American Journal of Computational Mathematics, **2011, 1, 256-263**
doi:10.4236/ajcm.2011.14031 Published Online December 2011 (http://www.SciRP.org/journal/ajcm)

Context-Dependent Data Envelopment Analysis with Interval Data

Mohammad Izadikhah
Department of Mathematics, Islamic Azad University, Arak, Iran
E-mail: *m_izadikhah@yahoo.com*, *m-izadikhah@iau-arak.ac.ir*
Received August 11, 2011; *revised September* 20, 2011; *accepted September* 28, 2011

Abstract

Data envelopment analysis (DEA) is a non-parametric method for evaluating the relative efficiency of decision making units (DMUs) on the basis of multiple inputs and outputs. The context-dependent DEA is introduced to measure the relative attractiveness of a particular DMU when compared to others. In real-world situation, because of incomplete or non-obtainable information, the data (Input and Output) are often not so deterministic, therefore they usually are imprecise data such as interval data, hence the DEA models becomes a nonlinear programming problem and is called imprecise DEA (IDEA). In this paper the context-dependent DEA models for DMUs with interval data is extended. First, we consider each DMU (which has interval data) as two DMUs (which have exact data) and then, by solving some DEA models, we can find intervals for attractiveness degree of those DMUs. Finally, some numerical experiment is used to illustrate the proposed approach at the end of paper.

Keywords: DEA, Context-Dependent, Interval Data, Interval Attractiveness, Interval Progress

1. Introduction

Data envelopment analysis (DEA), developed by Charnes *et al*. [1], usually evaluates decision making units (DMUs) from the angle of the best possible relative efficiency. If a DMU is evaluated to have the best possible relative efficiency of unity, then it is said to be DEA efficient; otherwise it is said to be DEA inefficient. Performance of inefficient DMUs depends on the efficient DMUs, that is, the inefficiency scores change only if the efficiency frontier is altered.

Although the performance of efficient DMUs is not influenced by the presence of inefficient DMUs, it is often influenced by the context. The context-dependent DEA [2-4] is introduced to measure the relative attractiveness of a particular DMU when compared to others. We know that the DMUs in the reference set can be used as benchmark targets for inefficient DMUs. The context-dependent DEA provides several benchmark targets by setting evaluation context [3]. The context-dependent DEA is introduced to measure the relative attractiveness of a particular DMU when compared to others. Relative attractiveness depends on the evaluation context constructed from alternative DMUs. The original DEA method evaluates each DMU against a set of efficient DMUs and cannot identify which efficient DMU is a better option with respect to the inefficient DMU. This is because all efficient DMUs have an efficiency score of one. The standard DEA models assume that all data are known exactly without any variation. However, this assumption may not be true. In the real world, some outputs and inputs may be only known as in forms of interval data, ordinal data and ratio interval data. If we incorporate such imprecise data information in to the standard linear CCR model, the resulting DEA model is a nonlinear and non-convex program, and is called imprecise DEA (IDEA), (Cooper *et al.* [5] and Kim *et al*. [6]). The approach in IDEA is to transform the non-linear and non-convex model to a linear programming equivalent, by imposing scale transformation on the data variable alterations (products of variables are replaced by new variables). Prior to IDEA, pertinent work was that of Cook *et al*. [7,8], which, is confined only to mixture of exact and ordinal data. They started by dealing with only one ordinal input [8] and then extended their model [7] to handle multiple cardinal and ordinal criteria. The basic idea in these models is to assign new auxiliary variables, one for every combination of ordinal variables and distinct ranks of it. The value for such an auxiliary variable corresponding to DMU_j and a rank position is 1, if

DMU_j is rated in the *k-th* place of ordinal variable and 0 otherwise. Extensions of these basic ideas are reported in [9,10]. Recently, Despotits and Smirlis [11] calculated upper and lower bounds for the efficiency scores of the DMUs with imprecise data. They developed an alternation approach for dealing with imprecise data. They transformed the non-linear DEA model to a linear programming equivalent by using a straightforward formulation, completely different than that in IDEA. Contrarily to IDEA their transformation on the variables are made on the basis of the original data set, without applying any scale transformations on data. Also, in Jahanshahloo *et al*. [12,13] the radius of stability for the DMUs with interval data is calculated. In this paper we concentrate on the context-dependent DEA with interval data. For this reason we consider each DMU_j, with interval data, as two DMUs that have exact data. Then by a procedure similar to that one in [3], the evaluation contexts are obtained by partitioning these DMUs into several levels of efficient frontier. Then by introducing some models based upon these efficient frontiers we can measure the relative attractiveness and progress of these DMUs and also we can determine the interval attractiveness and interval progress for each original DMU with interval data. Further by combination of these measures we can also characterize the performance of DMUs.

The rest of the paper is organized as follows: next section introduces the basic definitions of interval data and notations of the original context-dependent DEA. In Section 3, interval context-dependent DEA is presented. In Section 4 we illustrate our proposed DEA method with two numerical examples and some discussion. Finally, some conclusions are pointed out in the end of this paper.

2. Preliminaries

Now suppose we have n DMUs which utilize m inputs $x_{ij}, (i=1,\cdots,m)$ to produce s outputs $y_{rj}, (r=1,\cdots,s), (j=1,\cdots,n)$. Also, assume that input and output levels of each DMU are not known exactly. We define $X_j = (x_{1j},\cdots,x_{mj})$ and $Y_j = (y_{1j},\cdots,y_{sj})$, $j=1,\cdots,n$.

2.1. Interval Data

Let input and output values of any DMU be located in a certain interval, where x_{ij}^L and x_{ij}^U are the lower and upper bounds of the *i-th* input of the DMU_j, respectively, and y_{rj}^L, y_{rj}^U are the lower and upper bounds of the *r-th* output of the DMU_j, respectively, that is to say, $x_{ij}^L \le x_{ij} \le x_{ij}^U$ and $y_{rj}^L \le y_{rj} \le y_{rj}^U$. Such data are called interval data, because they are located in intervals. Note that always $x_{ij}^L \le x_{ij}^U$ and $y_{rj}^L \le y_{rj}^U$. If $x_{ij}^L = x_{ij}^U$, then the *i-th* input of the DMU_j has a definite value.

Interval problems are those whose parameter values are located in intervals, their exact values being unable to be identified.

Therefore we consider problems with data such as $x_{ij} \in \left[x_{ij}^L, x_{ij}^U\right]$ and $y_{rj} \in \left[y_{rj}^L, y_{rj}^U\right]$, where lower and upper bounds are known exactly, positive and finite.

2.2. Context-Dependent Data Envelopment Analysis with Exact Data

The original context-dependent DEA model is developed by using the following radial efficiency measure. Let I^1 be the set of all DMUs, and I^k and E^k interactively defined as $I^{k+1} = I^k - E^k$, where E^k consists of all the radially efficient DMUs by following linear programming:

$$
\begin{aligned}
\Phi_o^*(k) &= \max \Phi_o(k) \\
s.t.\quad & \sum_{j\in F(I^k)} \lambda_j X_j \le X_0 \\
& \sum_{j\in F(I^k)} \lambda_j Y_j \ge \Phi_o(k) Y_0 \\
& \lambda_j \ge 0, \qquad j \in F\left(E^k\right)
\end{aligned}
\tag{1}
$$

where $j \in F\left(I^k\right)$ means $\mathrm{DMU}_j \in I^k$. When k = 1, model (1) becomes the original output oriented CCR model and E^1 define the first-level efficient frontier. When k = 2, model (1) gives the second-level efficient frontier after the exclusion of the first-level efficient DMUs. And so on. In this manner we identify several levels of efficient frontiers. We call E^k the *k-th* level efficient frontier.

Assume that, $\mathrm{DMU}_o \in E^{k_0}$, $k_0 \in 1,\cdots,n$. We can calculate relative attractiveness measures for DMU_o with respect to *k-th* level efficient frontier. Based upon the evaluation context E^k, relative attractiveness measure of $\mathrm{DMU}_o \in E^{k_0}$ can obtained by the following context-dependent DEA:

$$
\begin{aligned}
H_o^*(k) &= \max H_o(k) \qquad k = 1,\dots,L - k_0 \\
s.t.\quad & \sum_{j\in F(E^{k+k_0})} \lambda_j X_j \le X_0 \\
& \sum_{j\in F(E^{k+k_0})} \lambda_j Y_j \ge H_o(k) Y_0 \\
& \lambda_j \ge 0, \qquad j \in F\left(E^{k+k_0}\right)
\end{aligned}
\tag{2}
$$

Then $A_o^*(k) \equiv 1/H_o^*(k)$ is called the (output oriented) attractiveness of DMU_o from a specific level E^k.

Model (2) is same as [3] with slightly change. In model (2) we set $k = 0,\cdots,L-k_0$ in order to considering E^{k_0} as evaluation context for $\mathrm{DMU}_o \in E^{k_0}$.

Note: *In this paper, only, attractiveness measure is used. By the same manner one can use progress measure* *for* DMUs *with interval data.*

3. Interval Context-Dependent DEA

Now, assume that there are n DMUs which produce s outputs y_{rj} by using m inputs x_{ij} such that $y_{rj} \in \left[y_{rj}^L, y_{rj}^U \right]$ and $x_{ij} \in \left[x_{ij}^L, x_{ij}^U \right]$. By considering each DMU_j as two DMUs with exact data, namely, $\left(X_j^L, Y_j^U \right)$ and $\left(X_j^U, Y_j^L \right)$ we will have $2n$ DMUs. Now, to identify L efficiency levels, we apply the procedure introduced in [3] for these $2n$ DMUs.

Let $J^1 = \left\{ \left(X_j^L, Y_j^U \right),\ j = 1, \cdots, n \right\}$ and $I^1 = \left\{ \left(X_j^U, Y_j^L \right),\ j = 1, \cdots, n \right\}$ be the sets of all these $2n$ DMUs. Now, we consider two following models:

$$
\begin{aligned}
&\bar{\phi}_o^*(k) = \max \bar{\phi}_o(k) \\
&\quad s.t. \\
&\quad \sum_{j \in F(J^k)} \lambda_j X_j^L + \sum_{j \in F(I^k)} \bar{\lambda}_j X_j^U \le X_o^L, \\
&\quad \sum_{j \in F(J^k)} \lambda_j Y_j^U + \sum_{j \in F(I^k)} \bar{\lambda}_j Y_j^L \ge \bar{\phi}_o(k) Y_o^U, \\
&\quad \lambda_j \ge 0, \qquad j \in F(J^k) \\
&\quad \bar{\lambda}_j \ge 0, \qquad j \in F(I^k)
\end{aligned} \tag{3}
$$

and

$$
\begin{aligned}
&\underline{\phi}_o^*(k) = \max \underline{\phi}_o(k) \\
&\quad s.t. \\
&\quad \sum_{j \in F(J^k)} \lambda_j X_j^L + \sum_{j \in F(I^k)} \bar{\lambda}_j X_j^U \le X_o^U, \\
&\quad \sum_{j \in F(J^k)} \lambda_j Y_j^U + \sum_{j \in F(I^k)} \bar{\lambda}_j Y_j^L \ge \underline{\phi}_o(k) Y_o^L, \\
&\quad \lambda_j \ge 0, \qquad j \in F(J^k) \\
&\quad \bar{\lambda}_j \ge 0, \qquad j \in F(I^k)
\end{aligned} \tag{4}
$$

Models (3) and (4) are similar to model (1) because these models simultaneously identify several levels of efficient frontiers as follows:

In the above models $j \in F\left(J^k\right)$ means $\left(X_j^L, Y_j^U \right) \in J^k$ and $j \in F\left(I^k\right)$ means $\left(X_j^U, Y_j^L \right) \in I^k$. Then we define $J^{k+1} = J^k - E_1^k$ and $I^{k+1} = I^k - E_2^k$, where $E_1^k = \left\{ \left(X_j^L, Y_j^U \right) \in J^k ; \bar{\phi}_o^*(k) = 1 \right\}$ and $E_2^k = \left\{ \left(X_j^U, Y_j^L \right) \in I^k ; \underline{\phi}_o^*(k) = 1 \right\}$, then to identify k th-level efficient DMUs, set $E^k = E_1^k \cup E_2^k$.

When $k = 1$, then first-level efficient frontier is defined by DMUs in E^1, that is, $E_1^1 \cup E_2^1$. When $k = 2$, models (3) and (4) give the second-level efficient frontier after the exclusion the first-level efficient DMUs. In this manner we can identify several levels of efficient frontiers, where $E_1^k \cup E_2^k$ consist the k th-level efficient frontier. By following steps we can identify these efficient frontiers using models (3) and (4):

***Step* 1.** Set $k = 1$, evaluate the DMUs belong to $J^1 \cup I^1$ using models (3) and (4) to obtain the first-level efficient DMUs, $E_1^k \cup E_2^k$.

***Step* 2.** Set $J^{k+1} = J^k - E_1^k$, $I^{k+1} = I^k - E_2^k$. (If $J^{k+1} = \varnothing$ and $I^{k+1} = \varnothing$ then stop).

***Step* 3.** Evaluate the new subset "inefficient" DMUs, J^{k+1}, using models(3) and (4) to obtain new sets of efficient DMUs E_1^{k+1} and E_2^{k+1}. Set $k = k + 1$ and go to step 2.

In the first section, we said that, the DMUs in the reference set can be used as benchmark targets for inefficient DMU. The context-dependent DEA provides several benchmark targets by setting evaluation context. DMUs can reach (if possible) to the nearest efficiency level as the first target to improve their efficiency.

Figure 1 plots the five levels of efficient frontiers of 5 DMUs with single interval input and single interval output (see **Table 1**). Assume that L levels of efficient frontiers are identified by the above algorithm. It can be seen that in this example $L = 5$.

Now based upon these evaluation contexts E^k, $\left(k = 1, \cdots, L \right)$, the context dependent DEA measures the relative attractiveness of each DMU (DMUs with interval data) as follows:

Suppose for DMU_o, by the above algorithm, we obtain $\left(X_o^U, Y_o^L \right) \in E^{l_2}$ and $\left(X_o^L, Y_o^U \right) \in E^{l_1}$, clearly $l_1 \le l_2$.

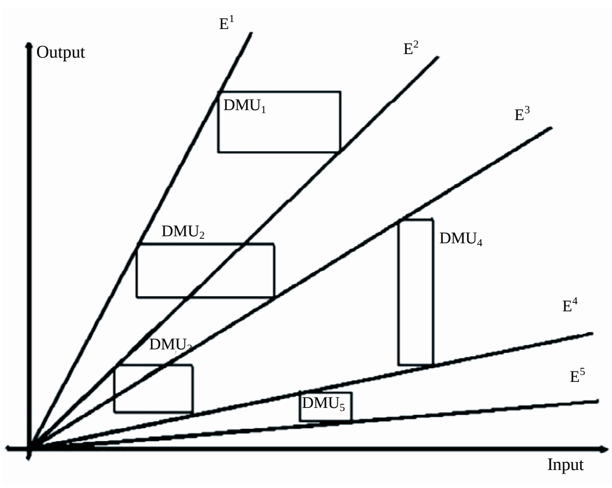

Figure 1. Levels of efficient frontiers.

Table 1. Data of numerical example.

DMU_j	Inputs x_j^L	x_j^U	Outputs y_j^L	y_j^U
1	3.5	6	6	7
2	2.1	5	3	4.2
3	1.5	3	0.6	1.5
4	7.5	8	1.6	4.5
5	5	6	0.5	1

Now, we introduce two following context dependent DEA models to obtain the relative attractiveness measure.

$$\begin{aligned}
&\overline{H}_o^*(k) = \max \overline{H}_o(k),\ k = 0,\cdots,L-l_2\\
&s.t.\\
&\sum_{j\in F\left(E_1^{k+l_2}\right)} \lambda_j X_j^L + \sum_{j\in F\left(E_2^{k+l_2}\right)} \overline{\lambda}_j X_j^U \le X_o^L,\\
&\sum_{j\in F\left(E_1^{k+l_2}\right)} \lambda_j Y_j^U + \sum_{j\in F\left(E_2^{k+l_2}\right)} \overline{\lambda}_j Y_j^L \ge \overline{H}_o(k) Y_o^U,\\
&\lambda_j \ge 0, \qquad j\in F\left(E_1^{k+l_2}\right)\\
&\overline{\lambda}_j \ge 0, \qquad j\in F\left(E_2^{k+l_2}\right)
\end{aligned} \tag{5}$$

and

$$\begin{aligned}
&\underline{H}_o^*(k) = \max \underline{H}_o(k),k = 0,\cdots,L-l_2\\
&s.t.\\
&\sum_{j\in F\left(E_1^{k+l_2}\right)} \lambda_j X_j^L + \sum_{j\in F\left(E_2^{k+l_2}\right)} \overline{\lambda}_j X_j^U \le X_o^U,\\
&\sum_{j\in F\left(E_1^{k+l_2}\right)} \lambda_j Y_j^U + \sum_{j\in F\left(E_2^{k+l_2}\right)} \overline{\lambda}_j Y_j^L \ge \underline{H}_o(k) Y_o^L,\\
&\lambda_j \ge 0, \qquad j\in F\left(E_1^{k+l_2}\right)\\
&\overline{\lambda}_j \ge 0, \qquad j\in F\left(E_2^{k+l_2}\right)
\end{aligned} \tag{6}$$

Clearly, $\overline{H}_o^*(k) \le 1$ and $\underline{H}_o^*(k) \le 1$ for $k = 0,\cdots,L-l_2$, and also, $\overline{H}_o^*(k+1) < \overline{H}_o^*(k)$ and $\underline{H}_o^*(k+1) < \underline{H}_o^*(k)$.

Theorem **1.** For DMU_o we have $\overline{H}_o^*(k) \le \underline{H}_o^*(k)$, $k = 0,\cdots,L-l_2$.

Proof. First assume that $\left(\overline{H}_o^*(k),\lambda^*,\overline{\lambda}^*\right)$ be optimal solution for model (5). Thus we have

$$\sum_{j\in F\left(E_1^{k+l_2}\right)} \lambda_j Y_j^U + \sum_{j\in F\left(E_2^{k+l_2}\right)} \overline{\lambda}_j Y_j^L \ge \overline{H}_o(k) Y_o^U \ge \overline{H}_o(k) Y_o^L$$

therefore $\left(\overline{H}_o^*(k),\lambda^*,\overline{\lambda}^*\right)$ is a feasible solution for model (6). Since model (6) is maximization and has an optimal value as $\underline{H}_o^*(k)$, then $\overline{H}_o^*(k) \le \underline{H}_o^*(k)$.

Corollary **1.** *If* $\overline{DMU} = \left(\overline{X},\overline{Y}\right)$ *has exact data, where* $X_o^L \le \overline{X} \le X_o^U$ *and* $Y_o^L \le \overline{Y} \le Y_o^U$, *then we must have* $\overline{H}^*(k) \in \left[\overline{H}_o^*(k),\underline{H}_o^*(k)\right], k = 0,\cdots,L-l_2$, *where* $\overline{H}^*(k)$ *is the relative attractiveness measure for* $\overline{DMU}$.

Assume that DMU_o has interval data, that is, $X_o \in \left[X_o^L,X_o^U\right]$ and $Y_o \in \left[Y_o^L,Y_o^U\right]$. By Corollary 1, $H^*(k) \in \left[\overline{H}_o^*(k),\underline{H}_o^*(k)\right], k = 0,\cdots,L-l_2$.

Definition **1.** If we call *k*-degree attractiveness of DMU_o (which is lie on the specific level $E^{l_2} = E_1^{l_2} \cup E_2^{l_2}$) by $A_o^*(k)$, then we have $A_o^*(k) \in \left[\frac{1}{\underline{H}_o^*(k)},\frac{1}{\overline{H}_o^*(k)}\right]$. That is, attractiveness score of DMU_o with respect to efficiency level E^k is $A_o^*(k)$.

Definition **2.** We define $\left[\frac{1}{\underline{H}_o^*(k)},\frac{1}{\overline{H}_o^*(k)}\right]$ as k-degree attractiveness of DMU_o.

Since the parameter values are located in intervals and their exact values being unable to be identified, hence value of $A_o^*(k)$ is unknown. Also, one can rank the DMUs in each level based upon their attractiveness scores.

4. Application

In order to illustrate the use of the methodology for determining the interval attractiveness developed here, first, in example 1 we use the data in **Table 1** and in example 2 we use an empirical data.

4.1. Example 1: Personal Selection Data

Assume that, we have 5 DMUs in one input and one output and these data are interval as shown in Table 1. By considering each DMU_j as two DMUs with exact data, namely, $\left(X_j^L,Y_j^U\right)$ and $\left(X_j^U,Y_j^L\right)$ we will have 10 DMUs. Now, to identify the efficient levels of these DMUs, we apply models (3) and (4).

Results are shown below:

- $E^1 = \left\{\left(X_1^L,Y_1^U\right),\left(X_2^L,Y_2^U\right)\right\}$
- $E^2 = \left\{\left(X_1^U,Y_1^L\right),\left(X_3^L,Y_3^U\right)\right\}$
- $E^3 = \left\{\left(X_2^U,Y_2^L\right),\left(X_4^L,Y_4^U\right)\right\}$
- $E^4 = \left\{\left(X_3^U,Y_3^L\right),\left(X_4^U,Y_4^L\right),\left(X_5^L,Y_5^U\right)\right\}$
- $E^5 = \left\{\left(X_5^U,Y_5^L\right)\right\}$

We see that for these DMUs, 5 efficient levels are identified, *i.e.*, $L = 5$. By models (5) and (6), interval attractiveness of each DMU can be calculated. In Table 2 interval attractiveness of each original DMUs (which have interval data) are shown in column 4. For example, in **Table 2**, first-degree interval attractiveness of DMU_1 with respect to efficiency level E^3 is [1.67, 3.33].

That is, if we choose exact value for input and output of DMU_1 and then calculate the attractiveness degree of DMU_1 with respect to efficiency level E^3, namely $A_1^*(3)$, we must have $1.67 \le A_1^*(3) \le 3.33$.

Discussion

In the above mentioned example, suppose that, we fix the data of these DMUs in their intervals, in the other words, assume that we have 5 DMUs, namely,

$\overline{\mathrm{DMU}}_j = \left(\overline{X}_j, \overline{Y}_j\right),\ j = 1, \cdots, 5,$ which have exact data such that $\overline{X}_j \in \left[X_j^L, X_j^U\right]$, $\overline{Y}_j \in \left[Y_j^L, Y_j^U\right]$. These data are as **Table 3**.

We will show that, the attractiveness score of these DMUs lies in the interval attractiveness presented in **Table 2**. Hence, we employ models (3) and (4) for these DMUs. Let $\overline{E}^k$ be the k th-level of efficient frontier. See **Figure 2** and compare $\overline{E}^k$ and E^k.

We define $S\left(E^k\right)$ as follows:

$$S\left(E^k\right) = \left\{(x,y) \middle| x \ge \sum_{j=1}^{n} \lambda_j X_j, y \le \sum_{j=1}^{n} \lambda_j Y_j, \lambda_j \ge 0, \left(X_j, Y_j\right) \in E^k \right\}$$

For example, **Figure 3** illustrates region of $S\left(\overline{E}^k\right)$. It can be seen that,

$$S\left(\overline{E}^5\right) = S\left(E^5\right) \subseteq S\left(E^4\right) \subseteq S\left(\overline{E}^4\right) \subseteq S\left(E^3\right)$$
$$= S\left(\overline{E}^3\right) \subseteq S\left(E^2\right) \subseteq S\left(\overline{E}^2\right) \subseteq S\left(\overline{E}^1\right) \subseteq S\left(E^1\right)$$

Assume that, we want to calculate the attractiveness degree of $\overline{\mathrm{DMU}}_1$ with respect to efficiency level $\overline{E}^4$, that is, we want to calculate $A_1^*(4)$. Since $\overline{\mathrm{DMU}}_1 \in \overline{E}^2$ and $S\left(E^3\right) \supseteq S\left(\overline{E}^4\right) \supseteq S\left(E^4\right)$, hence the attractiveness

Table 2. Interval attractiveness.

DMU_j	k-degree	Efficiency Levels	k-degree interval attractiveness
1	Zero-degree	E^2	[1,2]
	First-degree	E^3	[1.67,3.33]
	Second-degree	E^4	[5,10]
	Third-degree	E^5	[12,24]
2	Zero-degree	E^3	[1,3.33]
	First-degree	E^4	[3,10]
	Second-degree	E^5	[7.2,24]
3	Zero-degree	E^4	[1,5]
	First-degree	E^5	[2.4,12]
4	Zero-degree	E^4	[1,3]
	First-degree	E^5	[2.4,7.2]
5	Zero-degree	E^5	[1,2.4]

Table 3. DMUs with exact data.

DMU_j	Input	Output
1	5	6
2	3	4
3	2	1
4	7.5	4.5
5	6	0.5

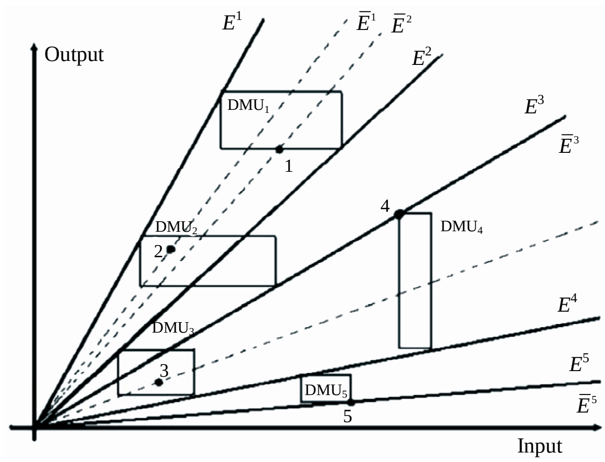

Figure 2. Levels of efficient froutiers for DMUs with exact data and interval data.

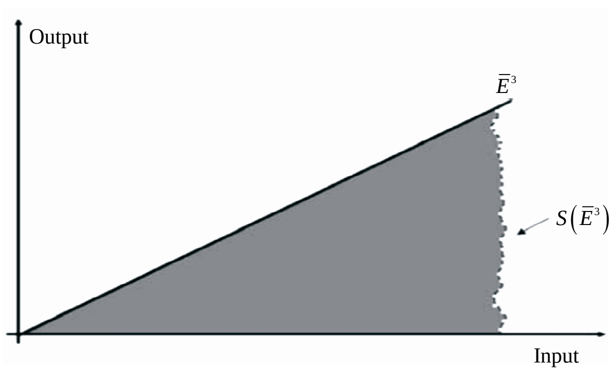

Figure 3. Region of $S\left(\overline{E}^3\right)$.

score of $\overline{\mathrm{DMU}}_1$ lies in the following interval:

$$\frac{1}{\underline{H}_1^*(4)} \le A_1^*(4) \le \frac{1}{\overline{H}_1^*(3)}$$

where $A_1^*(4)$ is the attractiveness degree of $\overline{\mathrm{DMU}}_1$ with respect to efficiency level $\overline{E}^4$, and $1/\left(\underline{H}_1^*(4)\right)$ and $1/\overline{H}_1^*(3)$ are taken from **Table 2**.

Therefore, without direct calculation of attractiveness scores of these DMUs (which have exact data) we must have

	$A_2^*(2)\in[1,3.33]$		
$A_1^*(3)\in[1.67,3.33]$	$A_2^*(3)\in[1,3.33]$		
$A_1^*(4)\in[1.67,10]$	$A_2^*(4)\in[1,10]$		$A_4^*(4)\in[1,3]$
$A_1^*(5)\in[12,24]$	$A_2^*(5)\in[7.2,24]$	$A_3^*(5)\in[2.4,12]$	$A_4^*(5)\in[2.4,7.2]$

However, if we calculate attractiveness of this set of DMUs we have the **Table 4** which shows that the above statement is true.

4.2. Example 2: Empirical Data

Consider a performance measurement problem of manufacturing industry, in which there are eight manufacturing industries from different cities (DMUs) participating in the evaluation, each consuming two inputs (Labor and working funds) and producing three outputs (Gross industrial output value, profit and taxes, and retail sales). The data are all estimated and are thus imprecise and only known within the prescribed bounds, which are listed in **Table 5**.

By using the DEA models (3) and (4) and by same manner as shown in example 1, we obtain the following levels of efficient frontiers:

- $E^1=\left\{\left(X_1^L,Y_1^U\right),\left(X_2^L,Y_2^U\right),\left(X_7^L,Y_7^U\right),\left(X_8^L,Y_8^U\right)\right\}$
- $E^2=\left\{\left(X_4^L,Y_4^U\right),\left(X_5^L,Y_5^U\right)\right\}$
- $E^3=\left\{\left(X_1^L,Y_1^U\right),\left(X_3^L,Y_3^U\right),\left(X_6^L,Y_6^U\right),\left(X_8^L,Y_8^U\right)\right\}$
- $E^4=\left\{\left(X_2^U,Y_2^L\right),\left(X_3^U,Y_3^L\right),\left(X_5^U,Y_5^L\right),\left(X_7^U,Y_7^L\right)\right\}$
- $E^5=\left\{\left(X_4^U,Y_4^L\right)\right\}$
- $E^6=\left\{\left(X_6^U,Y_6^L\right)\right\}$

Therefore, we see that for these DMUs, 6 efficient levels are identified, that is $L=6$.

If we apply models (5) and (6) for these DMUs, we obtain interval attractiveness of each DMU as **Table 6**.

For instance, consider DMU_1 which has interval data. By **Table 6**, one can see that interval attractiveness of DMU_1 with respect to efficiency level E^4 is [1.18, 1.5]. Nevertheless, if one can find the exact value of DMU_1 and again calculate the attractiveness score of DMU_1 with respect to E^4, it must lie in interval [1.18, 1.5]. For more details see discussion presented in example 1.

Note**:** All the computations in this example are carried out by a computer program using ***GAMS*** software.

5. Conclusions

We developed in this paper an approach for dealing with interval data in context dependent DEA. It is done by considering each DMU (which have interval data) as two DMUs (which have exact data) and then we obtain interval attractiveness for each DMU. For this reason, we introduced some DEA models for evaluating these $2n$ DMUs, and in the next step to obtain the interval attractiveness we merge the results of these models. Also we show that, if we choose n arbitrary DMUs with exact data, then the attractiveness of these DMUs are belong to that intervals. After this manager decided that, what combination of each interval is appropriate.

Although the proposed method presented in this paper is illustrated by a personal selection data, however, it can also be applied to many problems of decision manage-

Table 4. Exact attractiveness for exact data of Table 3.

DMU_j	Levels	Attractiveness
1	$\overline{E}^3$	2
	$\overline{E}^4$	2.4
	$\overline{E}^5$	15
2	$\overline{E}^2$	1.11
	$\overline{E}^3$	2.22
	$\overline{E}^4$	2.67
	$\overline{E}^5$	16.67
3	$\overline{E}^3$	6.25
4	$\overline{E}^4$	1.2
	$\overline{E}^5$	7.5

Table 5. Data for eight DMUs.

DMU	Input		Output		
	Labor	Working Funds	GIO	Profit and Taxes	Retail Sales
1	[66, 73]	[1354, 1540]	[3200, 3800]	[1100, 1200]	[1000,1150]
2	[54, 70]	[1205, 1425]	[3000, 3350]	[1000, 1150]	[800,930]
3	[65, 80]	[950, 985]	[2800, 3000]	[800, 900]	[650, 700]
4	[55, 63]	[850, 1000]	[2500, 2750]	[800, 850]	[600, 850]
5	[72, 85]	[1105, 1200]	[3050, 3700]	[950, 1150]	[1000, 1050]
6	[63, 80]	[1250, 1380]	[2700, 2900]	[800, 950]	[700, 750]
7	[57, 72]	[950, 1150]	[2950, 3250]	[950, 1200]	[800, 900]
8	[60, 71]	[800, 970]	[2700, 2800]	[800, 950]	[900, 1000]

Table 6. Interval attractiveness of Example 2.

DMU_j	k-degree	Efficiency Levels	k-degree interval attractiveness
1	Zero-degree	E^3	[1, 1.31]
	First-degree	E^4	[1.18, 1.5]
	Second-degree	E^5	[1.43, 1.83]
	Third-degree	E^6	[1.56, 2]
2	Zero-degree	E^4	[1, 1.51]
	First-degree	E^5	[1.2, 1.81]
	Second-degree	E^6	[1.42, 2.13]
3	Zero-degree	E^4	[1, 1.2]
	First-degree	E^5	[1.14, 1.26]
	Second-degree	E^6	[1.45, 1.63]
4	Zero-degree	E^5	[1, 1.67]
	First-degree	E^6	[1.39, 1.96]
5	Zero-degree	E^4	[1, 1.3]
	First-degree	E^5	[1.39, 1.58]
	Second-degree	E^6	[1.64, 1.86]
6	Zero-degree	E^6	[1, 1.51]
7	Zero-degree	E^4	[1, 1.58]
	First-degree	E^5	[1.17, 1.66]
	Second-degree	E^6	[1.42, 2.18]
8	Zero-degree	E^3	[1, 1.4]
	First-degree	E^4	[1.11, 1.5]
	Second-degree	E^5	[1.55, 2.08]
	Third-degree	E^6	[1.83, 2.5]

ment. By the same manner one can use the proposed procedure to calculate interval progress for each DMU.

6. References

[1] A. Charnes, W. W. Cooper and E. Rhodes, “Measuring the Efficiency of Decision Making Units,” *European Journal of Operational Research*, Vol. 2, No. 6, 1978, pp. 429-444. doi:10.1016/0377-2217(78)90138-8

[2] H. Morita, K. Hirokawa and J. Zhu, “A Slack-Based Measure of Efficiency in Context Dependent Data Envelopment Analysis,” *Omega*, Vol. 33, No. 4, 2005, pp. 357-362. doi:10.1016/j.omega.2004.06.001

[3] L. M. Seiford and J. Zhu, “Context-Dependent Data Envelopment Analysis: Measuring Attractiveness and Progress,” *Omega*, Vol. 31, No. 5, 2003, pp. 397-408. doi:10.1016/S0305-0483(03)00080-X

[4] J. Zhu, “Quantitative Models for Performance Evaluation and Benchmarking-Data Envelopment Analysis with Spreadsheets and DEA Excel Solver,” Kluwer, Boston, 2002.

[5] W. W. Cooper, K. S. Park and G. Yu, “IDEA and AR-IDEA: Models for Dealing with Imprecise Data in DEA,” *Management Science*, Vol. 45, No. 4, 1999, pp. 597-607. doi:10.1287/mnsc.45.4.597

[6] S. H. Kim, C. G. Park and K. S. Park, “An Application of Data Envelopment Analysis in Telephone Offices Evaluation with Partial Data,” *Computers and Operations Research*, Vol. 26, No. 1, 1999, pp. 59-72. doi:10.1016/S0305-0548(98)00041-0

[7] W. D. Cook, M. Kress and L. Seiford, “Data Envelopment Analysis in the Presence of Both Quantitative and Qualitative Factors,” *Journal of the Operational Research Society*, Vol. 47, No. 7, 1996, pp. 945-953.

[8] W. D. Cook, M. Kress and L. Seiford, “On the Use of Ordinal Data in Data Envelopment Analysis,” *Journal of the Operational Research Society*, Vol. 44, No. 2, 1993, pp. 133-140.

[9] W. D. Cook, J. Doyle, R. Green and M. Kress, “Multiple Criteria Modeling and Ordinal Data: Evaluation in Terms of Subset of Criteria,” *European Journal of Operational Research*, Vol. 98, No. 3, 1997, pp. 602-609. doi:10.1016/S0377-2217(96)00069-0

[10] J. Sarkis and S. Talluri, “A Decision Model for Evaluation of Flexible Manufacturing Systems in the Presence of Both Cardinal and Ordinal Factors,” *International Journal of Production Research*, Vol. 37, No. 13, 1999, pp. 2927-2938. doi:10.1080/002075499190356

[11] D. K. Despotis and Y. G. Smirlis, “Data Envelopment Analysis with Imprecise Data,” *European Journal of Operational Research*, Vol. 140, No. 1, 2002, pp. 24-36. doi:10.1016/S0377-2217(01)00200-4

[12] G. R. Jahanshahloo, F. Hosseinzadeh Lotfi and M. Moradi, “Sensitivity and Stability Analysis in DEA with interval Data,” *Applied Mathematics and Computation*, Vol. 156, No. 2, 2004, pp. 463-477. doi:10.1016/j.amc.2003.08.005

[13] M. Izadikhah and F. Hosseinzadeh Lotfi, “Sensitivity Analysis of Units with Interval Data in DEA,” *International Mathematical Forum*, No. 2, Vol. 42, 2007, pp. 2083-2097.

