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Research Article

Finite-Time Synchronization of Fractional-Order Complex-Valued Cohen-Grossberg Neural Networks with Mixed Time Delays and State-Dependent Switching

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This paper discussed the finite-time synchronization of fractional-order complex-valued Cohen-Grossberg neural networks (FCVCGNNs), which contain mixed time delays and state-dependent switching that make the model more comprehensive. Different from other methods, we use a method of nonseparating real and imaginary parts to get our conclusions. By applying fractional-order inequalities and the Lyapunov function, effective controllers with suitable conditions are derived. Additionally, the maximum time for the drive-response system to reach synchronization is also given. Finally, numerical examples are designed to illustrate the effectiveness of our obtained theoretical results.

1. Introduction

Inspired by the dynamic instability behaviors such as traveling waves, standing waves, resonance, and chaos, which exist in the short-term storage of visual or language patterns by neural networks, Cohen and Grossberg proposed a more general system that exhibits the absolute stability property [1]. Furthermore, some classical neural network models can be regarded as its special cases, such as cellar neural networks, bidirectional associative memory neural networks, and even the famous Hopfield neural networks. Recently, many in-depth studies have been carried out on Cohen-Grossberg neural networks (CGNNs), such as periodic solutions [2, 3] and stability [4, 5]. As a kind of dynamic characteristics, the synchronization of CGNNs has received extensive attention and deep research in recent years [6–8], especially, the finite-time synchronization has focused more attention due to its practical value [9–15]. For example, Kong et al. [9–12] designed different switching adaptive controllers to achieve the finite-time and fixed-time synchronization for fuzzy CGNNs with different characteristics.

Moreover, considering that the synapse values in the organism can be changed, corresponding to the mathematical model of the neural network, the connection weights should also be variable so that the network can be more in line with the actual biological characteristics. Hence, it is useful to establish a parameter switching system [16–18]. As for the CGNNs, there are few investigations on the CGNNs with state-dependent switching parameters.

As a branch of mathematical analysis, fractional calculus will originate from the contribution of Leibniz and L' Hospital regarding 300 years past [19]. Fractional order can be regarded as an expansion of integer order which can be employed more general and more precise in nonlinear systems. Some results have been achieved in image encryption [20], economics [21], and neural networks [22]. Due to its nonlocality and long-term memory characteristics, it can describe complex dynamics more accurately and simulate biological neural networks more realistically. Therefore, more and more dynamic behavior research on fractional neural networks such as stability [23, 24], synchronization [25, 26], and Hopf bifurcation [27, 28] has been studied in recent years.

It is noted that the studies mentioned above are limited to the real-valued neural networks, which have some inherent limitations. For example, the detection of symmetry problem and XOR problem can be solved by a single complex-valued neuron with the orthogonal decision boundaries. However, a single real-valued neuron fails to do so [29]. Thus, the dynamic behavior of fractional-order complex-valued neural networks is one of the most important and energetic research topics. Due to the complexity of the connection values and functions, it has very different and more complicated dynamical behaviors than real-valued ones. In recent years, the dynamical analysis of complex-valued neural networks has attracted much attention [7, 30–38].

For example, Rajchakit and Sriraman [35] investigated the robust passivity and stability of uncertain complex-valued impulsive neural network models through the Lyapunov function and LMI approach. Chanthorn et al. [36] employed a delay-dividing approach to study the robust stability of the uncertain stochastic complex-valued neural network with time delay. Moreover, Sriraman et al. [37] focused on the mean-square asymptotic stability of the discrete-time stochastic quaternion-valued neural networks. Humphries et al. [38] studied the global asymptotic stability problem for the fractional-order quaternion-valued bidirectional associative memory neural network models.

Time delay, which is always inevitable in neural networks, could cause system shock or even instability [39, 40]. And discrete-time delay, which always exists due to the limited transmission speed, has attracted more attention in neural networks research. In [41], the new improved fixed-time stability lemmas are proposed to attain the fixed-time synchronization of a class of discontinuous fuzzy inertial neural networks with time-varying delays. Both time-varying delays and linear fractional uncertainties are taken into consideration in a complex-valued neural network, and novel dissipativity criteria are designed in [42]. Kong and Zhu [43] studied the periodicity and finite time synchronization for a class of discontinuous inertial neural networks with time-varying delays. Besides, mixed time delays are more considered in the current research [44–46].

Taking the above factors into account, some relevant documents as following are about complex-valued neural networks: Pan and Zhang [30] designed two kinds of different exponential controllers to assure the finite-time synchronization for the delayed complex-valued neural networks. Zhang et al. [47] employed two different controllers to attain the finite-time synchronization of fractional-order CGNN with time delay and applied the nonseparation method to get the conclusion. The complex-valued neural networks are investigated by using the nonseparation method in [48–51], but their network model weights are all fixed, and they do not take into account the changes in weights. In [52], the more novel fixed-time stability principles are established to investigate the synchronization of a class of delayed discontinuous fuzzy CGNNs. And generally, most neural network models do not consider the effect of mixed time delays. To better illustrate the contributions of our work, we use Table 1 for comparison with other articles on NNs,

Table 1: Comparison with other papers.

Literature	MTDs	FO	CGNNs	PS	NSM	FTS
[10]	×	√	×	√	√	√
[30]	×	×	×	×	×	√
[54]	×	×	×	√	×	×
[55–57]	×	√	×	√	×	√
[58]	×	√	×	√	×	×
[48–50]	×	√	×	×	√	×
[47]	×	√	√	×	√	√
[33]	×	√	×	×	×	×
[51]	×	√	×	×	√	√
This paper	√	√	√	√	√	√

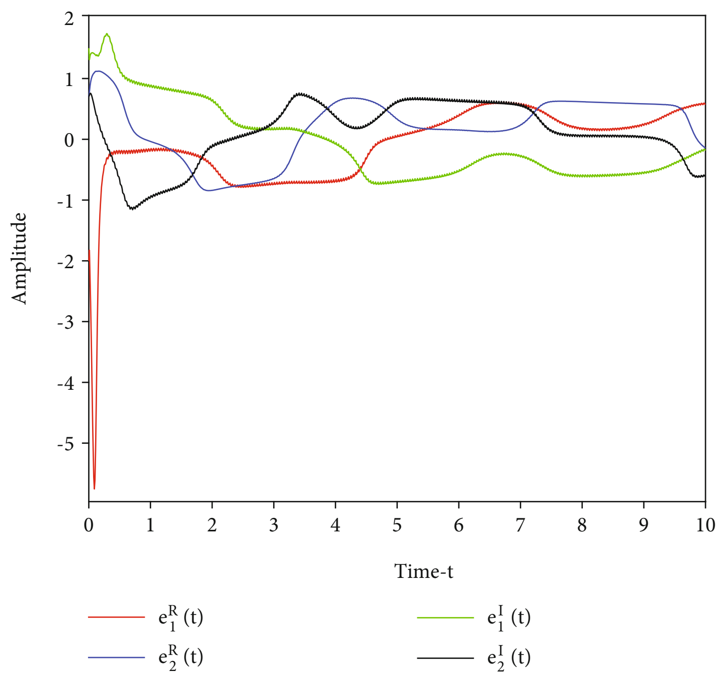
where mixed time delays, fractional order, Cohen-Grossberg Neural Networks, parameter switching, nonseparation method, and finite-time synchronization are abbreviated as MTDs, FO, CGNNs, PS, NSM, and FTS, respectively. √ means this item is included in that paper, and × means it is not.

The above analysis noted that compared with other neural networks, there are few investigations about the synchronization of CGNNs, and the models used in these studies are often CGNNs that do not contain mixed time delays. For example, [12, 14, 15] discussed the fixed-time synchronization of Cohen-Grossberg neural networks, but there are no mixed time delays in the model. As for the fractional-order complex-valued CGNNs, there are even fewer studies. Hence, it is necessary to focus on the finite-time synchronization of a more general CGNN. This is our first motivation.

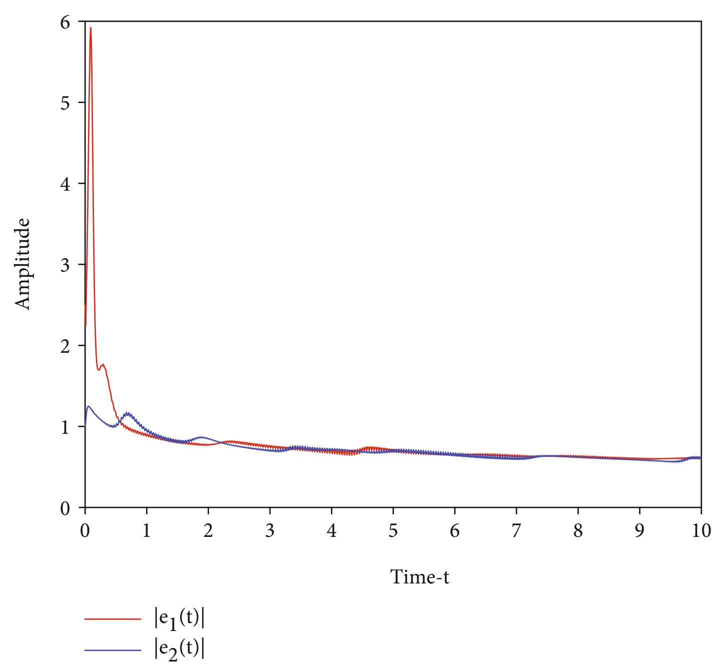
On the other hand, considering the complex-valued neural networks, the most common method is separate the complex-valued into two real parts, which has limitations in some situations. Thus, using a nonseparation method to ensure the finite-time synchronization of our model is the second motivation.

However, the finite-time synchronization of CGNN, which contains mixed time delays, fractional order, complex-valued, and switching parameters, has not been investigated. Inspired by the above discussion, this paper focuses on the finite-time synchronization of fractional-order complex-valued Cohen-Grossberg neural networks with mixed time delays and state-dependent switching. Based on the Lyapunov function and inequality theory, sufficient synchronization theorems are derived by a nonseparation method. Compared with some recent studies such as [11, 31, 33, 47, 53], we consider a more general model and use the nonseparation method, which is less restrictive in solving complex-valued neural networks to get our conclusion. The main contributions of this paper can be listed as follows:

(1) Different from the Cohen-Grossberg neural networks investigated in some references such as [10, 11, 47], the model studied in this paper is more general, including more factors such as mixed time



(a)



(b)

Figure 1: The error trajectories of FCVCGNNs without the controller. (a) The curves of real parts and imaginary parts. (b) The curves of modulus of error.

delays, complex value, and variable parameters, which greatly improve the universality of the system. The finite-time synchronization of this kind of CGNN is yet to be fully investigated. Moreover, we give the maximum setting time. Our paper contributes to this field of research

(2) Based on the fact that the complex function cannot always be divided into two parts in practical application, the nonseparation method is adopted in this paper. This method reduces the workload of analysis and derivation and improves the breadth of the derivation conclusions

(3) Compared with the nonseparation method used in [33, 48–50], we extend the use of this method to the derivation of complex-valued neural networks with different connection weights

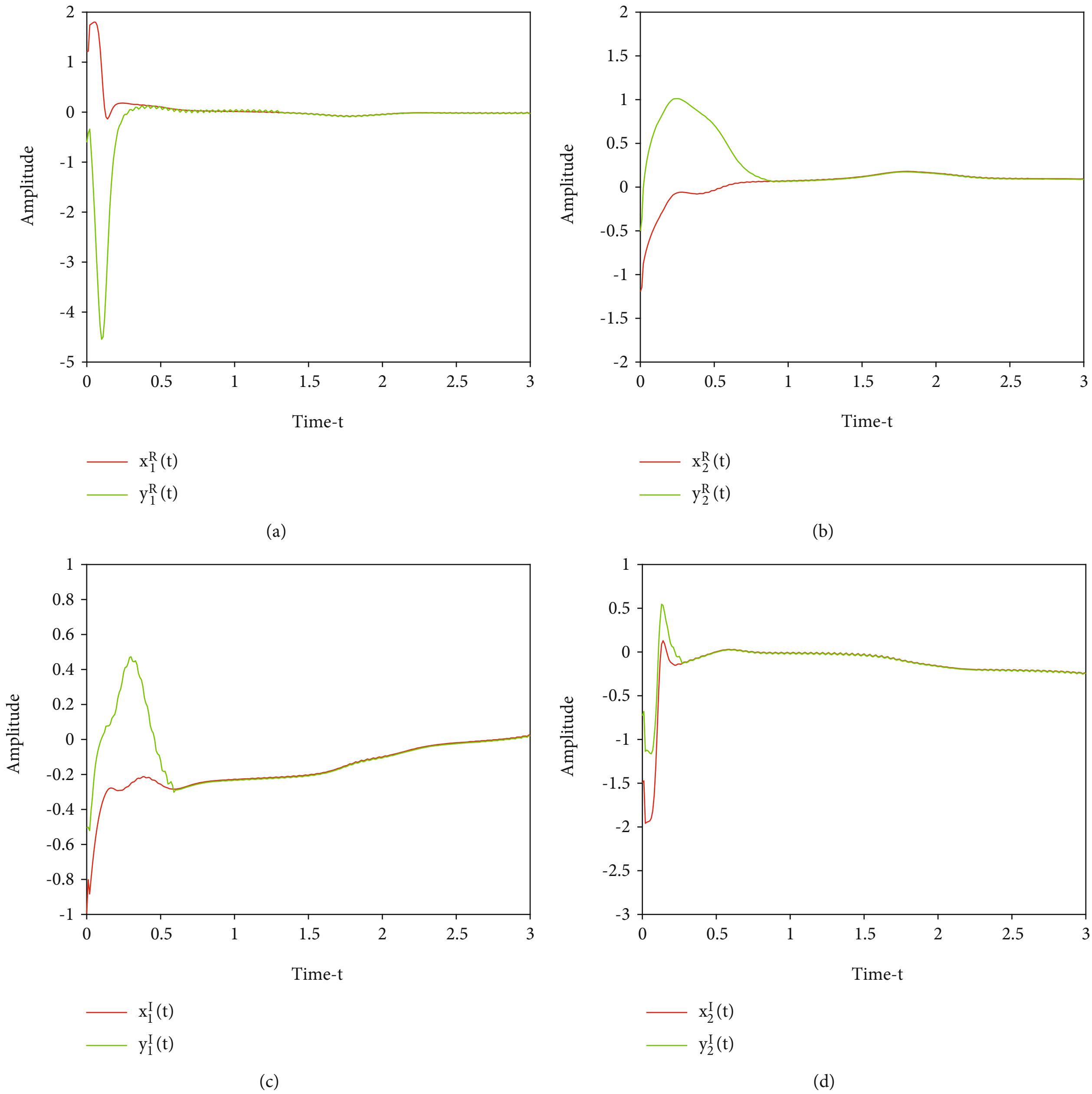


Figure 2: Continued.

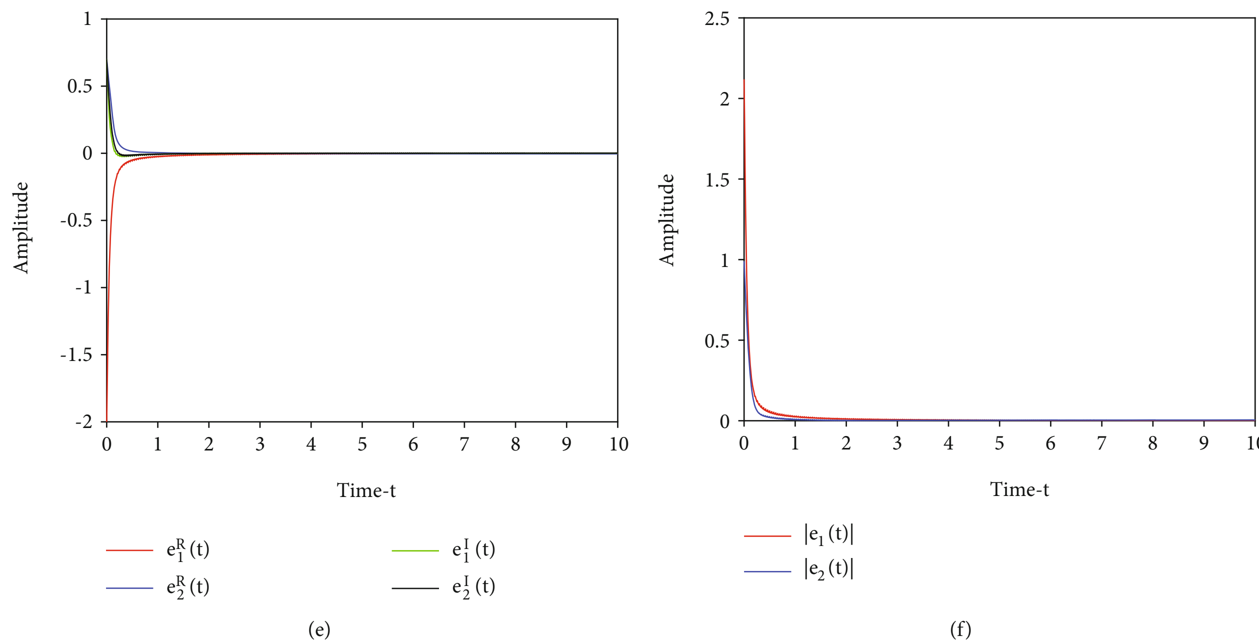


FIGURE 2: (a, b) The real parts of synchronization state trajectories with controller (24). (c, d) The imaginary parts of synchronization state trajectories with controller (24). (e) The error trajectories with controller (24). (f) The error modulus trajectories with controller (24).

The organization of this paper is as follows. In Section 2, some definitions and lemmas are introduced and give the model of our neural network. In Section 3, using feedback and an adaptive controller, we make the drive-response system of FCVCGNNs achieve synchronization in a finite time; moreover, we also get its setting time. In Section 4, some numerical examples are given to illustrate the validity of the proposed results. Some conclusions are drawn in Section 5.

Notation: throughout this paper, $\mathbb{R}, \mathbb{C}$ represent the real and complex domains, respectively. Let $z = a + bi$ be a complex number, where $a, b \in \mathbb{R}$ are the real and imaginary parts of z, $\bar{z} = a - bi$ is the conjugate of z, and $|z| = \sqrt{z \cdot \bar{z}}$ is the module of z.

2. Model Description and Preliminaries

In this section, some basic definitions of finite-time synchronization and fractional calculus are given; what is more, some lemmas and assumptions needed in the later proof are included. Also, we give the model description of studied FCVCGNNs.

2.1. Model Description. Generally, FCVCGNNs with mixed time delays can be described as follows:

$$\begin{cases} D^{\alpha}x_p(t) = -d_p(x_p(t))\Big(h_p(x_p(t)) - \sum_{q=1}^{n} a_{pq}(x_q(t)) f_q(x_q(t)) \\ \quad - \sum_{q=1}^{n} b_{pq}(x_q(t-\delta(t)))g_q(x_q(t-\delta(t))) - \sum_{q=1}^{n} c_{pq}(x_q(t))\int_{t-\tau(t)}^{t} f_q(x_q(s))ds - I_p\Big), x_p(t) \\ = \varphi_p(t), t \in [-\rho, 0], \end{cases} \tag{1}$$

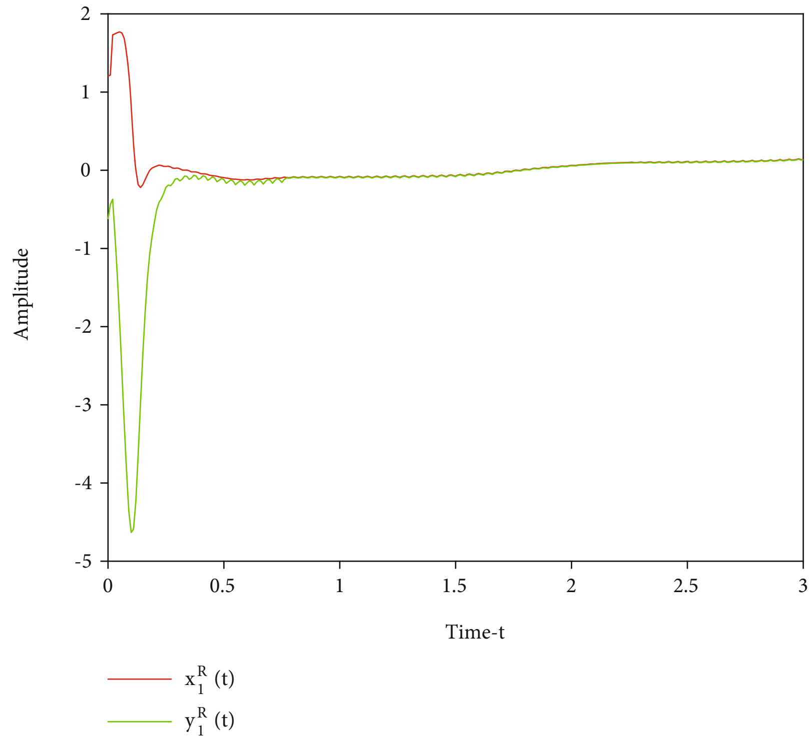
where $p = 1, 2, \cdots, n$ is the order of neurons in a neural network, $x_p(t) \in \mathbb{C}$ is the state of the pth neuron at time t, $d_p(\cdot)$ represents the amplification function, $h_p(\cdot)$ is the behaved function, $f_q(\cdot), g_q(\cdot)$ are the activation functions of the qth neuron with and without time delay, respectively, $\tau(t)$ and $\delta(t)$ represent discrete and distributed time delay, ρ is the max value between them, $I_p \in \mathbb{C}$ is the external input for network, and $a_{pq}, b_{pq}, c_{pq} \in \mathbb{C}$ denotes the state-dependent connection strengths of the p-th and q-th neuron and satisfy:

$$a_{pq}(\cdot) = \begin{pmatrix} \check{a}_{pq}, & \text{if} |\cdot| \le \kappa_p, \\ \hat{a}_{pq}, & \text{if} |\cdot| > \kappa_p, \end{pmatrix} b_{pq}(\cdot) = \begin{pmatrix} \check{b}_{pq}, & \text{if} |\cdot| \le \kappa_p, \\ \hat{b}_{pq}, & \text{if} |\cdot| > \kappa_p, \end{pmatrix}$$
$$c_{pq}(\cdot) = \begin{pmatrix} \check{c}_{pq}, & \text{if} |\cdot| \le \kappa_p, \\ \hat{c}_{pq}, & \text{if} |\cdot| > \kappa_p, \end{pmatrix} \tag{2}$$

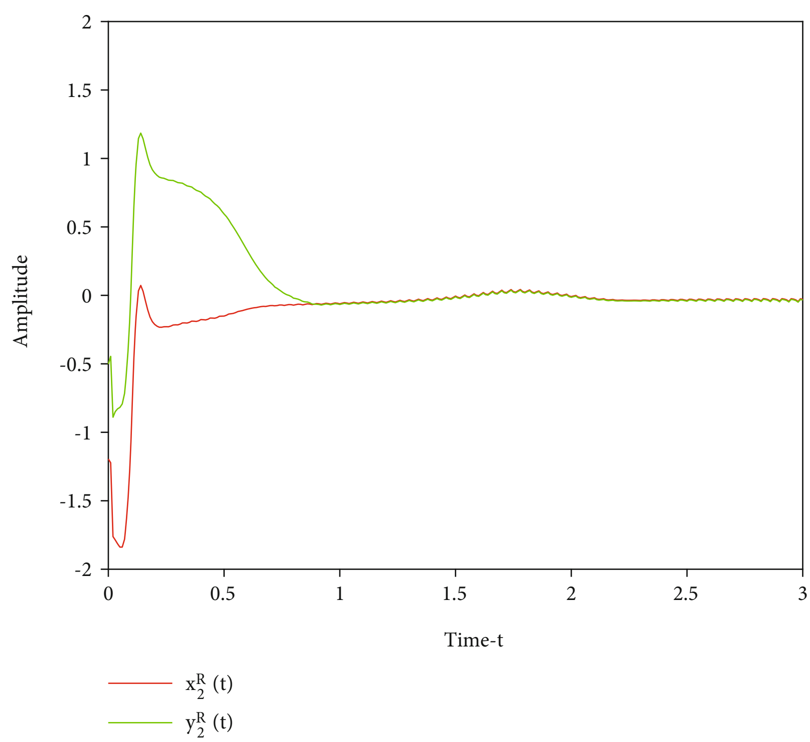
where $\cdot$ denotes $x_q(t)$ or $y_q(t)$ which represents the state of the pth neuron, $a_{pq}, \hat{a}_{pq}, \check{b}_{pq}, \hat{b}_{pq}, \check{c}_{pq}, \hat{c}_{pq}$ are known constants, and $\kappa_p > 0$ is a threshold level.

Through the theories of differential inclusions and set-valued map, drive system (1) can be transformed as follows:

$$\begin{cases} D^{\alpha}x_p(t) = -d_p(x_p(t))\Big(h_p(x_p(t)) - \sum_{q=1}^{n} co\left[a_{pq}(x_q(t))\right] f_q(x_q(t)) \\ \quad - \sum_{q=1}^{n} co\left[b_{pq}(x_q(t-\delta(t)))\right] g_q(x_q(t-\delta(t))) \\ \quad - \sum_{q=1}^{n} co\left[c_{pq}(x_q(t))\right] \int_{t-\tau(t)}^{t} f_q(x_q(s))ds - I_p\Big), x_p(t) = \varphi_p(t), t \in [-\rho, 0], \end{cases} \tag{3}$$

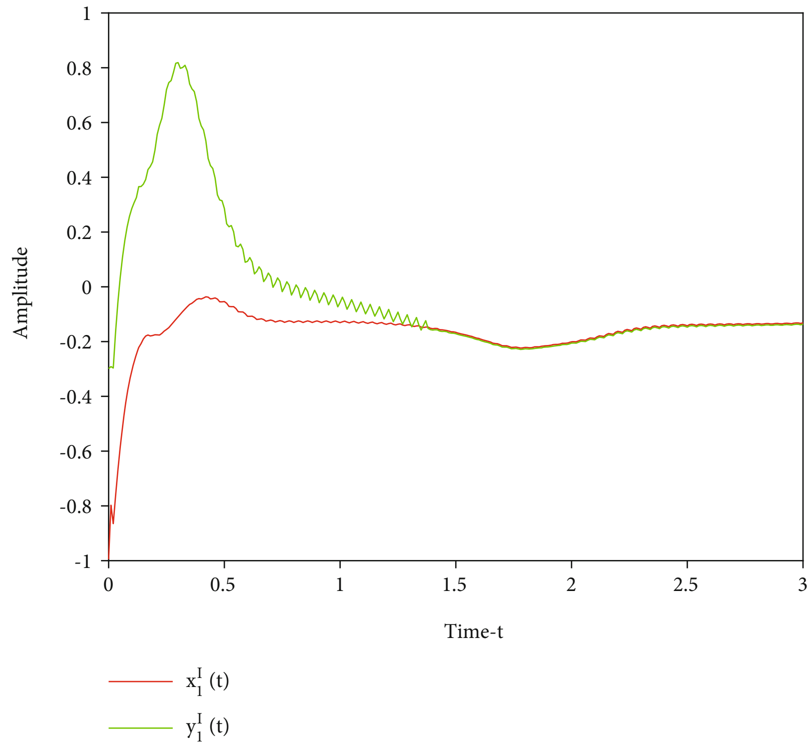


(a)



(b)

Figure 3: Continued.



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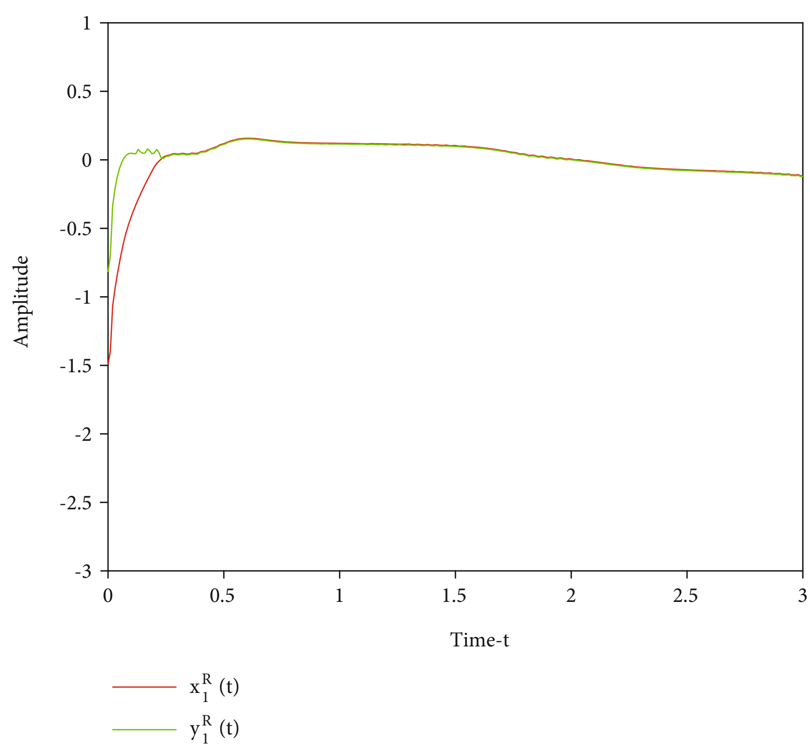

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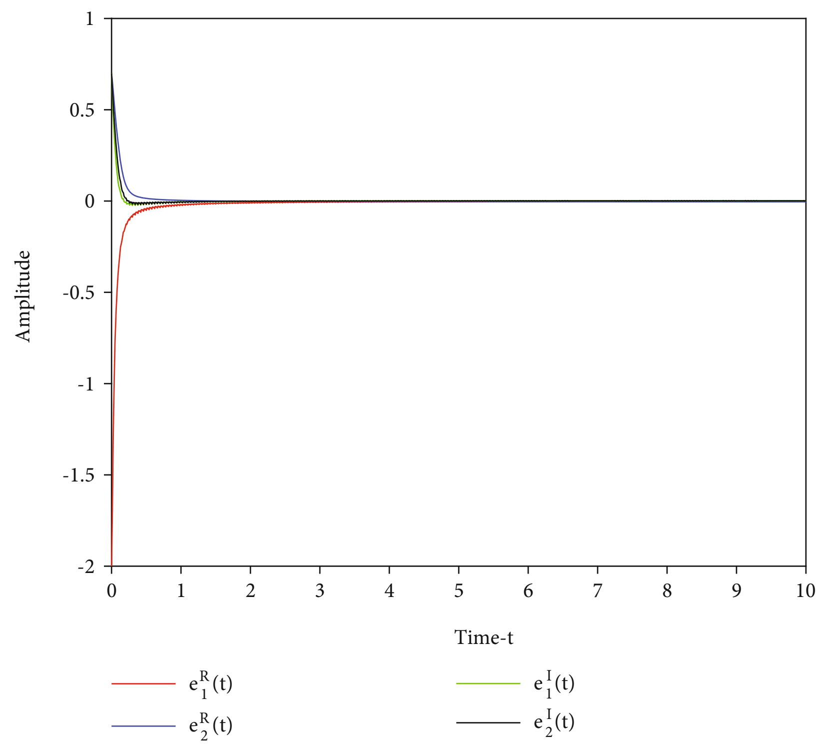
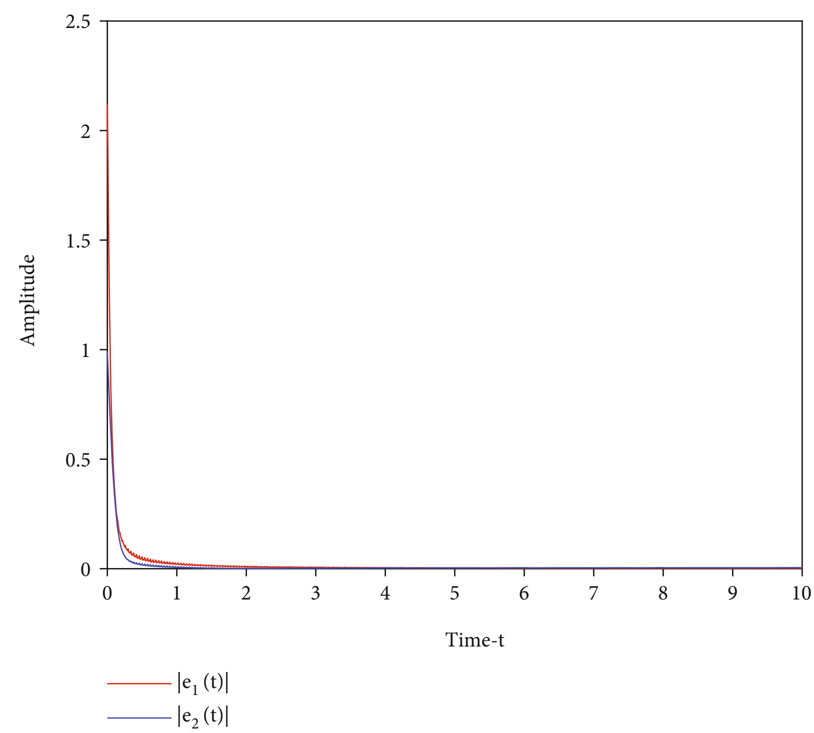
Figure 3: Continued.



(e)



(f)

Figure 3: (a, b) The real parts of synchronization state trajectories with controller (42). (c, d) The imaginary parts of synchronization state trajectories with controller (42). (e) The error trajectories with controller (42). (f) The error modulus trajectories with controller (42).

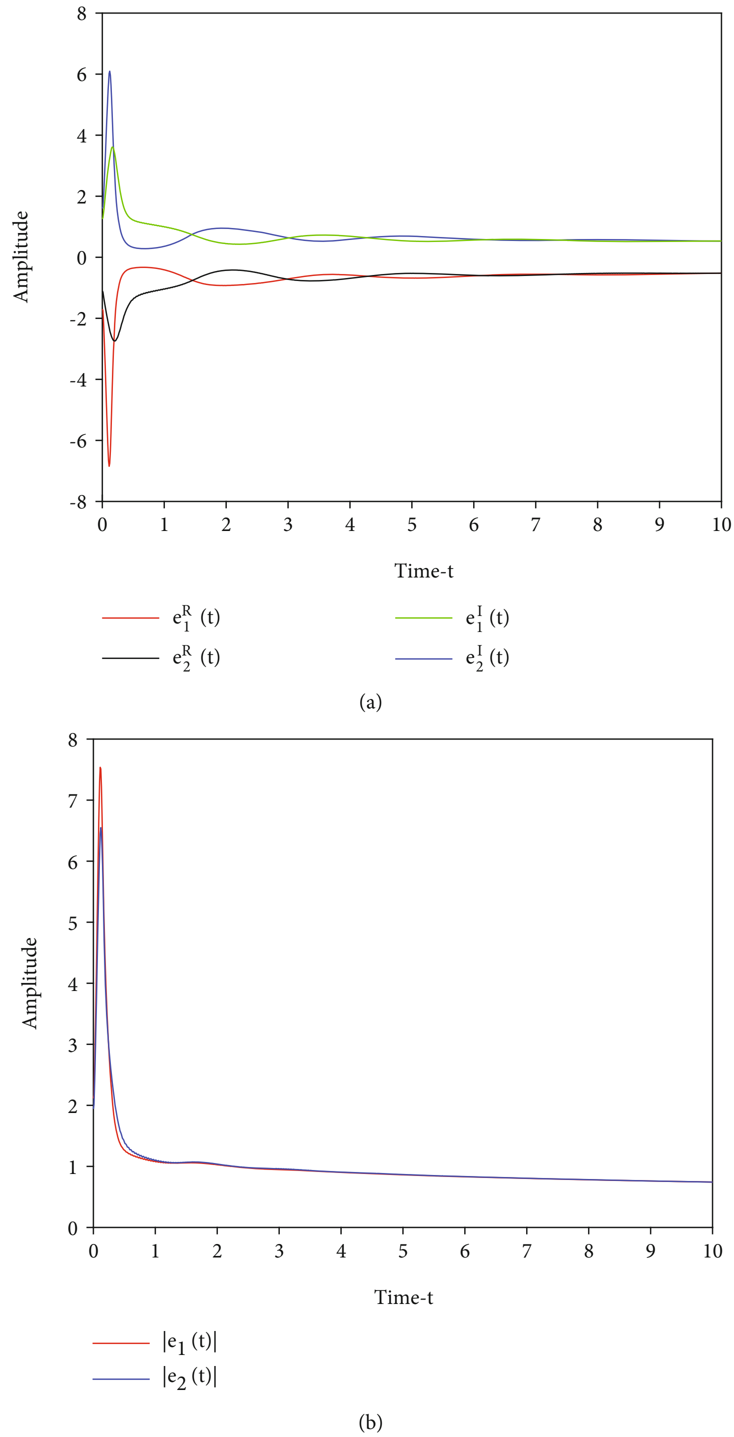
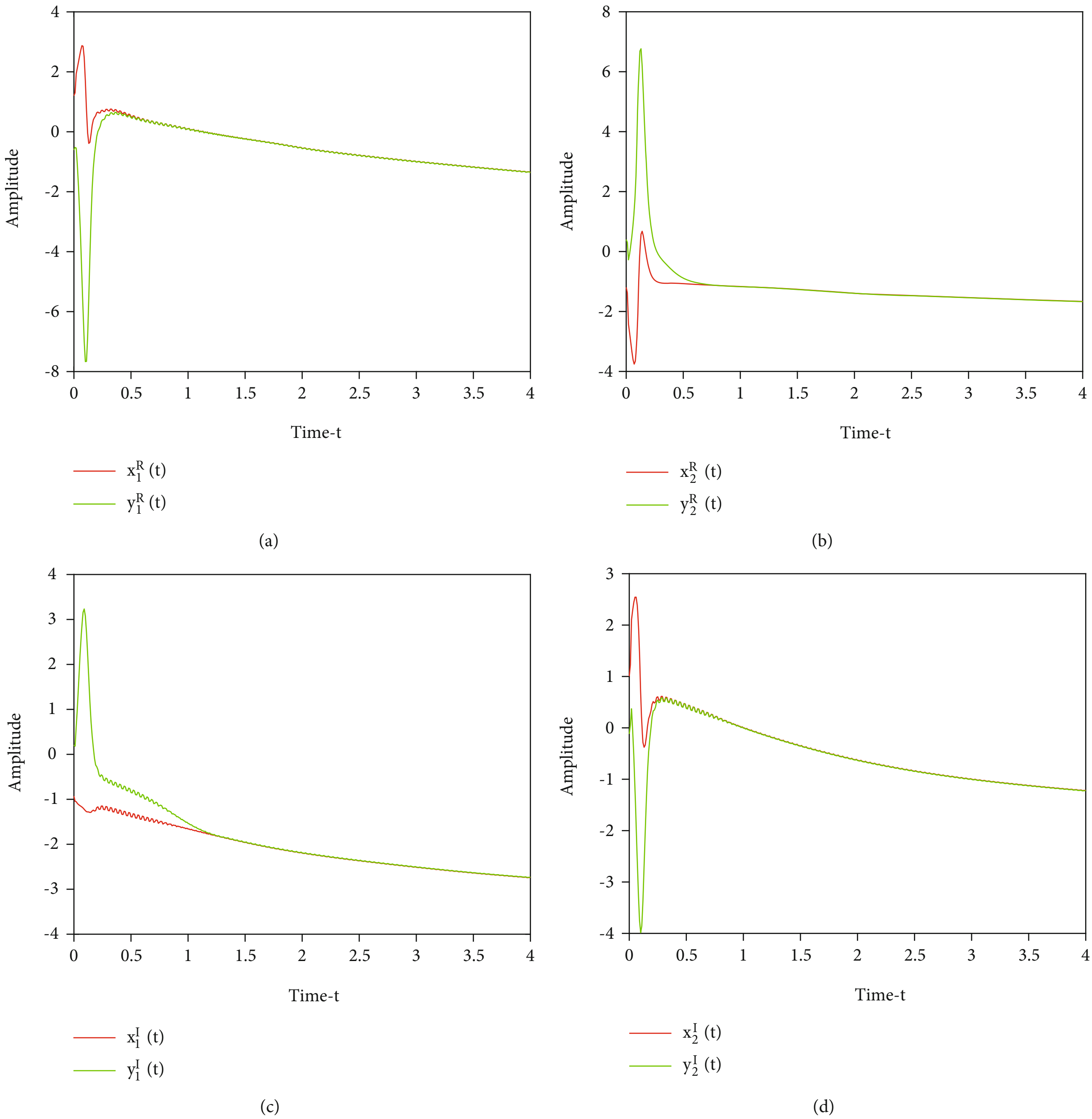


FIGURE 4: The error trajectories of FCVCGNNs without the controller. (a) The curves of real parts and imaginary parts. (b) The curves of modulus of error.



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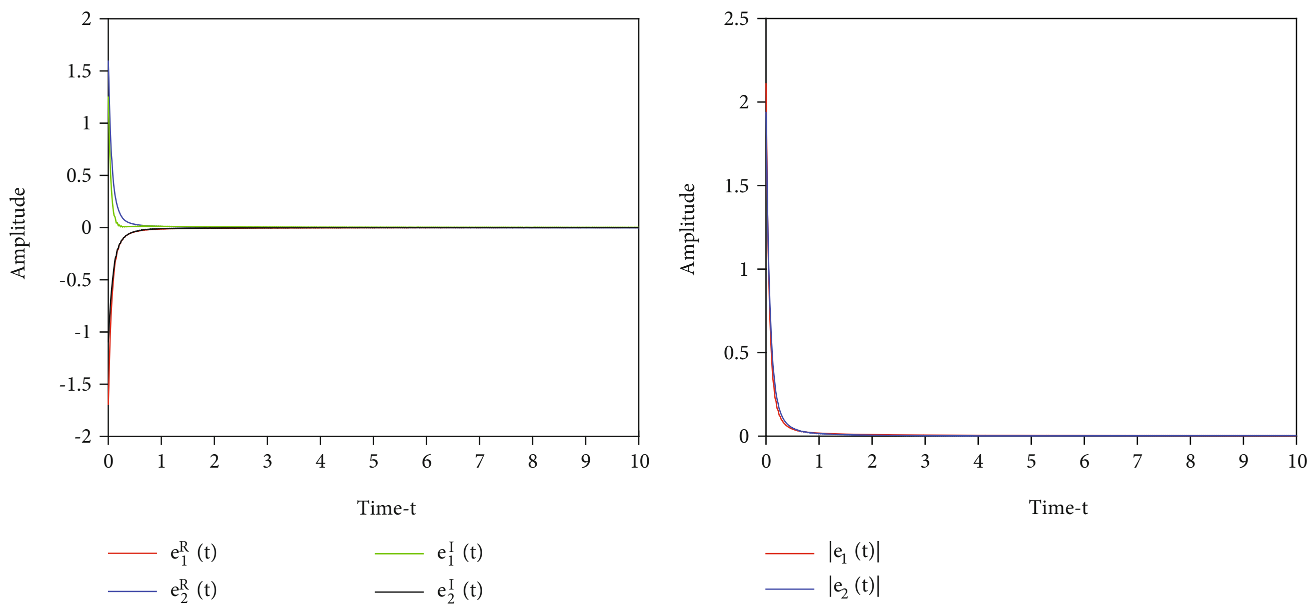


(e) (f)

Figure 5: (a, b) The real parts of synchronization state trajectories with controller (24). (c, d) The imaginary parts of synchronization state trajectories with controller (24). (e) The error trajectories with controller (24). (f) The error modulus trajectories with controller (24).

where co indicates the closure convex hull, $co[a_{pq}(x_q(t))] = co[\check{a}_{pq}, \widehat{a}_{pq}]$, and $co[b_{pq}(x_q(t-\delta(t)))] = co[\check{b}_{pq}, \widehat{b}_{pq}]$, $co[c_{pq}(x_q(t))] = co[\check{c}_{pq}, \widehat{c}_{pq}]$.

Or equivalently, there exists ${}^{\backprime}a_{pq} \in co[a_{pq}(x_q(t))]$, ${}^{\backprime}b_{pq} \in co[b_{pq}(x_q(t-\delta(t)))]$,

${}^{\backprime}c_{pq} \in co[c_{pq}(x_q(t))]$, $a'_{pq} \in co[a_{pq}(y_q(t))]$, $b'_{pq} \in co[b_{pq}(y_q(t-\delta(t)))]$, $c'_{pq} \in co[c_{pq}(y_q(t))]$, such that

$$\begin{cases} D^\alpha x_p(t) = -d_p(x_p(t)) \left[h_p(x_p(t)) - \sum_{q=1}^{n} {}^{\backprime}a_{pq}(x_q(t)) f_q(x_q(t)) \right. \\ \left. \quad - \sum_{q=1}^{n} {}^{\backprime}b_{pq}(x_q(t)) g_q(x_q(t-\delta(t))) - \sum_{q=1}^{n} {}^{\backprime}c_{pq}(x_q(t)) \int_{t-\tau.(t)}^{t} f_q(x_q(s)) ds - I_p \right], x_p(t) \\ = \varphi_p(t), t \in [-\rho, 0]. \end{cases} \tag{4}$$

Similarly, the corresponding response system can be described as

$$\begin{cases} D^\alpha y_p(t) = -d_p\left(y_p(t)\right) \left[h_p\left(y_p(t)\right) - \sum_{q=1}^{n} a'_{pq}\left(y_q(t)\right) f_q\left(y_q(t)\right) \right. \\ \left. \quad - \sum_{q=1}^{n} b'_{pq}\left(y_q(t)\right) g_q\left(y_q(t-\delta(t))\right) - \sum_{q=1}^{n} c'_{pq}\left(y_q(t)\right) \int_{t-\tau(t)}^{t} f_q\left(y_q(s)\right) ds - I_p \right] \\ \quad + u_p(t), y_p(t) = \psi_p(t), t \in [-\rho, 0] \end{cases} \tag{5}$$

For convenience, we can simplify systems (4) and (5) as follows:

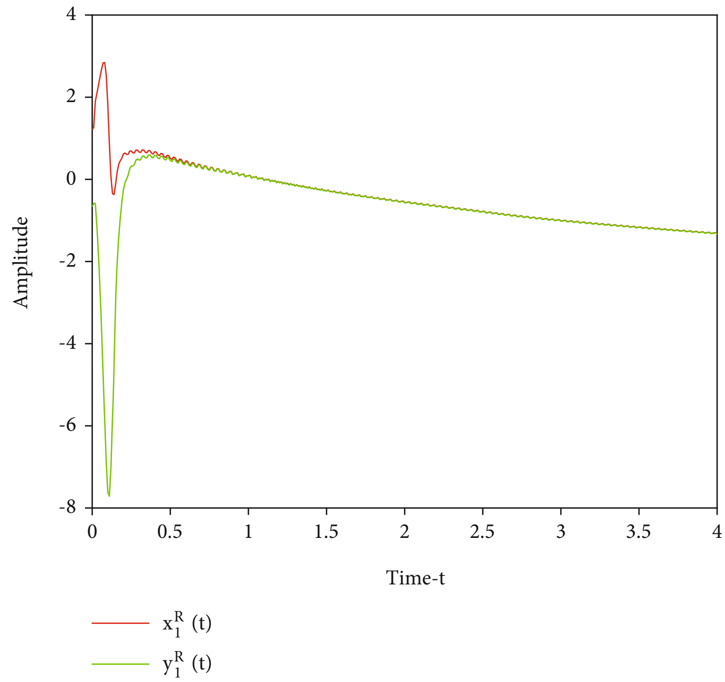
$$D^\alpha x_p(t) = -d_p(x_p(t)) \left[h_p(x_p(t)) - {}^{\backprime}A f_q(x_q(t)) - {}^{\backprime}B g_q(x_q(t-\delta(t))) \right. \\ \left. - {}^{\backprime}C \int_{t-\tau(t)}^{t} f_q(x_q(s)) ds - I_p \right],$$

$$D^\alpha y_p(t) = -d_p\left(y_p(t)\right) \left[h_p\left(y_p(t)\right) - A' f_q\left(y_q(t)\right) - B' g_q\left(y_q(t-\delta(t))\right) \right. \\ \left. - C' \int_{t-\tau(t)}^{t} f_q\left(y_q(s)\right) ds - I_p \right] + U_p(t), \tag{6}$$

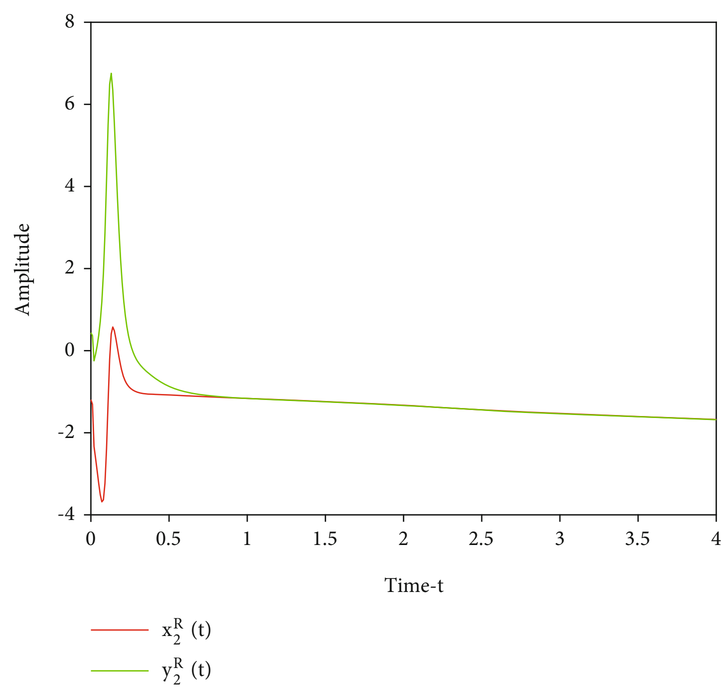
where ${}^{\backprime}A = ({}^{\backprime}a_{pq})_{n\times n} \in \mathbb{C}^{n\times n}$, $A' = (a'_{pq})_{n\times n} \in \mathbb{C}^{n\times n}$, ${}^{\backprime}B = ({}^{\backprime}b_{pq})_{n\times n} \in \mathbb{C}^{n\times n}$, $B' = (b'_{pq})_{n\times n} \in \mathbb{C}^{n\times n}$, ${}^{\backprime}C = ({}^{\backprime}c_{pq})_{n\times n} \in \mathbb{C}^{n\times n}$, $C' = (c'_{pq})_{n\times n} \in \mathbb{C}^{n\times n}$, $A = \max\{|{}^{\backprime}A|, |A'|\}$, and $B = \max\{|{}^{\backprime}B|, |B'|\}$, $C = \max\{|{}^{\backprime}C|, |C'|\}$.

The error system between drive system and response system is defined as

$$e_p(t) = y_p(t) - x_p(t). \tag{7}$$

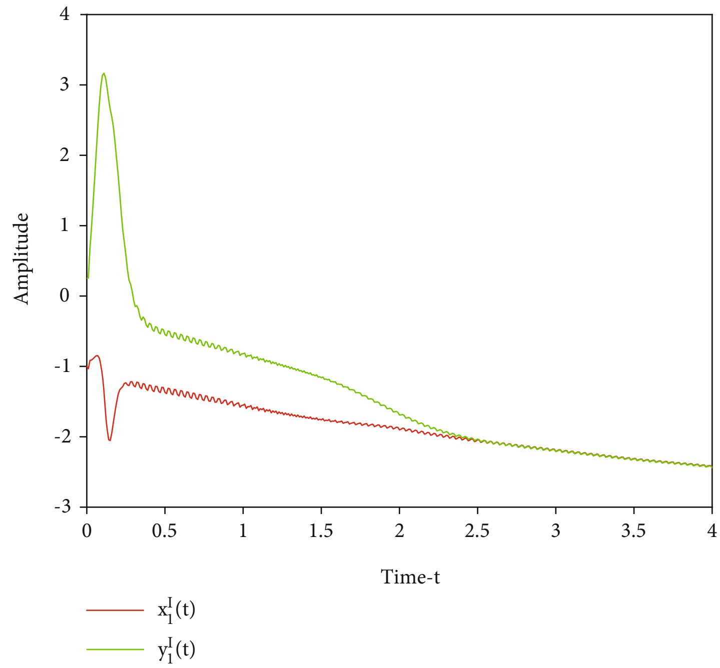


(a)



(b)

Figure 6: Continued.



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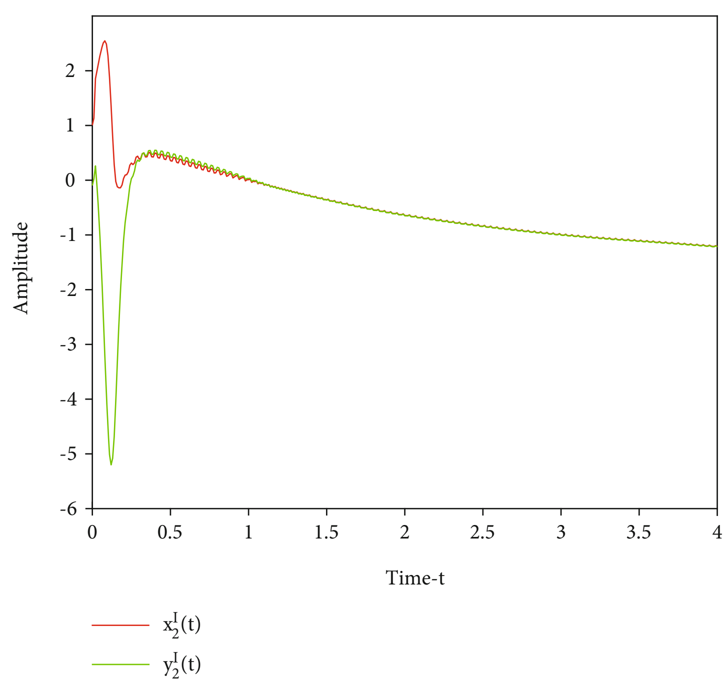

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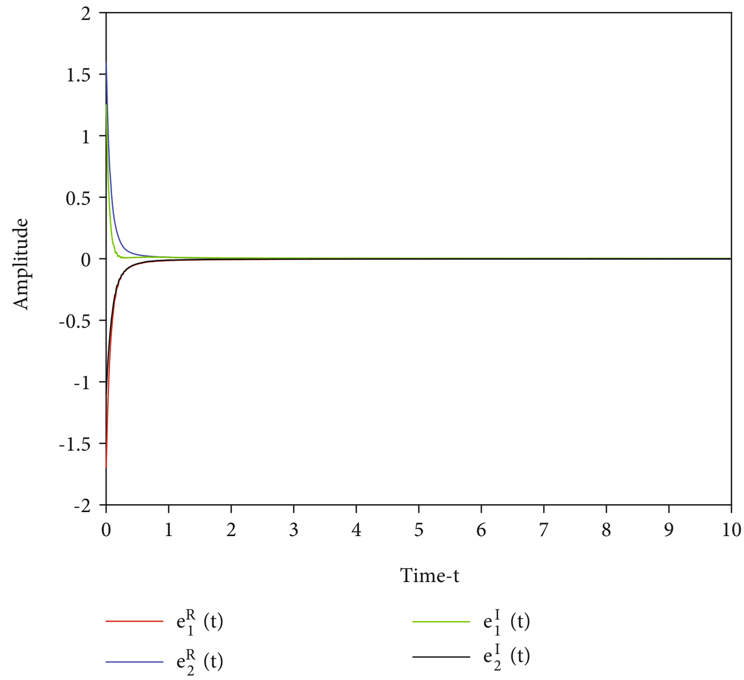

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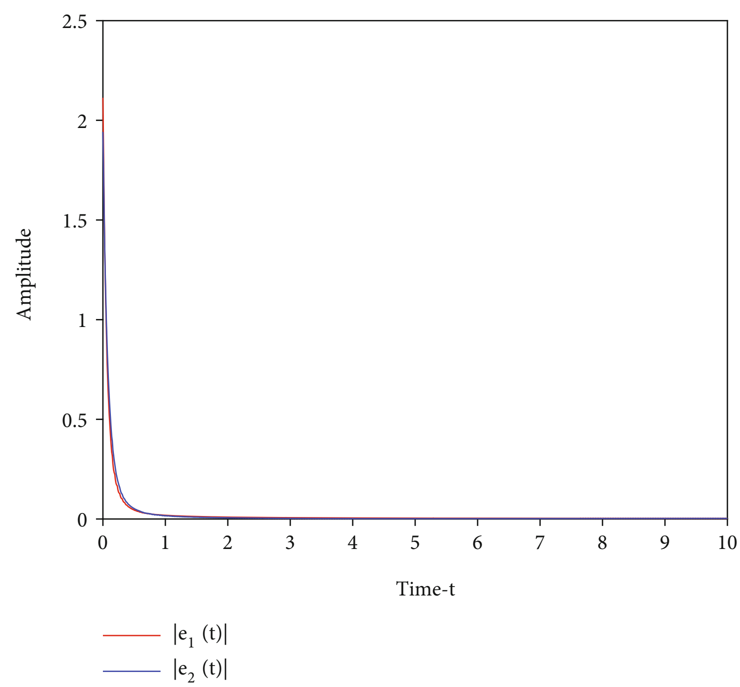

(f)

Figure 6: (a, b) The real parts of synchronization state trajectories with controller (42). (c, d) The imaginary parts of synchronization state trajectories with controller (42). (e) The error trajectories with controller (42). (f) The error modulus trajectories with controller (42).

Then, we can get

$$
\begin{aligned}
D^{\alpha}e_p(t) ={}& -\left[d_p\left(y_p(t)\right)h_p\left(y_p(t)\right) - d_p\left(x_p(t)\right)h_p\left(x_p(t)\right)\right] + \left[d_p\left(y_p(t)\right)\cdot A'\cdot f_q\left(y_q(t)\right)\right. \\
&\left. - d_p\left(x_p(t)\right)\cdot {}^{\backprime}A\cdot f_q\left(x_q(t)\right)\right] + \left[d_p\left(y_p(t)\right)\cdot B'\cdot g_q\left(y_q(t-\delta(t))\right) - d_p\left(x_p(t)\right)\right. \\
&\left.\cdot {}^{\backprime}B\cdot g_q\left(x_q(t-\delta(t))\right)\right] + \left[d_p\left(y_p(t)\right)\cdot C'\cdot \int_{t-\tau(t)}^{t} f_q\left(y_q(s)\right)ds - d_p\left(x_p(t)\right)\cdot {}^{\backprime}C\right. \\
&\left.\cdot \int_{t-\tau(t)}^{t} f_q\left(x_q(s)\right)ds\right] + \left[d_p\left(y_p(t)\right)\cdot I_p - d_p\left(x_p(t)\right)\cdot I_p\right] + U_p(t) \\
={}& -\left[d_p\left(y_p(t)\right)h_p\left(y_p(t)\right) - d_p\left(x_p(t)\right)h_p\left(x_p(t)\right)\right] + \left[d_p\left(y_p(t)\right)\cdot A'\cdot f_q\left(y_q(t)\right)\right. \\
& - d_p\left(x_p(t)\right)\cdot A'\cdot f_q\left(y_q(t)\right) + d_p\left(x_p(t)\right)\cdot A'\cdot f_q\left(y_q(t)\right) - d_p\left(x_p(t)\right) \\
&\left.\cdot {}^{\backprime}A\cdot f_q\left(x_q(t)\right)\right] + \left[d_p\left(y_p(t)\right)\cdot B'\cdot g_q\left(y_q(t-\delta(t))\right) - d_p\left(x_p(t)\right)\cdot B'\right. \\
& \cdot g_q\left(y_q(t-\delta(t))\right) + d_p\left(x_p(t)\right)\cdot B'\cdot g_q\left(y_q(t-\delta(t))\right) - d_p\left(x_p(t)\right)\cdot {}^{\backprime}B \\
&\left.\cdot g_q\left(x_q(t-\delta(t))\right)\right] + \left[d_p\left(y_p(t)\right)\cdot C'\cdot \int_{t-\tau(t)}^{t} f_q\left(y_q(s)\right)ds - d_p\left(x_p(t)\right)\cdot C'\right. \\
& \cdot \int_{t-\tau(t)}^{t} f_q\left(y_q(s)\right)ds + d_p\left(x_p(t)\right)\cdot C'\cdot \int_{t-\tau(t)}^{t} f_q\left(y_q(s)\right)ds - d_p\left(x_p(t)\right)\cdot {}^{\backprime}C \\
&\left.\cdot \int_{t-\tau(t)}^{t} f_q\left(x_q(s)\right)ds\right] + \left[d_p\left(y_p(t)\right)\cdot I_p - d_p\left(x_p(t)\right)\cdot I_p\right] + U_p(t),
\end{aligned}
\tag{8}
$$

and the initial condition of the error system is

$$\theta_p(t) = \psi_p(t) - \varphi_p(t) \tag{9}$$

Throughout this paper, the assumptions below will be available in the following proof.

Assumption 1. In the complex field, for function f, d, g, there exists positive constants M_f, M_d, M_g, L_d, L_f such that

$$
\begin{aligned}
&\left|d_p(x)\right| \le M_d, \quad \left|f_q(x)\right| \le M_f, \quad \left|g_q(x)\right| \le M_g, \\
&|d(y)-d(x)| \le L_d|y-x|, |f(y)-f(x)| \le L_f|y-x|,
\end{aligned}
\tag{10}
$$

where $x, y \in \mathbb{C}$.

Assumption 2. For well-behaved function $h_i(\cdot)$ and amplication function $d_i(\cdot)$, for all $x \ne y$, there exists positive constant w_i, such that

$$\frac{|h_i(y)d_i(y) - h_i(x)\, d_i(x)|}{|y-x|} \le w_i, \quad i = 1, 2, \cdots, n. \tag{11}$$

2.2. Definitions and Lemmas

Definition 1. (See [19]). The Caputo fractional derivative with fractional-order α for a function $y(t)$ can be described as follows:

$${}_{t_0}^{C}D_t^{\alpha}y(t) = \frac{1}{\Gamma(m-\alpha)}\int_{t_0}^{t}\frac{y^{(m)}(\tau)}{(t-\tau)^{1+\alpha-m}}d\tau, \tag{12}$$

where $t \ge t_0$, and m is a positive integer satisfying $m-1 < \alpha < m$. $\Gamma(\cdot)$ is the Gamma function, and it can be described by $\Gamma(x) = \int_0^{+\infty} e^{-t}t^{x-1}dt$. Particularly, when $0 < \alpha < 1$,

$${}_{t_0}^{C}D_t^{\alpha}y(t) = \frac{1}{\Gamma(1-\alpha)}\int_{t_0}^{t}\frac{y'(\tau)}{(t-\tau)^{\alpha}}d\tau. \tag{13}$$

Remark 2. In practical applications, the initial value of the system is hard to know accurately; so, the Caputo fractional-order that does not require a precise initial value is selected here.

Definition 3. (See [19]). The fractional integral of order α for a function $y(t)$ can be described as follows:

$$I_{t_0,t}^{\alpha}y(t) = \frac{1}{\Gamma(\alpha)}\int_{t_0}^{t}(t-s)^{\alpha-1}y(s)ds. \tag{14}$$

Particularly, if $\alpha \in (0, 1)$, then $I_{t_0,t}^{\alpha}(D_{t_0,t}^{\alpha}f(t)) = V(t) - V(t_0)$.

Property: the Caputo derivative satisfies the following equality:

$${}_{t_0}D_t^{\alpha}\left({}_{t_0}D_t^{-\beta}f(t)\right) = {}_{t_0}D_t^{\alpha-\beta}f(t), \tag{15}$$

in which, $\alpha \ge \beta \ge 0$.

Consider the following drive-response system,

$$
\begin{aligned}
&\text{drive system}: \dot{x} = f(x,t), \qquad x_0 = x(0), \\
&\text{response system}: \dot{y} = f(y,t), \quad y_0 = y(0).
\end{aligned}
\tag{16}
$$

Definition 4. (See [59]). Drive-response system is said to be finite time synchronization if there exists a suitable controller U, and the setting time T satisfies

$$\lim_{t \longrightarrow T}|y(t) - x(t)| = 0, \tag{17}$$

and when $t > T$, the error system will always be 0.

Definition 5. (See [60]). The error system is said to be global Mittag-Leffler stable, if there exist constants $\vartheta > 0, \mu > 0$ satisfying

$$|y(t)| \le [\zeta(y(0))E_{\alpha}(-\vartheta t^{\alpha})]^{\mu}. \tag{18}$$

Lemma 6. (See [60]). *Assume that a positive, continuous function $V(t)$ satisfies*

$$D^{\alpha}V(t) \le -\beta V(t) + \theta, \forall t \ge 0, \tag{19}$$

where β, θ are two constants that satisfy $\beta > 0, \theta > = 0$, when $0 < \alpha < 1$, then

$$V(t) \le V(0)E_{\alpha}(-\beta t^{\alpha}) + \theta t^{\alpha}E_{\alpha,\alpha+1}(-\beta t^{\alpha}). \tag{20}$$

Lemma 7. (See [50]). *Let $x \in \mathbb{C}$ be a continuous and analytic* *function, and then*

$$D^{\alpha}x(t)x(\bar{t}) \leq x(\bar{t}) \cdot D^{\alpha}x(t) + x(t) \cdot D^{\alpha}x(\bar{t}), \forall \quad t > 0, \tag{21}$$

where $0 < \alpha < 1$.

Lemma 8. (See [50]). *Let* $\alpha, \beta \in \mathbb{C}$, *and then the following inequality holds:*

$$\alpha\bar{\beta} + \beta\bar{\alpha} \leq \alpha\bar{\alpha} + \beta\bar{\beta}. \tag{22}$$

Corollary 9. *Moreover, we can get*

$$(\alpha + \beta) \cdot \alpha \bar{+} \beta = (\alpha + \beta) \cdot \left(\bar{\alpha} + \bar{\beta}\right) = \alpha\bar{\alpha} + \beta\bar{\beta} + \alpha\bar{\beta} + \beta\bar{\alpha} \leq 2\left(\alpha\bar{\alpha} + \beta\bar{\beta}\right). \tag{23}$$

Remark 10. Compared with the nonseparating method used in [30, 33, 48, 50], we extend the conclusion of Lemma 8 to broaden its usage, as shown in Corollary 9. The neural networks with mismatched parameters, which cannot use the non-separation method to achieve the proof process in references above, can be easily achieved with our method.

3. Main Results

In this section, the finite-time synchronization of a kind of FCVCGNNs is considered, which contains mixed time delays and state-dependent switching. Several sufficient conditions are derived based on the feedback and adaptive control strategies.

3.1. Feedback Controller. In order to synchronize the drive system and response system within a limited time, we select the following feedback controller:

$$u_p(t) = -k_1 e_p(t) - \frac{k_2 \cdot e_p(t)}{\left|e_p(t)\right|^2}, \tag{24}$$

where k_1, k_2 are positive constants.

Theorem 11. *Suppose that Assumptions 1 and 2 are satisfied, the response system (5) and drive system (4) can achieve finite-time synchronization under the feedback controller (24) if the following conditions hold:*

$$\beta < 0, \quad \theta < 0, \tag{25}$$

where

$$\begin{aligned} \beta &= \left(w^2 - 2k_1\right) + 5 + 2 \cdot \left|A'\right|^2 \cdot M_f^2 \cdot L_d^2 + 2 \cdot |A|^2 \\ &\quad \cdot M_d^2 \cdot L_f^2 + 2 \cdot \left|B'\right|^2 \cdot M_g^2 \cdot L_d^2 + 2 \cdot \left|C'\right|^2 \cdot \rho^2 \\ &\quad \cdot M_f^2 \cdot L_d^2 + \left|I_p\right|^2 \cdot L_d^2 + 4 \cdot M_d^2 \cdot |B|^2 \cdot L_g^2, \\ \theta &= 4 \cdot M_d^2 \cdot |B|^2 \cdot L_g^2 \cdot |\psi(s)|^2 + 2 \cdot \rho^2 \cdot M_f^2 \\ &\quad \cdot M_d^2 \cdot \left|C' - \grave{C}\right|^2 - 2k_2. \end{aligned} \tag{26}$$

Proof. Consider the following Lyapunov function:

$$T = \left(\frac{V(0)\Gamma(\alpha + 1)}{n \cdot \nu}\right)^{\frac{1}{\alpha}}. \tag{27}$$

□

The fractional order derivative of $V(t)$ along the trajectories (8) is

$$\begin{aligned} D^{\alpha}V(t) &\leq \sum_{p=1}^{n}\left[e_p(t)D^{\alpha}e_p(\bar{t}) + e_p(\bar{t})D^{\alpha}e_p(t)\right] \\ &= \sum_{p=1}^{n} e_p(t) \cdot \Big[\Big(d_p\left(y_p(t)\right)h_p\left(y_p(t)\right)\bar{}\, d_p(x_p(t))h_p(x_p(t))\Big) \\ &\quad + d_p\left(y_p(\bar{t})\right) \cdot A' \cdot f_q\left(y_q(t)\right) d_p(x_p(t)) \cdot A' \cdot f_q\left(y_q(t)\right) + d_p(x_p(\bar{t})) \cdot A' \\ &\quad \cdot f_q\left(y_q(t)\right) d_p(x_p(t)) \cdot \grave{A} \cdot f_q(x_q(t)) + d_p\left(y_p(\bar{t})\right) \cdot B' \cdot g_q\left(y_q(t\delta(t))\right) \\ &\quad + \left(d_p(x_p(t)) \cdot B' \bar{\cdot}\, g_q\left(y_q(t\delta(t))\right)\right) + d_p(x_p(\bar{t})) \cdot B' \cdot g_q\left(y_q(t\delta(t))\right) \\ &\quad + \left(d_p(x_p(t)) \cdot \grave{B} \bar{\cdot}\, g_q(x_q(t\delta(t)))\right) + d_p\left(y_p(\bar{t})\right) \cdot C' \cdot \int_{t\tau(t)}^{t} f_q\left(y_q(s)\right)ds \\ &\quad + \left(d_p(x_p(t)) \cdot C' \cdot \bar{\int}_{t\tau(t)}^{t} f_q\left(y_q(s)\right)ds\right) + d_p(x_p(\bar{t})) \cdot C' \cdot \int_{t\tau(t)}^{t} f_q\left(y_q(s)\right)ds \\ &\quad + \left(d_p(x_p(t)) \cdot \grave{C} \cdot \bar{\int}_{t\tau(t)}^{t} f_q(x_q(s))ds\right) + d_p\left(y_p(t)\right) \cdot \bar{I}_p d_p(x_p(t)) \cdot I_p \\ &\quad - \left(k_1 e_p(t) \bar{+} \frac{k_2 \cdot e_p(t)}{\left|e_p(t)\right|^2}\right)\Bigg] + \sum_{p=1}^{n} e_p(\bar{t}) \cdot \Big[d_p\left(y_p(t)\right) \cdot A' \cdot f_q\left(y_q(t)\right) \\ &\quad - d_p(x_p(t)) \cdot A' \cdot f_q\left(y_q(t)\right) + d_p(x_p(t)) \cdot A' \cdot f_q\left(y_q(t)\right) - d_p(x_p(t)) \cdot \grave{A} \\ &\quad \cdot f_q(x_q(t)) + d_p\left(y_p(t)\right) \cdot B' \cdot g_q\left(y_q(t - \delta(t))\right) - d_p(x_p(t)) \cdot B' \\ &\quad \cdot g_q\left(y_q(t - \delta(t))\right) + d_p(x_p(t)) \cdot B' \cdot g_q\left(y_q(t - \delta(t))\right) - d_p(x_p(t)) \cdot \grave{B} \\ &\quad \cdot g_q(x_q(t - \delta(t))) + d_p\left(y_p(t)\right) \cdot C' \cdot \int_{t-\tau(t)}^{t} f_q\left(y_q(s)\right)ds - d_p(x_p(t)) \cdot C' \\ &\quad \cdot \int_{t-\tau(t)}^{t} f_q\left(y_q(s)\right)ds + d_p(x_p(t)) \cdot C' \cdot \int_{t-\tau(t)}^{t} f_q\left(y_q(s)\right)ds - d_p(x_p(t)) \cdot \grave{C} \\ &\quad \cdot \int_{t-\tau(t)}^{t} f_q(x_q(s))ds + d_p\left(y_p(t)\right) \cdot I_p - d_p(x_p(t)) \cdot I_p - k_1 e_p(t) - \frac{k_2 \cdot e_p(t)}{\left|e_p(t)\right|^2}\Big]. \end{aligned} \tag{28}$$

According to Assumptions 1 and 2 and Lemmas 7 and 8, we can get the following inequality:

$$\begin{aligned} &\Big(d_p\left(y_p(t)\right)h_p\left(y_p(t)\right)\bar{}\, d_p(x_p(t))h_p(x_p(t))\Big) \cdot e_p(t) \\ &\quad + \Big[-\Big(d_p\left(y_p(t)\right)h_p\left(y_p(t)\right) - d_p(x_p(t))h_p(x_p(t))\Big)\Big] \cdot e_p(\bar{t}) \\ &\leq \Big(d_p\left(y_p(t)\right)h_p\left(y_p(t)\right)\bar{}\, d_p(x_p(t))h_p(x_p(t))\Big) \\ &\quad \times \Big[-\Big(d_p\left(y_p(t)\right)h_p\left(y_p(t)\right) - d_p(x_p(t))h_p(x_p(t))\Big)\Big] \\ &\quad + e_p(t) \cdot e_p(\bar{t}) = e_p(t) \cdot e_p(\bar{t}) + \left|d_p\left(y_p(t)\right)h_p\left(y_p(t)\right) - d_p(x_p(t))h_p(x_p(t))\right|^2 \\ &\leq e_p(t) \cdot e_p(\bar{t}) + w^2 \cdot e_p(t) \cdot e_p(\bar{t}) = \left(1 + w^2\right) \cdot e_p(t) \cdot e_p(\bar{t}). \end{aligned} \tag{29}$$

Similarly, we have

$$
\begin{aligned}
&e_p(t)\cdot\Big[d_p\big(y_p(\bar{t})\big)\cdot A'\cdot f_q\big(y_q(t)\big)d_p(x_p(t))\cdot A'\cdot f_q\big(y_q(t)\big)\\
&\quad+d_p(x_p(\bar{t}))\cdot A'\cdot f_q\big(y_q(t)\big)d_p(x_p(t))\cdot {}^{`}A\cdot f_q(x_q(t))\Big]+e_p(\bar{t})\\
&\quad\cdot\Big[d_p\big(y_p(t)\big)\cdot A'\cdot f_q\big(y_q(t)\big)-d_p(x_p(t))\cdot A'\cdot f_q\big(y_q(t)\big)\\
&\quad+d_p(x_p(t))\cdot A'\cdot f_q\big(y_q(t)\big)-d_p(x_p(t))\cdot {}^{`}A\cdot f_q(x_q(t))\Big]\\
&\le e_p(t)\cdot e_p(\bar{t})+\Big[d_p\big(y_p(\bar{t})\big)\cdot A'\cdot f_q\big(y_q(t)\big)d_p(x_p(t))\cdot A'\cdot f_q\big(y_q(t)\big)\\
&\quad+d_p(x_p(\bar{t}))\cdot A'\cdot f_q\big(y_q(t)\big)d_p(x_p(t))\cdot {}^{`}A\cdot f_q(x_q(t))\Big]\\
&\quad\times\Big[d_p\big(y_p(t)\big)\cdot A'\cdot f_q\big(y_q(t)\big)-d_p(x_p(t))\cdot A'\cdot f_q\big(y_q(t)\big)\\
&\quad+d_p(x_p(t))\cdot A'\cdot f_q\big(y_q(t)\big)-d_p(x_p(t))\cdot {}^{`}A\cdot f_q(x_q(t))\Big]\\
&\le e_p(t)\cdot e_p(\bar{t})+2\Big[\Big(d_p\big(y_p(t)\big)-d_p(x_p(t))\Big)\cdot A'\cdot f_q\big(y_q(t)\big)\\
&\quad\times\Big(d_p\big(y_p(t)\big)\bar{d}_p(x_p(t))\Big)\cdot A'\cdot f_q\big(y_q(t)\big)\Big]+2\Big[d_p(x_p(t))\cdot A\\
&\quad\cdot\Big(f_q\big(y_q(t)\big)-f_q(x_q(t))\Big)\times d_p(x_p(t))\cdot A\cdot\Big(\bar{f}_q\big(y_q(t)\big)\bar{\bar{f}}_q(x_q(t))\Big)\Big]\\
&=e_p(t)\cdot e_p(\bar{t})+2\cdot\left|A'\right|^2\cdot M_f^2\cdot L_d^2\cdot e_p(t)\cdot e_p(t)+2\cdot M_d^2\cdot|A|^2\cdot L_f^2\cdot e_p(t)\cdot e_p(\bar{t}),
\end{aligned}
$$

$$
\begin{aligned}
&e_p(t)\cdot\Big[d_p\big(y_p(\bar{t})\big)\cdot B'\cdot g_q\big(y_q(t\delta(t))\big)d_p(x_p(t))\cdot B'\cdot g_q\big(y_q(t\delta(t))\big)\\
&\quad+d_p(x_p(\bar{t}))\cdot B'\cdot g_q\big(y_q(t\delta(t))\big)d_p(x_p(t))\cdot {}^{`}B\cdot g_q(x_q(t\delta(t)))\Big]+e_p(\bar{t})\\
&\quad\cdot\Big[\Big(d_p\big(y_p(t)\big)\cdot B'\cdot g_q\big(y_q(t-\delta(t))\big)-d_p(x_p(t))\cdot B'\cdot g_q\big(y_q(t-\delta(t))\big)\\
&\quad+d_p(x_p(t))\cdot B'\cdot g_q\big(y_q(t-\delta(t))\big)-d_p(x_p(t))\cdot {}^{`}B\cdot g_q(x_q(t-\delta(t)))\Big)\Big]\\
&\le e_p(t)\cdot e_p(\bar{t})+2\Big[\Big(d_p\big(y_p(t)\big)-d_p(x_p(t))\Big)\cdot B'\cdot g_q\big(y_q(t-\delta(t))\big)\\
&\quad\times\Big(d_p\big(y_p(t)\big)\bar{d}_p(x_p(t))\Big)\cdot B'\cdot g_q\big(y_q(t\delta(t))\big)\Big]\\
&\quad+2\Big[d_p(x_p(t))\cdot B\cdot\Big(g_q\big(y_q(t-\delta(t))\big)-g_q(x_q(t-\delta(t)))\Big)\\
&\quad\times d_p(x_p(t))\cdot B\cdot\Big(g_q\big(y_q(\bar{t}\delta(t))\big)g_q(x_q(t\delta(t)))\Big)\Big]\\
&=e_p(t)\cdot e_p(\bar{t})+2\cdot\left|B'\right|^2\cdot M_g^2\cdot L_d^2\cdot e_p(t)\cdot e_p(\bar{t})+2\cdot M_d^2\cdot|B|^2\cdot L_g^2\cdot e_p(t-\delta(t))\\
&\quad\cdot e_p(t\bar{\delta}(t)),
\end{aligned}
$$

$$
\begin{aligned}
&e_p(t)\cdot\Big[d_p\big(y_p(\bar{t})\big)\cdot C'\cdot\int_{t\tau(t)}^{t}f_q\big(y_q(s)\big)ds+d_p(x_p(\bar{t}))\cdot C'\cdot\int_{t\tau(t)}^{t}f_q\big(y_q(s)\big)ds\\
&\quad+d_p(x_p(\bar{t}))\cdot C'\cdot\int_{t\tau(t)}^{t}f_q\big(y_q(s)\big)ds+d_p(x_p(t))\cdot {}^{`}C\cdot\int_{t\tau(t)}^{\bar{t}}f_q(x_q(s))ds\Big]\\
&\quad+e_p(\bar{t})\cdot\Big[d_p\big(y_p(t)\big)\cdot C'\cdot\int_{t-\tau(t)}^{t}f_q\big(y_q(s)\big)ds-d_p(x_p(t))\cdot C'\\
&\quad\cdot\int_{t-\tau(t)}^{t}f_q\big(y_q(s)\big)ds+d_p(x_p(t))\cdot C'\cdot\int_{t-\tau(t)}^{t}f_q\big(y_q(s)\big)ds-d_p(x_p(t))\cdot {}^{`}C\\
&\quad\cdot\int_{t-\tau(t)}^{t}f_q(x_q(s))ds\Big]\le e_p(t)\cdot e_p(\bar{t})+2\Big[\Big(d_p\big(y_p(t)\big)-d_p(x_p(t))\Big)\cdot C'\cdot M_f\cdot\rho\\
&\quad\times\Big(d_p\big(y_p(t)\big)\bar{d}_p(x_p(t))\Big)\cdot C'\cdot M_f\cdot\rho\Big]+2\Big[d_p(x_p(t))\cdot M_f\cdot\rho\cdot\Big(C'-{}^{`}C\Big)\\
&\quad\cdot d_p(x_p(t))\cdot\bar{M}_f\cdot\rho\cdot\Big(C'{}^{`}C\Big)\Big]\\
&=e_p(t)\cdot e_p(\bar{t})+2\cdot\left|C'\right|^2\cdot\rho^2\cdot M_f^2\cdot L_d^2\cdot e_p(t)\cdot e_p(\bar{t})+2\cdot\rho^2\cdot M_f^2\cdot M_d^2\cdot\left|C'-{}^{`}C\right|^2,
\end{aligned}
$$

$$
\begin{aligned}
&e_p(t)\cdot d_p\big(y_p(t)\big)\cdot \bar{I}_p d_p(x_p(t))\cdot I_p+e_p(\bar{t})\cdot\Big[d_p\big(y_p(t)\big)\cdot I_p-d_p(x_p(t))\cdot I_p\Big]\\
&\le e_p(t)\cdot e_p(\bar{t})+d_p\big(y_p(t)\big)\cdot \bar{I}_p d_p(x_p(t))\cdot I_p\times\Big[d_p\big(y_p(t)\big)\cdot I_p-d_p(x_p(t))\cdot I_p\Big]\\
&=e_p(t)\cdot e_p(\bar{t})+\left|I_p\right|^2\cdot L_d^2\cdot e_p(t)\cdot e_p(\bar{t}).
\end{aligned}
\tag{30}
$$

Submitting formulas (29) and (30) into formula (28),

$$
\begin{aligned}
D^\alpha V(t)\le\sum_{p=1}^{n}\Big\{&\left(w^2-2k_1\right)e_p(t)\cdot e_p(\bar{t})+e_p(t)\cdot e_p(\bar{t})+e_p(t)\\
&\cdot e_p(\bar{t})+2\cdot\left|A'\right|^2\cdot M_f^2\cdot L_d^2\cdot e_p(t)\cdot e_p(\bar{t})+2\cdot M_d^2\\
&\cdot|A|^2\cdot L_f^2\cdot e_p(t)\cdot e_p(\bar{t})+e_p(t)\cdot e_p(\bar{t})+2\cdot\left|B'\right|^2\\
&\cdot M_g^2\cdot L_d^2\cdot e_p(t)\cdot e_p(\bar{t})+2\cdot M_d^2\cdot|B|^2\cdot L_g^2\\
&\cdot e_p(t-\delta(t))\cdot e_p(t\bar{\delta}(t))+e_p(t)\cdot e_p(\bar{t})+2\cdot\left|C'\right|^2\\
&\cdot\rho^2\cdot M_f^2\cdot L_d^2\cdot e_p(t)\cdot e_p(\bar{t})+2\cdot\rho^2\cdot M_f^2\cdot M_d^2\\
&\cdot\left|C'-{}^{`}C\right|^2e_p(t)\cdot e_p(\bar{t})+\left|I_p\right|^2\bullet L_d^2\bullet e_p(t)\cdot e_p(\bar{t})-2k_2\Big\}\\
\le\sum_{p=1}^{n}\Big\{&\Big[\left(w^2-2k_1\right)+5+2\cdot\left|A'\right|^2\cdot M_f^2\cdot L_d^2+2\cdot|A|^2\\
&\cdot M_d^2\cdot L_f^2+2\cdot\left|B'\right|^2\cdot M_g^2\cdot L_d^2+2\cdot\left|C'\right|^2\cdot\rho^2\cdot M_f^2\\
&\cdot L_d^2+\left|I_p\right|^2\cdot L_d^2\Big]e_p(t)\cdot e_p(\bar{t})+2\cdot\rho^2\cdot M_f^2\cdot M_d^2\\
&\cdot\left|C'-{}^{`}C\right|^2-2k_2+2\cdot M_d^2\cdot|B|^2\cdot L_g^2\cdot e_p(t-\delta(t))\cdot e_p(t\bar{\delta}(t))\Big\}.
\end{aligned}
\tag{31}
$$

On the other hand, $|e_p(t-\delta(t))|\le\sup_{-\delta(t)\le s\le t}|e_p(t)|\le\sup_{-\delta(t)\le s\le 0}|e_p(t)|+\sup_{0\le s\le t}|e_p(t)|=|\psi(s)|+|e_p(t)|$.

Based on Lemma 8,

$$
\begin{aligned}
&e_p(t-\delta(t))\cdot e_p(t\bar{\delta}(t))\\
&\quad=\left|e_p(t-\delta(t))\right|^2\le\left(|\psi(s)|+\left|e_p(t)\right|\right)^2\\
&\quad=|\psi(s)|^2+\left|e_p(t)\right|^2+2\cdot|\psi(s)|\cdot\left|e_p(t)\right|\\
&\quad\le 2|\psi(s)|^2+2\left|e_p(t)\right|^2.
\end{aligned}
\tag{32}
$$

Submitting formula (32) into formula (31),

$$
\begin{aligned}
D^\alpha V(t)\le\sum_{p=1}^{n}\Big\{&e_p(t)\cdot e_p(\bar{t})\Big[\left(w^2-2k_1\right)+5+2\cdot\left|A'\right|^2\cdot M_f^2\\
&\cdot L_d^2+2\cdot|A|^2\cdot M_d^2\cdot L_f^2+2\cdot\left|B'\right|^2\cdot M_g^2\cdot L_d^2+2\\
&\cdot\left|C'\right|^2\cdot\rho^2\cdot M_f^2\cdot L_d^2+\left|I_p\right|^2\cdot L_d^2\Big]+2\cdot M_d^2\cdot|B|^2\cdot L_g^2\\
&\cdot\left(2|\psi(s)|^2+2\left|e_p(t)\right|^2\right)+2\cdot\rho^2\cdot M_f^2\cdot M_d^2\cdot\left|C'-{}^{`}C\right|^2-2k_2\Big\}\\
\le\sum_{p=1}^{n}\Big\{&\Big[\left(w^2-2k_1\right)+5+2\cdot\left|A'\right|^2\cdot M_f^2\cdot L_d^2+2\cdot|A|^2\\
&\cdot M_d^2\cdot L_f^2+2\cdot\left|B'\right|^2\cdot M_g^2\cdot L_d^2+2\cdot\left|C'\right|^2\cdot\rho^2\cdot M_f^2\\
&\cdot L_d^2+\left|I_p\right|^2\cdot L_d^2+4\cdot M_d^2\cdot|B|^2\cdot L_g^2\Big]e_p(t)\cdot e_p(\bar{t})+4\\
&\cdot M_d^2\cdot|B|^2\cdot L_g^2\cdot|\psi(s)|^2+2\cdot\rho^2\cdot M_f^2\cdot M_d^2\cdot\left|C'-{}^{`}C\right|^2-2k_2\Big\}.
\end{aligned}
\tag{33}
$$

Or we can simplify formula (33) as

$$D^{\alpha}V(t) \le \beta V(t) + \theta. \tag{34}$$

By using Lemma 6, we have

$$V(t) \le V(0)E_{\alpha}(-\beta t^{\alpha}). \tag{35}$$

Based on Definition 5, it indicates that the error system is Mittag-Leffler stable. Therefore, the drive system and response system can achieve synchronization under the feedback controller (24).

Remark 12. Given the condition without distributed time delay, the drive system (4) becomes:

$$\begin{aligned} D^{\alpha}x_p(t) = -d_p\left(x_p(t)\right)\Big[h_p\left(x_p(t)\right) - \grave{A}f_q\left(x_q(t)\right) \\ - \grave{B}g_q\left(x_q(t-\delta(t))\right) - I_p\Big]. \end{aligned} \tag{36}$$

At the same time, the response system (5) becomes

$$\begin{aligned} D^{\alpha}y_p(t) = -d_p\Big(y_p(t)\Big)\Big[h_p\Big(y_p(t)\Big) - A'f_q\Big(y_q(t)\Big) \\ - B'g_q\Big(y_q(t-\delta(t))\Big) - I_p\Big] + U_p(t), \end{aligned} \tag{37}$$

Based on Theorem 11, we can get Corollary 13.

Corollary 13. *Suppose that Assumptions 1 and 2 are satisfied, the response system (37) and drive system (36) can achieve finite-time synchronization under the feedback controller (24) if the following conditions hold:*

$$\beta < 0, \quad \theta < 0, \tag{38}$$

where

$$\begin{aligned} \beta &= \left(w^2 - 2k_1\right) + 5 + 2\cdot\left|A'\right|^2\cdot M_f^2\cdot L_d^2 + 2\cdot|A|^2\cdot M_d^2\cdot L_f^2 \\ &\quad + 2\cdot\left|B'\right|^2\cdot M_g^2\cdot L_d^2 + \left|I_p\right|^2\cdot L_d^2 + 4\cdot M_d^2\cdot|B|^2\cdot L_g^2, \\ \theta &= 4\cdot M_d^2\cdot|B|^2\cdot L_g^2\cdot|\psi(s)|^2 - 2k_2. \end{aligned} \tag{39}$$

Remark 14. Suppose the complex domain degenerates to the real domain, then the drive system (4) and response system (5) can also attain synchronization. Based on Theorem 11, we can get Corollary 15.

Corollary 15. *Suppose that Assumptions 1 and 2 are satisfied, the response system (5) and drive system (4) can achieve finite-time synchronization under the feedback controller (24) if the following conditions hold:*

$$\beta < 0, \theta < 0, \tag{40}$$

where

$$\begin{aligned} \beta &= 2(w - k_1) + 2\cdot\left|A'\right|\cdot M_f\cdot L_d + 2\cdot|A|^2\cdot M_d\cdot L_f \\ &\quad + 2\cdot\left|B'\right|\cdot M_g\cdot L_d + \left|I_p\right|\cdot L_d + 2\cdot M_d\cdot|B|\cdot L_g \\ &\quad + 2\cdot\left|C'\right|\cdot\rho\cdot M_f\cdot L_d, \\ \theta &= 2\cdot M_d\cdot|B|\cdot L_g\cdot|\psi(s)| + 2\cdot\rho\cdot M_f\cdot M_d \\ &\quad \cdot\left|C' - \grave{C}\right| - 2k_2. \end{aligned} \tag{41}$$

Remark 16. Compared with the separation method, which needs to divide the complex-valued system into two real parts used in references [30, 56, 57, 61], this nonseparation method only needs to consider the whole system. It is no doubt that this method reduces the computational effort. Moreover, considering the fact that the complex function cannot always be divided into two parts in practical application, the method used in this paper is more realistic.

Remark 17. Compared with [30, 48–50], which only use Lemma 7 in the process of proof to get their results, we can see this paper uses a different way to get our results in (29) and (30). It is easy to note that the method used in the above literature can only solve the problem that the conjugate terms only consist of multiplication because the connection weights in these studies are the same. Hence, based on Lemma 8, Corollary 9 is derived, which reduces the limitation of use in this nonseparation method. And the approach applied in this article can provide a new method to study the synchronization of other complex-valued neural networks, especially the mismatched parameters of complex-valued neural networks.

3.2. Adaptive Controller. By using the above lemmas and definitions, we design a suitable adaptive controller as follows:

$$u_p(t) = -k(t)e_p(t) - \frac{k_2\cdot e_p(t)}{\left|e_p(t)\right|^2}, \tag{42}$$

where $D^{\alpha}k(t) = k_1 e_p(t)\cdot e_p\bar{(t)}$ and k_1, k_2 are positive constants.

$$k_2 > 2\cdot M_d^2\cdot|B|^2\cdot L_g^2\cdot|\psi(s)|^2 + \rho^2\cdot M_f^2\cdot M_d^2\cdot\left|C' - \grave{C}\right|^2, \tag{43}$$

and we can derive the setting time which is

$$T = \left(\frac{V(0)\Gamma(\alpha+1)}{n\cdot v}\right)^{\frac{1}{\alpha}}, \tag{44}$$

where

$$-\nu = 4 \cdot M_d^2 \cdot |B|^2 \cdot L_g^2 \cdot |\psi(s)|^2 + 2 \cdot \rho^2 \cdot M_f^2 \cdot M_d^2 \cdot \left|C' - {}^{`}C\right|^2 - 2k_2. \tag{45}$$

Theorem 18. *Suppose that Assumption 1 and 2 are satisfied, the response system (5) and drive system (4) can achieve finite-time synchronization under the adaptive controller (42) if the following condition holds:*

Proof. The Lyapunov function is chosen as

$$V(t) = \sum_{p=1}^{n} e_p(t) \cdot e_{\bar{p}}(t) + \sum_{p=1}^{n} \frac{1}{k_1} \cdot (k - k^*)^2. \tag{46}$$

□

The fractional order derivative of $V(t)$ is

$$D^\alpha V(t) \le \sum_{p=1}^{n} \left(e_p(t) D^\alpha e_{\bar{p}}(t) + e_{\bar{p}}(t) D^\alpha e_p(t)\right) + \sum_{p=1}^{n} \frac{2}{k_1} \cdot (k - k^*) \cdot D^\alpha k(t). \tag{47}$$

Based on formula (33), we can get

$$\begin{aligned} D^\alpha V(t) \le \sum_{p=1}^{n} \Big\{ &\Big[\left(w^2 - 2k(t)\right) + 5 + 2 \cdot \left|A'\right|^2 \cdot M_f^2 \cdot L_d^2 + 2 \cdot |A|^2 \cdot M_d^2 \cdot L_f^2 + 2 \cdot \left|B'\right|^2 \cdot M_g^2 \cdot L_d^2 + 2 \cdot \left|C'\right|^2 \cdot \rho^2 \\ &\cdot M_f^2 \cdot L_d^2 + \left|I_p\right|^2 \cdot L_d^2 + 4 \cdot M_d^2 \cdot |B|^2 \cdot L_g^2\Big] e_p(t) \cdot e_{\bar{p}}(t) + 4 \cdot M_d^2 \cdot |B|^2 \cdot L_g^2 \cdot |\psi(s)|^2 + 2 \cdot \rho^2 \cdot M_f^2 \cdot M_d^2 \\ &\cdot \left|C' - {}^{`}C\right|^2 - 2k_2 + (2k(t) - 2k^*) e_p(t) \cdot e_{\bar{p}}(t) \Big\} \\ = \sum_{p=1}^{n} \Big\{ &\Big[\left(w^2 - 2k^*\right) + 5 + 2 \cdot \left|A'\right|^2 \cdot M_f^2 \cdot L_d^2 + 2 \cdot |A|^2 \cdot M_d^2 \cdot L_f^2 + 2 \cdot \left|B'\right|^2 \cdot M_g^2 \cdot L_d^2 + 2 \cdot \left|C'\right|^2 \\ &\cdot \rho^2 \cdot M_f^2 \cdot L_d^2 + \left|I_p\right|^2 \cdot L_d^2 + 4 \cdot M_d^2 \cdot |B|^2 \cdot L_g^2\Big] e_p(t) \cdot e_{\bar{p}}(t) + 4 \cdot M_d^2 \cdot |B|^2 \cdot L_g^2 \cdot |\psi(s)|^2 + 2 \cdot \rho^2 \cdot M_f^2 \\ &\cdot M_d^2 \cdot \left|C' - {}^{`}C\right|^2 - 2k_2 \Big\}, \end{aligned} \tag{48}$$

where $k^* = 1/2w^2 + 5/2 + |A'|^2 \cdot M_f^2 \cdot L_d^2 + |A|^2 \cdot M_d^2 \cdot L_f^2 + |B'|^2 \times M_g^2 \cdot L_d^2 + |C'|^2 \cdot \rho^2 \cdot M_f^2 \cdot L_d^2 + 1/2 \cdot |I_p|^2 \cdot L_d^2 + 2 \cdot M_d^2 \cdot |B|^2 \cdot L_g^2$.

Then, formula (48) can be simplified as $D^\alpha V(t) \le \sum_{p=1}^{n} \{4 \cdot M_d^2 \cdot |B|^2 \cdot L_g^2 \cdot |\psi(s)|^2 + 2 \cdot \rho^2 \cdot M_f^2 \cdot M_d^2 \cdot |C' - {}^{`}C|^2 - 2k_2\} \le -n \cdot \nu$.

There exists a function $F(t) \ge 0$ satisfying

$$D^\alpha V(t) + F(t) = -n \cdot \nu. \tag{49}$$

According to Definition 3, fractional integrals are taken on both sides of the equation:

$$I_{0,t}^\alpha (D^\alpha V(t)) + I_{0,t}^\alpha F(t) = I_{0,t}^\alpha (-n \cdot \nu), \tag{50}$$

where

$$\begin{aligned} I_{0,t}^\alpha F(t) &= \frac{1}{\Gamma(\alpha)} \int_0^t (t-s)^{\alpha-1} F(s) ds \ge 0, I_{0,t}^\alpha (-n \cdot \nu) \\ &= \frac{1}{\Gamma(\alpha)} \int_0^t (t-s)^{\alpha-1} (-n \cdot \nu) ds = \frac{(-n \cdot \nu) t^\alpha}{\Gamma(\alpha+1)}. \end{aligned} \tag{51}$$

Then, formula (50) degenerates to

$$\begin{aligned} &V(t) - V(0) + I_{0,t}^\alpha F(t) = \frac{(-n \cdot \nu) t^\alpha}{\Gamma(\alpha+1)}, \\ &-V(0) \le \frac{(-n \cdot \nu) t^\alpha}{\Gamma(\alpha+1)}, \\ &t \le \left(\frac{V(0)\Gamma(\alpha+1)}{n \cdot \nu}\right)^{1/\alpha}. \end{aligned} \tag{52}$$

Naturally, we conclude that the system (4) and (5) can attain synchronization under the adaptive controller (42) in a finite time $T = (V(0)\Gamma(\alpha+1)/n \cdot \nu)^{1/\alpha}$.

Remark 19. In [47], the finite-time synchronization of CGNNs in the real number field was investigated. In addition, there are few studies involving CGNNs synchronization. In [30, 55–57], the finite-time synchronization of complex-valued neural networks was discussed via the separation method. However, the finite-time synchronization of fractional-order CGNNs with mixed time delays is rarely involved; moreover, this paper discusses synchronization by employing the non-separation method.

Remark 20. In particular, HNNs can be regarded as a special form of CGNNs. When $d_p = 1, h_p = d_p \cdot x_p(t)$, CGNNs degenerate to HNNs. Therefore, it can be considered that the network synchronization studied in this article is more widely.

Remark 21. Formula (52) demonstrates that the settling time is related to fractional order α and controller value k_2; thus, we can minimize the settling time by regulating these values.

4. Illustrative Example

In this section, four examples are given to illustrate the effectiveness of the designed controllers.

$$\begin{cases} D^{\alpha}x_p(t) = -d_p(x_p(t)) \cdot \left[h_p(x_p(t)) - \sum_{q=1}^{2} a_{pq}(x_q(t))f_q(x_q(t)) - \sum_{q=1}^{2} b_{pq}(x_q(t-\delta(t)))g_q(x_q(t-\delta(t))) - \sum_{q=1}^{2} c_{pq}(x_q(t)) \int_{t-\tau(t)}^{t} f_q(x_q(s))ds - I_p\right], \\ D^{\alpha}y_p(t) = -d_p\left(y_p(t)\right) \cdot \left[h_p\left(y_p(t)\right) - \sum_{q=1}^{2} a_{pq}\left(y_q(t)\right)f_q\left(y_q(t)\right) - \sum_{q=1}^{2} b_{pq}\left(y_q(t-\delta(t))\right)g_q\left(y_q(t-\delta(t))\right) - \sum_{q=1}^{2} c_{pq}\left(y_q(t)\right) \int_{t-\tau(t)}^{t} f_q\left(y_q(s)\right)ds - I_p\right] + u_p(t), p = 1, 2, \end{cases} \tag{53}$$

Example 1. Consider the 2-dimensional fractional order complex-valued CGNNs (53) as shown in following:

where $\alpha = 0.9$, $h(\cdot) = 1 + i$, $d(\cdot) = f(\cdot) = g(\cdot) = \tan h(\cdot) + i \tan h(\cdot)$, $\tau(\cdot) = 0.5 + 0.5 \sin(t)$, $\delta = 0.1$, $q = 1, 2$.

From Assumptions 1 and 2, we have $w = 1$, $M_d = M_f = M_g = L_d = L_f = L_g = 1$.

The connection weights can be modeled as

$$\begin{aligned} a_{11} &= \begin{cases} -0.3 + 0.3i, & |x_1| \le 1. \\ -0.4 + 0.4i, & |x_1| > 1. \end{cases} & a_{12} &= \begin{cases} -0.3 - 0.3i, & |x_1| \le 1. \\ -0.2 - 0.2i, & |x_1| > 1. \end{cases} \\ a_{21} &= \begin{cases} -0.3 - 0.3i, & |x_2| \le 1. \\ -0.4 - 0.4i, & |x_2| > 1. \end{cases} & a_{22} &= \begin{cases} 0.4 - 0.4i, & |x_2| \le 1. \\ 0.3 - 0.3i, & |x_2| > 1. \end{cases} \\ b_{11} &= \begin{cases} 0.2 - 0.1i, & |x_1| \le 1. \\ 0.3 + 0.2i, & |x_1| > 1. \end{cases} & b_{12} &= \begin{cases} -0.2 + 0.2i, & |x_1| \le 1. \\ 0.3 - 0.1i, & |x_1| > 1. \end{cases} \\ b_{21} &= \begin{cases} -0.3 + 0.2i, & |x_2| \le 1. \\ 0.2 - 0.1i, & |x_2| > 1. \end{cases} & b_{22} &= \begin{cases} 0.1 - 0.1i, & |x_2| \le 1. \\ -0.2 + 0.2i, & |x_2| > 1. \end{cases} \\ c_{11} &= \begin{cases} -0.05 - 0.05i, & |x_1| \le 1. \\ -0.01 + 0.01i, & |x_1| > 1. \end{cases} & c_{12} &= \begin{cases} -0.05 - 0.05i, & |x_1| \le 1. \\ -0.01 + 0.01i, & |x_1| > 1. \end{cases} \\ c_{21} &= \begin{cases} 0.01 - 0.01i, & |x_2| \le 1. \\ -0.05 - 0.05i, & |x_2| > 1. \end{cases} & c_{22} &= \begin{cases} -0.05 - 0.05i, & |x_2| \le 1. \\ 0.01 - 0.01i, & |x_2| > 1. \end{cases} \end{aligned} \tag{54}$$

The initial condition of system (53) is $x_1(0) = 1.2 - 1.8i$, $x_2(0) = 1.5 - 1.5i$, $y_1(0) = 0.8 + 0.5i$, $y_2(0) = -0.5 - 0.8i$, when we consider the drive-response systems without controller, it can be seen from Figure 1 that the response system cannot be synchronized with the drive system; so, the curves of the error and error modulus cannot reach 0.

Under the state feedback controller (24), referring to formula (26), through simple calculation, we can get

$$\begin{aligned} 4.1 = k_1 &> \frac{1}{2}w^2 + \frac{5}{2} + \left|A'\right|^2 \cdot M_f^2 \cdot L_d^2 + |A|^2 \cdot M_d^2 \cdot L_f^2 \\ &\quad + \left|B'\right|^2 \cdot M_g^2 \cdot L_d^2 + \left|C'\right|^2 \cdot \rho^2 \cdot M_f^2 \cdot L_d^2 + \frac{1}{2}I_p^2 \\ &\quad \cdot L_d^2 + 2 \cdot M_d^2 \cdot |B|^2 \cdot L_g^2 = \frac{1}{2} \cdot 1^2 + \frac{5}{2} + 2 \cdot 0.4^2 \\ &\quad \cdot 1^2 \cdot 1^2 + 2 \cdot 0.4^2 \cdot 1^2 \cdot 1^2 + \left(0.3^2 + 0.2^2\right) \\ &\quad \cdot 1^2 \cdot 1^2 + 2 \cdot 0.05^2 \cdot 1^2 \cdot 1^2 \cdot 1^2 + \frac{1}{2} \cdot 0.1^2 \\ &\quad \cdot 1^2 + 2 \cdot 1^2 \cdot \left(0.3^2 + 0.2^2\right) \cdot 1^2 = 4.04, \\ 2.6 = k_2 &> 2 \cdot M_d^2 \cdot |B|^2 \cdot L_g^2 \cdot |\psi(s)|^2 + \rho^2 \cdot M_f^2 \cdot M_d^2 \\ &\quad \cdot \left|C' - {}^{\textrm{`}}C\right|^2 = 2 \cdot 1^2 \cdot \left(0.3^2 + 0.2^2\right) \cdot 1^2 \\ &\quad \cdot \left(0.4^2 + 2^2 + 2.3^2 + 0.7^2\right) + 1^2 \cdot 1^2 \cdot 1^2 \\ &\quad \cdot \left(0.06^2 + 0.04^2\right) = 2.59. \end{aligned} \tag{55}$$

Substituting the calculated k_1, k_2 into controller (24), we can obtain a state feedback controller that satisfies the condition of Theorem 11. Therefore, the system (53) can attain synchronization. Figures 2(a)–2(d) describe the state trajectory diagrams in the real and imaginary parts of the drive response system, respectively, and the error signal is given in Figure 2(e). Figure 2(f) shows the modulus of error under the controller (24).

Example 2. Consider system (53) under the adaptive controller (42) and according to (43), we can get the values of controller that $k_2 = 2.6$.

Furthermore, the maximum setting time can be calculated from formula (44) as follows:

$$T = \left(\frac{-V(0)\Gamma(\alpha+1)}{n\cdot v}\right)^{\frac{1}{\alpha}} = \left(\frac{-V(0)\Gamma(1.9)}{-0.28}\right)^{\frac{1}{0.9}} = 5.68. \quad (56)$$

Substituting the calculated k_2 into formula (42), the adaptive controller that satisfies the condition of Theorem 18 is obtained. Therefore, the system (53) can attain synchronization. Figures 3(a)–3(d) describe the state trajectory diagrams in the real and imaginary parts of the drive response system, respectively, and the error signal is given in Figure 3(e). Figure 3(f) shows the modulus of error under the controller (42).

Remark 22. Compared with other references such as [10, 30, 47, 56] the models studied in this article are more general. In other words, they can be regarded as special cases of this paper, so Theorems 11 and 18 are also applicable.

Example 3. Consider system (53) with $d(\cdot) = f(\cdot) = g(\cdot) = 1 - e^{-z}/1 - e^{-z}, h(\cdot) = 1/1 - e^{-z}$, the initial condition of system (53) is $x_1(0) = 1.2 - 0.5i, x_2(0) = -1.2 + 0.4i, y_1(0) = -1 + 0.2i, y_2(0) = 1 - 0.1i$, and others are same as example 1. Here, activation functions cannot be expressed explicitly by separating real and imaginary parts, which can be regarded as an entire. By simple computing, we can get that $M_d = M_f = M_g = \sqrt{2}, L_d = L_f = 2/3, w = 1$; similarly to example 1, we can get $k_1 = 4 > 3.93, k_2 = 1.9 > 1.893$.

When we consider the drive-response systems without controller, it can be seen from Figure 4 that the response system cannot be synchronized with the drive system; so, the curves of the error and error modulus cannot reach 0.

Substituting the calculated k_1, k_2 into formula (24), we can obtain a state feedback controller that satisfies the condition of Theorem 11. Therefore, the system (29) can attain synchronization. Figures 5(a)–5(d) describe the state trajectory diagrams in the real and imaginary parts of the drive response system, respectively, and the error signal is given in Figure 5(e). Figure 5(f) shows the modulus of error under the controller (24).

Example 4. Consider system (53) under the adaptive controller (42) and according to formula (43) and the calculation above, we can get the values of controller that $k_2 = 1.9$.

Substituting the calculated k_2 into controller (42), we can obtain an adaptive controller that satisfies the condition of Theorem 18. Therefore, the system (53) can attain synchronization. Figures 6(a)–6(d) describe the state trajectory diagrams in the real and imaginary parts of the drive response system, respectively, and the error signal is given in Figure 6(e). Figure 6(f) shows the modulus of error under the controller (42).

Remark 23. In this section, we use four examples to illustrate the correctness of our conclusions. Examples 1 and 2 use feedback and adaptive controllers, respectively. Furthermore, Examples 3 and 4 change the activation function into an inseparable form to realize the finite-time synchronization of the network by using the controller. Compared with the separation method used in [30, 56, 57, 61], the nonseparation method used in this paper is universal to solve the complex-valued activation functions, which are difficult to divide.

5. Conclusions and Prospects

In this paper, the finite-time synchronization of a special kind of Cohen-Grossberg neural network, which consists of fractional-order, couple-valued, mixed time delays, and state-dependent switching, is investigated. Different from other papers, we did not separate the complex system into real parts and imaginary parts but adopted a nonseparation method. By applying set-valued map, differential inclusion theory, fractional calculus theory, suitable state-feedback controller, and adaptive controller are designed to achieve the synchronization in a finite time. Finally, numerical simulations are also given to illustrate the effectiveness of our strategy.

Further research mainly includes two aspects. Due to the complexity and high cost of the constant control method in large-scale networks, a method that consumes less energy will be developed. Impulsive control or event control is used to achieve the finite-time synchronization of our model. Moreover, the nonseparation method used in this article can be applied to achieve the synchronization of other complex-valued CGNNs with specific qualities, such as fuzzy CGNNs and CGNNs with impulses.

Data Availability

The datasets generated and/or analyzed during the current study are available from the corresponding author upon reasonable request.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

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