Qajar decoration is usually unmistakable. Simple, rather strident tiled geometric or epigraphic designs in small glazed bricks were especially popular. Reflecting glass now complemented polychrome tilework, adding play of light to play of color (See Figure 8). The facetted surface of muqarnas vaults was the ideal vehicle for this late but still novel expression of a classic preoccupation of Iranian architecture. The dissolution of surface by resplendent ornament.

Figure 8. The designers *Muhammad Hasan-e-Memar* and *Muhammad Reza Kashi Paz-e-Shirazi* used extensively stained glass on the façade and other traditional elements such as *panj kāseh-i* (five concaves), The *Nasīr al-Mulk* Mosque in *Shiraz, Iran*.

5.5.2 Books Authored in Qajar dynasty

Mathematics text books before establishing *Dar ul-Fonun* involved the book entitled *Kanz Al-Hisab* (manuscript 1256 AH) which a translation and the description for Handbook of *Khulasat al-hisab*, written by *Shaykh‑i Bahāʾī*, compiled by *Farhad Mirza* [Note 49] Qajar, reviewed by *Ali Mohammad Isfahani*. The author mentions that the book was compiled to teach students and learn rudimentary issues of mathematics during *Mohammad Shah Qajar* kingship. *Vajed Ali Khan*, in his *Matla-i ul-Ulum* which was compiled in 13th AH century, has dedicated one chapter to techniques of an architect, mentioning that: those who are aware of principles of geometry, they'd better know that this technique can be finished in the best way possible (Ali Khan, 1998). The Dar al-Funun as a channel

for the flow of scientific, cultural and intellectual innovation imported from Western Europe to Qajar society and as an experimental space in which long-established ideas and practices were challenged. It also sheds new light on the secularization of education in Iran, which led to the Ulama's loss of control over educational affairs.

6. Conclusion

Comprehensive studies of relationship between architecture, mathematics and geometry indicated that there has been a close relation between these two fields. With changes occurred in mathematics, we can observe developments in architecture. This issue confirms theories of scientists, including *Özdural* and *Necipoglu* who believed in relationship between mathematics and architecture. Although for different reasons, this relation can be pale, it was never vanished and no one can deny it. During some dynasties, unprecedented progresses in mathematics, compiling books and mathematical achievements related to architecture strengthened this relation due to reasons such as rulers' support of mathematicians. Though, in some eras, this relation became weaker and pale due to social and political reasons and even people's different attitude toward time sciences and kind of science employment.

Evidence remained from pre-Islam era indicates that some traces of geometry can be seen in designing cities, palaces and royal gardens. The relationship between Islamic era mathematicians and architecture field was substantiated through first Abbasid caliphs for urban construction and development. Meanwhile due to need for mathematics and lack of knowledge of craftsmen and architects, a bed was provided to entrance of mathematicians to architecture and eminent mathematicians, and scientists took the most important seats of construction affairs. The 4th century is considered as a turning point of relationship between mathematics and architecture area due to *Abū al-Wafā' Būzjānī*'s works in geometry and practical arithmetic. In this century emphasis of *Baghdad school* on balancing theoretical sciences and practical works had drastic effects on architectural works. On the rise of Seljuk government, in 6th century, paying attention to applied sciences boomed more and more. The presence of experienced and resourceful Viziers, including *Khawja Nizam Al-Mulk Tusi* caused geometry influence architecture in this era. On the rise of Ilkhanate ruling, 7th century, using geometry in building affairs increased due to need for acceleration and accuracy in designs. Concurrent with Timurid ruling, 9th century, is the peak of art history and architecture boom, since in this era the more employment of geometry and mathematics can be seen architectural designs. Scientists including *al-Kāshī* it proceeded on theoretical application of mathematics in architecture in form of practical and simple solutions through writings about application of geometry in building and authoring theoretical expression. On the rise of Safavid dynasty, the peak of geometry application in decoration of monuments can be seen. Actions, including using *Khayyām*'s triangle proportions in *Nizam Al-Mulk* dome of Isfahan's Grand Mosque [Note 50] and using it in order to decorate this monument are samples of such actions. In dynasties after Safavid and especially in Qajar dynasty mathematics receded from practical geometry issues due to stepping Western and theoretical methods, tending toward positivist and pure issues. Doing so, the connection chain of mathematics and architecture, which was close in different historical dynasties, disconnected gradually.
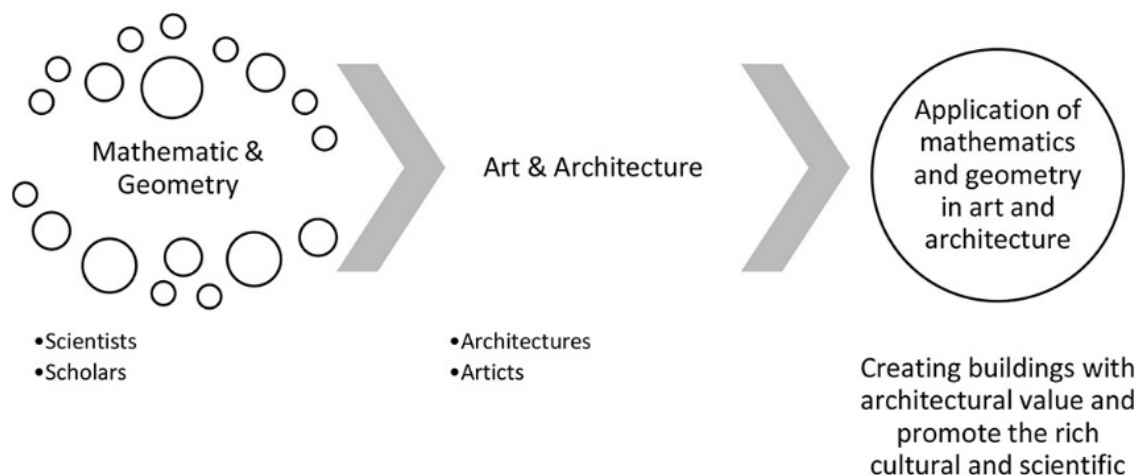

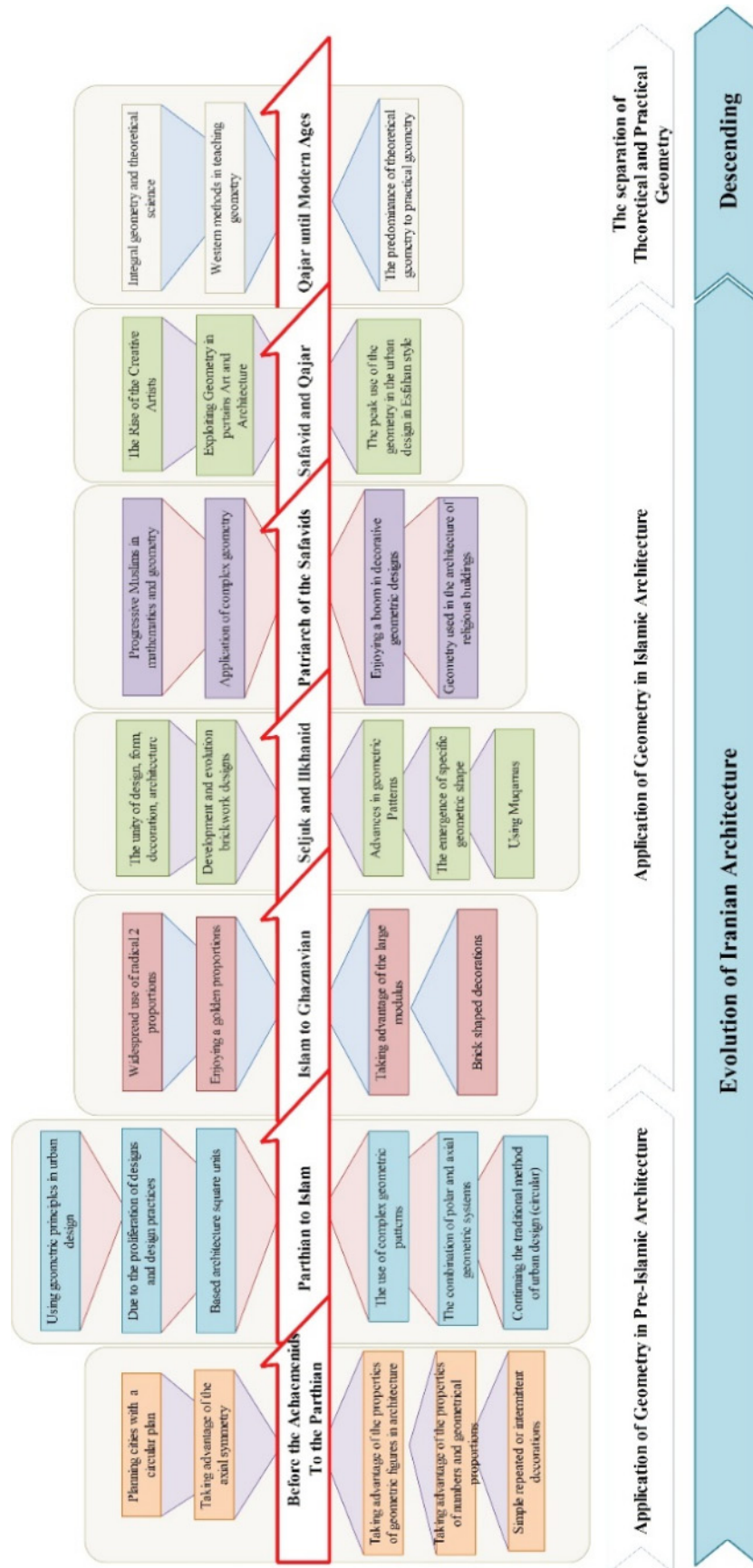Figure 9. The relationship of mathematics and geometry with art and architecture

Figure 10. Diagram of the conclusion

According to various studies in the field, it can be concluded that during history development and promotion of mathematics and geometry had direct relationship with art and architecture, in a way that with development in theoretical sciences, mathematics, geometry and architecture developed. Another consequential finding was that many mathematicians such as Khayyām themselves have directly supervised the construction of consequential government buildings in their era. Consequently, and cadence in these sciences caused recession and remission in architecture. Since the peak boom of mathematics, geometry from 4th century to 9th century is concurrent with peak of architecture in Iran (Figure 9, 10). According to the findings of the theoretical and practical aspects of research, this paper can be the basis for future studies on the roots of Persian architecture, a strong relationship between Persian architecture progression and evolution of mathematic geometry has been reported in this literature. However Future studies on the current topic are therefore recommended. More research on this topic needs to be undertaken to an association between Architecture, and mathematic geometry is more clearly understood.

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Notes

Note 1. Ibrāhīm al-Khayyām Nīshāpūrī (18 May 1048 – 4 December 1131; Persian, was a Persian polymath, philosopher, mathematician, astronomer and poet. He also wrote treatises on mechanics, geography, mineralogy, music, and Islamic theology.

Note 2. Būzjānī was one of the leading astronomers and mathematicians of the Middle Ages, with significant contributions in observational astronomy? His achievements in trigonometry paved the way for more precise astronomical calculations (2 (Moussa, 2011, p. 188).

Note 3. Muḥammad ibn Mūsā al-Khwārizmī Earlier transliterated as Algoritmi or Algaurizin, (c. 780 – c. 850) was a Persian mathematician, astronomer and geographer during the Abbasid Empire, a scholar in the House of Wisdom in Baghdad.

Note 4. Abū ʿAlī al-Ḥasan ibn al-Ḥasan ibn al-Haytham, scientist, polymath, mathematician, astronomer and philosopher who made significant contributions to the principles of optics, astronomy, mathematics, meteorology, visual perception and the scientific method (Selin, 1997).

Note 5. Ghiyāth al-Dīn Jamshīd al-Kāshī was one of the best Persian astronomers and mathematicians in the Islamic world. He was born in 1380, in Kashan, in central Iran (Azarian, 1998).

Note 6. The Achaemenid Empire was first Persian Empire, an empire based in Western Asia, founded in the 6th century BC by Cyrus the Great.

Note 7. Herodotus was an ancient Greek historian who was lived in the fifth-century BC (c. 484–425 BC). Widely referred to as "The Father of History" (first conferred by Cicero), he was the first historian known to collect his materials systematically and critically, and then to arrange them into a historiography narrative.

Note 8. According to Herodotus, Ecbatana was chosen as the Medes capital in the late 8th century B.C.E. by Deioces.

Note 9. Dr Abdolazim-Amir Shah-Karami the advisor to the Office of Conservation, Revitalisation and Registration of Historical Buildings of the Iranian Cultural Heritage and Tourism Organisation (ICHTI).

Note 10. Persepolis (New Persian: Takht-e Jamshid or Pārseh) literal meaning "city of Persians," was the ceremonial capital of the Achaemenid Empire (ca. 550–330 BC) (Michael Woods, 2008).

Note 11. An apadana is a large hypostyle hall, the best-known examples being the great audience hall and portico at Persepolis and the palace of Susa. The Persepolis Apadana belongs to the oldest building phase of the city of Persepolis, the first half of the 5th century BC, as part of the original design by Darius the Great. Its construction completed by Xerxes I (Cool Root, 1985).

Note 12. Pasargadae, the capital of Cyrus the Great (559–530 BC) and also his last resting place, was a city in ancient Persia (modern-day Iran), located near the city of Shiraz (in Pasargad County) and is today an archaeological site and one of Iran's UNESCO World Heritage Sites.

Note 13. Charbagh or Chahar Bagh is a Persian-style garden layout. The quadrilateral garden is divided by walkways or flowing water into four smaller parts. In Persian, "Chār" means 'four' and "bāgh" means 'garden' (Ruggles, 2008).

Note 14. Also known as the Arsacid Empire was a major Iranian political and cultural power in ancient Iran.

Note 15. Motif is an element of a pattern, image or part of one, or theme. A motif may be repeated in a design or composition, often many times, or may just occur once in a work.

Note 16. Sasanian palace at Buzpar (say Boshpar), close to the achemenid tomb aka Gur Dokhtar.

Note 17. Sasanid Empire was the last Iranian empire before the rise of Islam, ruled by the Sasanian dynasty from 224 CE to 651 CE.

Note 18. The Ghaznavid dynasty (Persian: غزنویان) was a Turko-Persian Muslim dynasty of Turkic slave origin at their greatest extent ruling large parts of Persia, and much of Transoxania, and the northern parts of the Indian subcontinent from 977 to 1186.

Note 19. The Topkapı Scroll (Turkish: Topkapı Parşömeni) is a Timurid dynasty pattern scroll in the collection of the Topkapı Palace museum. The scroll is a valuable source of information, consisting of 114 patterns that may have been used both indirectly and directly by architects to create the tiling patterns in many mosques around the world, including the quasicrystal *Girih* tilings from Darb-e Imam (Necipoglu, 1992).

Note 20. The Baghdad School was a relatively short-lived yet influential school of Islamic art developed during the late 12th century in the capital Baghdad of the ruling Abbasid Caliphate.

Note 21. Abū Naṣr Muḥammad ibn Muḥammad Fārābī was a renowned scientist and philosopher of the Islamic Golden Age. He was also a cosmologist, logician, and musician. Through his commentaries and treatises, Al-Farabi became well-known among medieval Muslim intellectuals as "The Second Teacher," that is, the successor to Aristotle, "The First Teacher."

Note 22. He was a Persian mathematician and astronomer who worked in Baghdad. He made important innovations in spherical trigonometry, and his work on arithmetics for businessmen contains the first instance of using negative numbers in a medieval Islamic text.

Note 23. In Arabic: "فيما يحتاج اليه العمال والصناع من الاشكال الهندسيه"

Note 24. Samanid amir, Ismail Samani's tomb, one of the most esteemed sights of Central Asian architecture, was built in the 9th (10th) century (between 892 and 943).

Note 25. This tall monument for the Prince Shams ul-Ma'ali, Amir Qabus ibn Wushmgir ordered to build during his life, in the year 397 the lunar Hegira, and the year 375 the solar Hegira.

Note 26. A style of architecture dating from the 11th century to the Mongol invasion period, which includes the methods and devices of The Samanids, Ghaznavids, and Seljukids (Fallahfar, 1999, p. 106).

Note 27. He was a Persian historian and author. He wrote the famous work of Persian literature Tarikh-e Mas'oudi ("Masoudian History," also known as "Tārīkh-e Bayhaqī").

Note 28. Was a Turkish Sunni Muslim dynasty. (10th century)

Note 29. Was a breakaway state of the Mongol Empire, which was ruled by the Mongol House of Hulagu. (1256–1335)

Note 30. The Abbasid Caliphs give the name title wazir to a minister formerly called Katib (secretary) who was at first merely a helper, afterwards became the representative and successor of the Dapir (official scribe or secretary) of the Sassanian kings (Nicholson, 1998, p. 257).

Note 31. The minister of Toghrol and Alp Arsalān Sāmānid Dynasty.

Note 32. Abu Ali Hasan ibn Ali Tusi (1018 - 1092), better known by his honorific title of Nizam al-Mulk (Persian: نظام‌الملک "Order of the Realm") was a Persian scholar and vizier of the Seljuk Empire.

Note 33. Muzaffer Esfazari, the author in 12th century.

Note 34. Protrusion and Niche of the plan

Note 35. This dynasty was founded by Timur (Tamerlane) in the 14th century. The Timurids lost control of most of Persia to the Safavid dynasty in 1501, but members of the dynasty continued to rule parts of Central Asia and parts of India.

Note 36. Safavid was one of the most significant ruling dynasties of Persia. They ruled one of the greatest Persian empires after the Muslim conquest of Persia (1501–1736).

Note 37. Ibn Khaldūn (1332 AD–1406 AD) was an Arab Muslim historiographer and historian, regarded to be among the founding fathers of the modern sociology, and historiography and economics.

Note 38. He was a medieval Persian physician from Amol, Iran.

Note 39. Between 1335 and 1342 Amoli also composed a large and widely read Persian encyclopedia on the classification of knowledge titled (Nafa'is al-funun fi 'ara'is al-'uyun)

Note 40. Ghiyāth al-Dīn Jamshīd Masʿūd al-Kāshī was a Persian astronomer and mathematician.

Note 41. The Qajar dynasty was a Persianized native Iranian royal family of Turkic origin, which ruled Persia (Iran) from 1785 to 1925.

Note 42. Tahmasp I (1514 – 1576) was an influential Shah of Iran, who enjoyed the longest reign of any member of the Safavid dynasty

Note 43. Shāh 'Abbās the Great (1571 –1629) was the 5th Safavid Shah Emperor of Iran, and generally considered the greatest ruler of the Safavid dynasty.

Note 44. Shaykh‐i Bahāʾī (1547 - 1 1621) was a scholar, philosopher, architect, mathematician, astronomer and a poet in 16th-century Iran.

Note 45. Umar al-Khayyām Nīshāpūrī (1048 –1131) was a Persian polymath, philosopher, mathematician, astronomer and poet. He also wrote treatises on mechanics, geography, mineralogy, music, and Islamic theology.

Note 46. Wasma'a Khalid Chorbachi is an American artist.

Note 47. Written by Farhad Mirza bin Waliahad.

Note 48. Established in 1851, was the first university and modern institution of higher learning in Persia.

Note 49. Farhād mīrzā Moʿtamad-al-dawla (1818-1888), Qajar prince-governor and bibliophile.

Note 50. Jamé Mosque of Isfahan

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